

## **Probabilistic Methods for Risk Assessment of Airframe Digital Twin Structures**

**Harry Millwater**

University of Texas at San Antonio & AeroMatter, LLC  
San Antonio, TX  
USA

[harry.millwater@utsa.edu](mailto:harry.millwater@utsa.edu)

**Juan Ocampo**

St. Mary's University & AeroMatter, LLC  
San Antonio, TX.  
USA

[jocampo@stmarytx.edu](mailto:jocampo@stmarytx.edu)

**Nathan Crosby**

AeroMatter, LLC  
San Antonio, TX.  
USA

[nathan.crosby@aeromatter.org](mailto:nathan.crosby@aeromatter.org)

**Sohrob Mottaghi**

Federal Aviation Administration  
Atlantic City, NJ.  
USA

[Sohrob.Mottaghi@faa.gov](mailto:Sohrob.Mottaghi@faa.gov)

**Michael Reyer**

Federal Aviation Administration  
Kansas City, MO.  
USA

[michael.reyer@faa.gov](mailto:michael.reyer@faa.gov)

### ***ABSTRACT***

*The development and deployment of an effective airframe Digital Twin (ADT) promises improved structural diagnoses and prognoses to enhance safety and economic considerations. A core component of an effective DT is the accurate and efficient prediction of the single flight probability-of-failure (SFPOF) that encompasses state-of-the-art fracture mechanics, usage, random variable modeling, and probabilistic methods. The FAA-funded Smart|DT software and its ecosystem provides the computation of the probability-of-failure, the remaining useful life, and the effects of inspections and repairs combined with a usage module, a probabilistic*

## Probabilistic Methods for Risk Assessment of Airframe Digital Twin Structures

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*database that encompasses material variability and NDE data, example problems, and training material. The components of the Smart|DT ecosystem are described below along with near-term objectives and future challenges. Numerical examples are presented to demonstrate the computations. Although originally developed for general aviation, the technology with Smart|DT is generic and can be applied equally to commercial and defense airframes.*

### 1.0 INTRODUCTION

A key concept of the digital twin philosophy for aircraft is that the geometry, material properties, loading, inspection methods, in-service induced damage, etc., are tail number specific [Tuegel, et al, 2011; Glaessgen and Stargel, 2021]. Therefore, a realization of digital twin requires state-of-the-art modeling of the as-built structure, real-time usage data, accurate evaluation of the reliability of the structure, and model updating to induced damage and maintenance actions. This article discusses the **required probability computations** for assessing the health of the airframe DT and providing appropriate information for forecasting and decision making. The ADT is one component of the larger aircraft digital twin.

The ADT is based on the well-established damage tolerance analysis methodology with a quantification and assessment of the effects of uncertainty in loading, materials, geometric properties, among others. While damage tolerance analysis is well established, probabilistic damage tolerance is still challenging due to the requirement of obtaining sufficiently good probabilistic distribution information of inputs, the computational requirements for computing the single flight probability-of-failure, and the updating of the probabilistic results due to in-service information. While current approaches have provided an excellent success story and continue to ensure the safety and reliability of airframe, advances are required in order to fully realize the benefits of the ADT.

The ADT framework is shown in Figure 1-1 **[NGC final report]**. The framework consists of usage awareness and load forecasting, structural analysis and damage progression, damage state awareness, and probabilistic computations of the single flight probability-of-failure, the remaining useful life, and the crack size distributions.

An essential component of the ADT is the quantification and propagation of uncertainties in all aspects of the methodology. In this regard, the Federal Aviation Administration has funded a probabilistic damage tolerance analysis software program, Smart|DT, that provides an essential computational components of the airframe digital twin. The components of the Smart|DT ecosystem and its capabilities are summarized below.

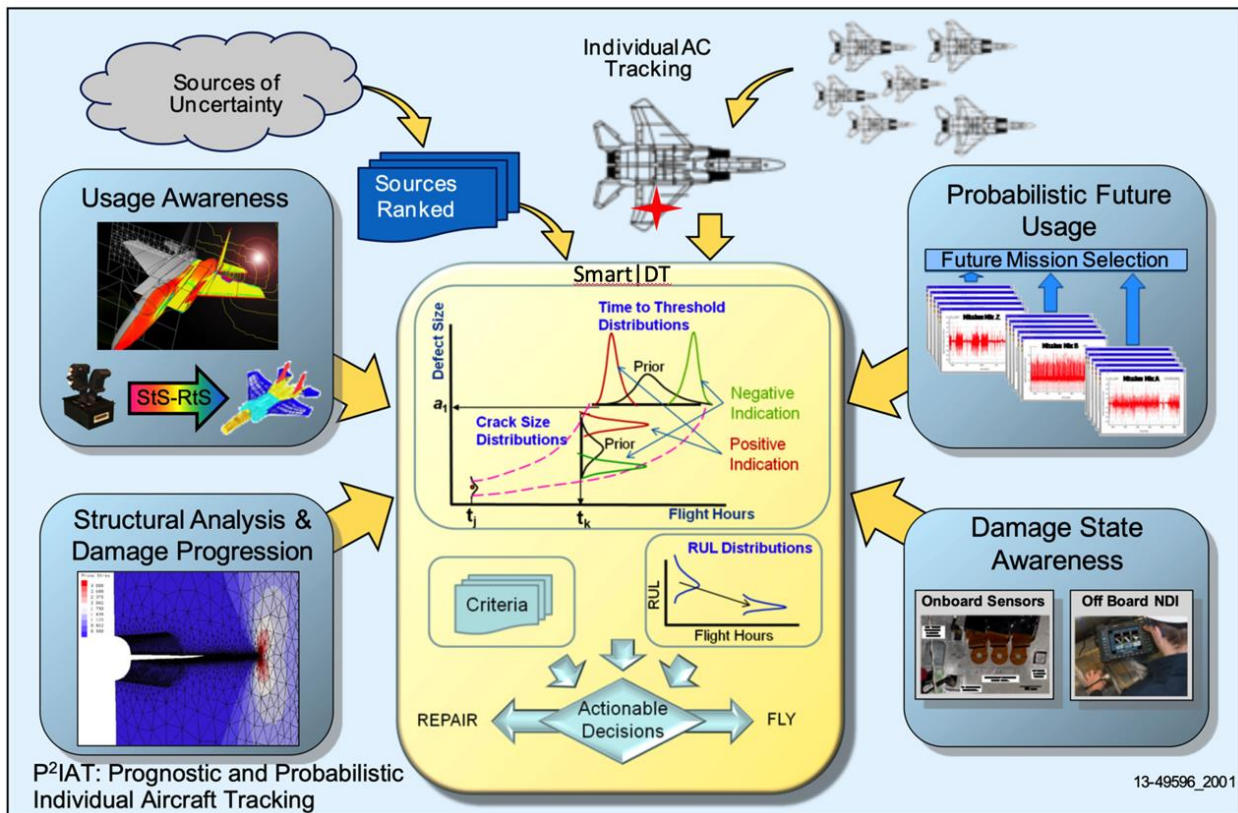


Figure 1-1: Schematic of the ADT Framework [Adapted from Ref. NGC Final Report].

## 2.0 SMART|DT ECOSYSTEM COMPONENTS

Figure 2-1 shows a schematic of the components of the Smart|DT software. Further information can be found in reference [Millwater 2019]. Pre-existing data, e.g., usages, random variables (material properties, initial defect sizes, geometry parameters), and POD curves, are provided with the software and selectable from pull-down menus. A variety of probability distributions are available such as normal, lognormal, Weibull, extreme value, and tabular. User-defined values can be added through manual input or user-defined JSON files.

The methodology integrates the following components:

### 2.1 Loading Data:

Exceedance curves (9 included), load limit factors, and flight duration and velocity matrices. The extreme value distribution for the max. load per flight is computed from the generated usage. Algorithms to predict future usage and missing flight information are not included at this time but can be integrated within the current framework. [Anagnostou and Engel, 2017; Wang, et al., 2017].

### 2.2 Material Data:

Material properties (crack growth rate, fracture toughness, and yield & ultimate strength). Probabilistic material

## **Probabilistic Methods for Risk Assessment of Airframe Digital Twin Structures**

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data are provided for 9 aluminium alloys (40 heat treatments & 68 crack orientations), 11 steel alloys (20 heat treatments & 27 crack orientations), and 2 titanium alloys (4 heat treatments & 6 crack orientations). A variety of sources of publicly data were used to construct a probabilistic distribution [Skinn et al., 1994; Mettu et al., 1999; Miedlar, 2002; Rice 2003; Rice et al., 2003; White et al., 2002; Forman et al., 2005].

### **2.3 Initial Defect Size:**

Data was gathered from a variety of sources [Wieland & Millwater, 2002; Millwater and Wieland, 2010] to develop initial defect sizes probability distributions for commercial (pressurized fuselage & wing), military fighter (A-7D, F-4, T-28), and military transport (Wing).

### **2.4 Geometry Data:**

Hole size and edge distance probability distributions are provided from references [Wieland & Millwater, 2002; Bhachu et. al, 2013].

### **2.5 Inspection and Repair:**

Smart|DT incorporates a specially developed “weighted branch integration” method to facilitate inspection and repair simulations [Ocampo and Millwater, 2015]. This capability allows Smart|DT to consider altered fracture mechanics after repairs. To simulation the inspection process, Smart|DT contains over 100 Probability of Detection (POD) curves (visual, eddy current, die penetrant, magneto optical imaging, ultrasonic, and X-ray methods) [Hurst and Gamble, 2018].

### **2.7 Fracture Mechanics Capabilities:**

Smart|DT interfaces with external crack growth codes through a user-defined crack growth file. That is, users can define the crack growth behavior using proprietary or commercial software such as Nasgro [<https://www.swri.org/consortia/nasgro>] and Afgrow [<https://www.afgrow.net>]; however, using a predefined crack growth curve precludes the possibility of considering random variables that affect the crack growth behavior such as dadN variability and geometric effects. As an alternative, Smart|DT contains an ultrafast crack growth code, HyperGrow, that uses an equivalent spectrum approximation along with an adaptive step size crack growth solver. HyperGrow can compute over 10 thousand crack growth simulations per second on a desktop computer. [Millwater, et al., 2018, Ocampo, et al., 2019] Efforts are ongoing to provide a runtime link between Smart|DT and the Nasgro software. Near-term efforts include the capability to incorporate machine learning-based surrogate models for progressive fracture mechanics.

### **2.8 Fleet Management:**

Smart|DT contains a fleet management capability such that a population of aircraft with associated flight time information can be provided. Smart|DT will integrate the fleet demographics together with the SFPOF to predict the probability of a failure event within a user-specified flight interval.

### **2.9 Probability Computations:**

Smart|DT contains the ability to compute the single flight probability-of-failure, the remaining useful life, and the crack size distributions. The specially-developed Adaptive Multiple Importance Sampling (AMIS) algorithm

is a fast and effective method for computing the SFPOF.

### **2.10 Optimized Inspections:**

Smart|DT contains the ability to determine the most effective inspection schedule given: i) a risk threshold and a single inspection type, or ii) risk threshold and a multiple inspection types. For case i), the times of inspection will be returned that will ensure that the SFPOF is below a user-defined threshold. For case ii), the times of inspection and the inspection types will be returned. The AMIS algorithm, discussed below, is especially efficient at considering multiple inspection scenarios.

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### **2.10 Software Integration:**

Although it has its own graphical user interface, Smart|DT has been developed such that it is scriptable and modular, that is, it can be easily integrated within a comprehensive aircraft digital twin. In particular, input data can be passed using standard text and JSON files and results output in a variety of formats. State-of-the-art computational algorithms have been implemented in a parallel format using the industry standard OPENMP format. Smart|DT runs on Windows, MacOS, and Linux systems.



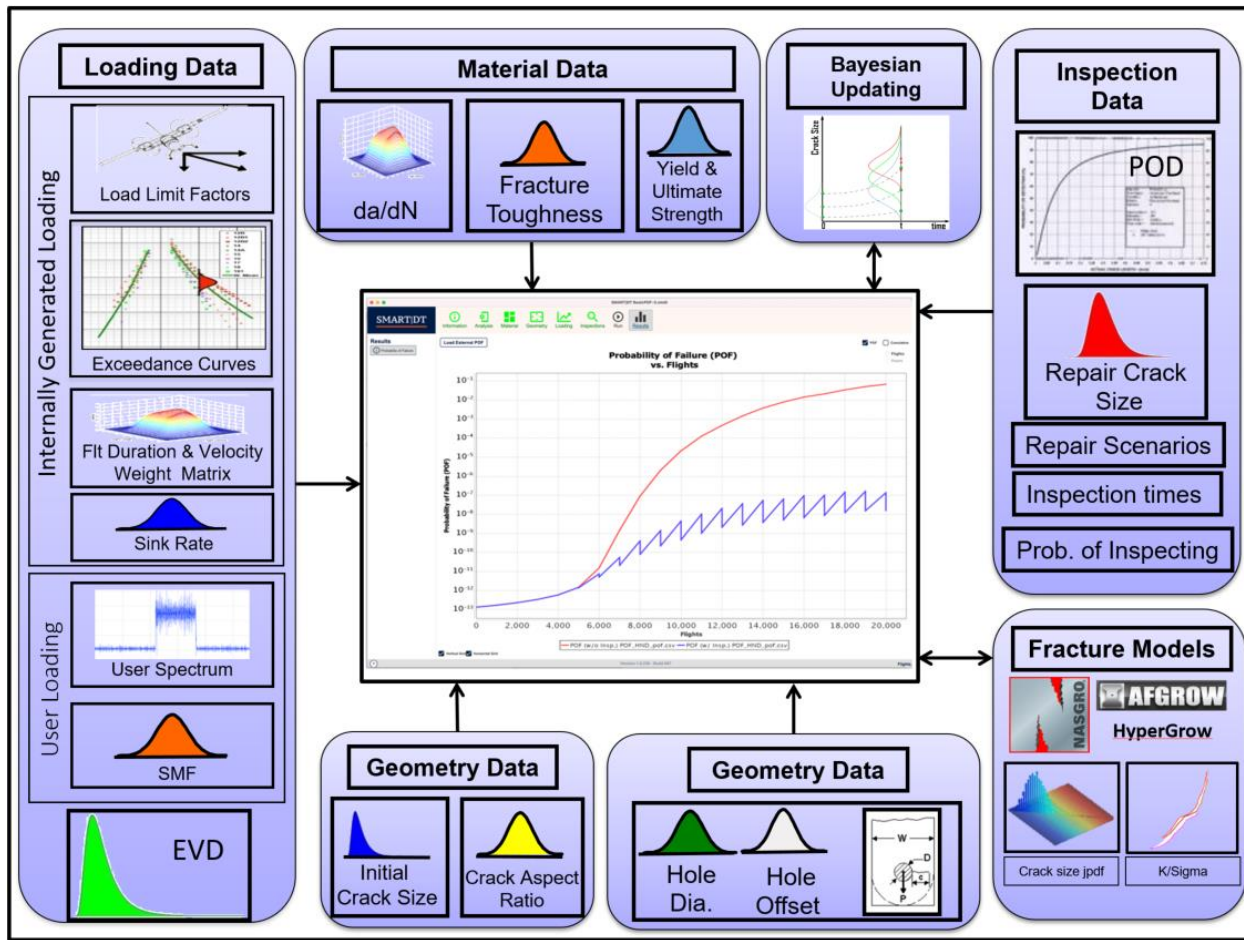


Figure 2-1: Schematic of Smart|DT Components.

### 3.0 PROBABILISTIC COMPUTATIONS

There are several important probabilistic computations that are required for an ADT. These are the single flight probability-of-failure (SFPOF) with and without inspections and repairs, the remaining useful life (RUL), crack size distributions (CSD), and the percent of cracks detected.

#### 3.1 Single Flight Probability-of-Failure (SFPOF):

Figure 3-1 shows an example of an SFPOF calculation as a function of flight hours. Failure occurs when the max stress per flight exceeds the residual strength ( $\sigma_{EVD} \geq \sigma_{RS}$ ) with residual strength governed by unstable fracture ( $K_I \geq K_C$ ) or net section yield. Figure 3-2 shows a schematic of process – the residual strength decreases with flight hours due to crack growth and the orange region shows the probability of the max stress exceeding the residual strength. However, the residual strength is a random variable since it depends on other random variables such as the initial crack size, fracture toughness, dadN variability, geometric parameters, etc. As a result, a probability integral must be solved, as described below.



There are two formulations in use “Lincoln” and “Freudenthal” – named for their authors. Lincoln assumes that the component survives until time T, whereas Freudenthal accounts for the probability of failure of the component at all times before time T. As a result, Lincoln is more conservative and predicts higher SFPOF values. In the equations below, t represents flight hours,  $F_{EVD}$  is the cumulative distribution function for the max stress per flight ( $\sigma_{EVD}$ ), x represents the set of random variables, e.g., initial crack size, fracture toughness, dadN variability, geometric parameters, etc.

$$POF_{Lincoln}(t) = P[\sigma_{Max} > \sigma_{RS}(t)] = \int [1 - F_{EVD}(\sigma_{RS}(t, x))] f_X(x) dx$$

$$POF_{Freudenthal}(t_n) = \frac{\int [\prod_{i=1}^{n-1} F_{EVD}(\sigma_{RS}(x, t_i))] (1 - F_{EVD}(\sigma_{RS}(x, t_n))) f_X(x) dx}{\int \prod_{i=1}^n F_{EVD}(\sigma_{RS}(x, t_i)) f_X(x) dx}$$

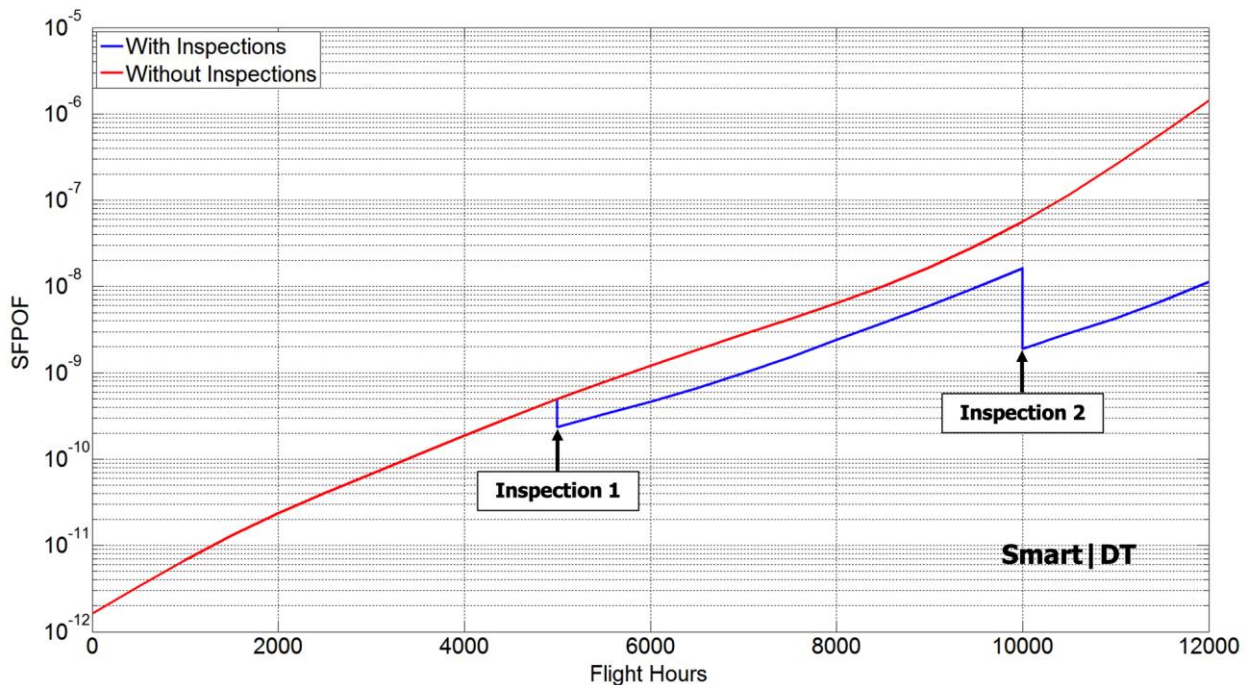


Figure 3-1: Example of Single Flight Probability-of-Failure Calculations.

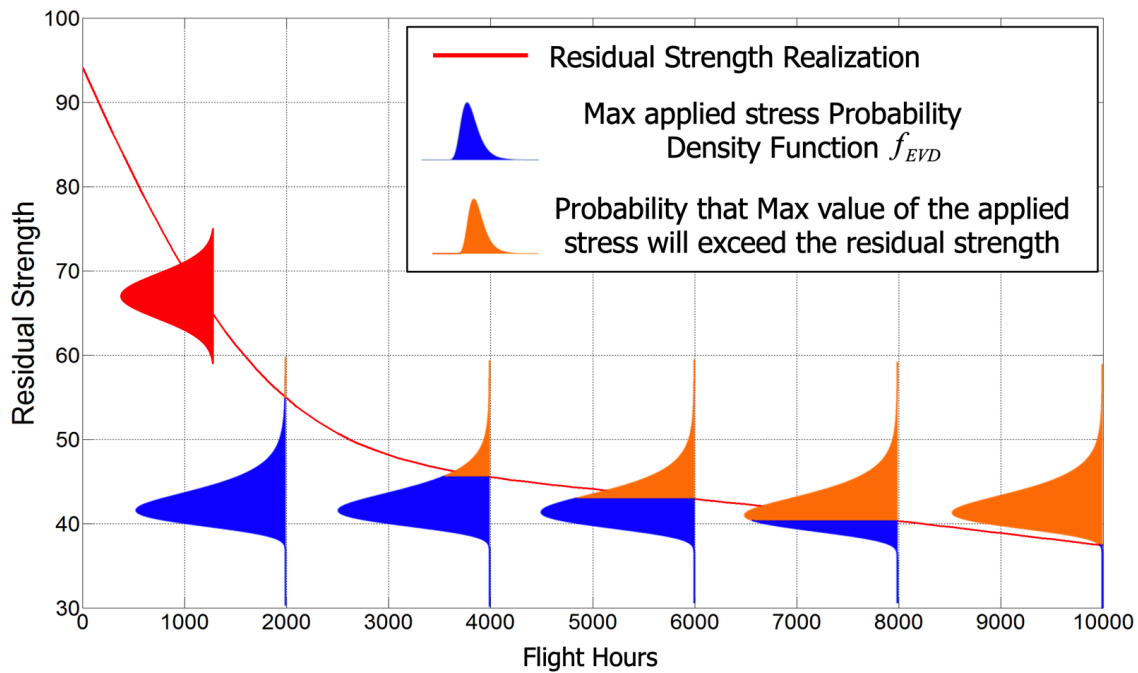


Figure 3-2: Schematic of Probability Calculation – Max Stress Per Flight Exceeding the Residual Strength ( $\sigma_{EVD} \geq \sigma_{RS}$ ).

### 3.2 Adaptive Multiple Importance Sampling Method (AMIS):

The calculation of the SFPOF is challenging due to the small probability values, typically  $\sim 10^{-7}$  or smaller, and the incorporation of inspection and repair options. As a result, a careful selection of the numerical method used to compute the SFPOF is required. Numerical integration methods are adequate but limited to only a few (2 or 3) random variables. Standard Monte Carlo sampling requires an inordinate number of samples,  $\sim 10^9$ , to compute accurate results. First and second order reliability methods (FORM/SORM) need special considerations to handle the multimodal crack size distributions present after inspections and repairs. As a result, Smart|DT uses a newly-developed Adaptive Multiple Importance Sampling method (AMIS) developed specifically to compute the SFPOF with accuracy and efficiency [Crosby, 2021].

Experience has shown that AMIS can compute the SFPOF with  $\sim 6$  orders of magnitude of fewer samples than standard Monte Carlo sampling. In addition, AMIS stores and reuses crack growth analyses. This allows it to efficiently handle inspections and repairs because it can reuse  $n_{\text{reuse}}$  of the fracture results. This reuse capability facilitates an optimized inspection capability described below. A summary of the important components of AMIS is given below.

Determines important sampling region for efficient sampling
Constructs a mixture density across all analysis times
User-defined convergence
Efficiently reuses crack growth analyses for inspection and repair scenarios
Facilitates inspection schedule optimization

Table 3-1: Summary of the Important Components of AMIS.



### 3.3 Remaining Useful Life (RUL):

The remaining useful life is a probabilistic estimation of the flight hours until a critical crack size is reached. Computing the RUL is a straightforward computation that can be accomplished easily even when using standard Monte Carlo sampling as sufficient results can be obtained with only ~100 samples. The result is an expected value result of the RUL along with quantile values. Figure 3-3 shows an example of the RUL computed at seven different crack sizes. A histogram of the remaining useful life and its statistics: mean ( $\mu$ ), and standard deviation ( $\sigma$ ) are provided.

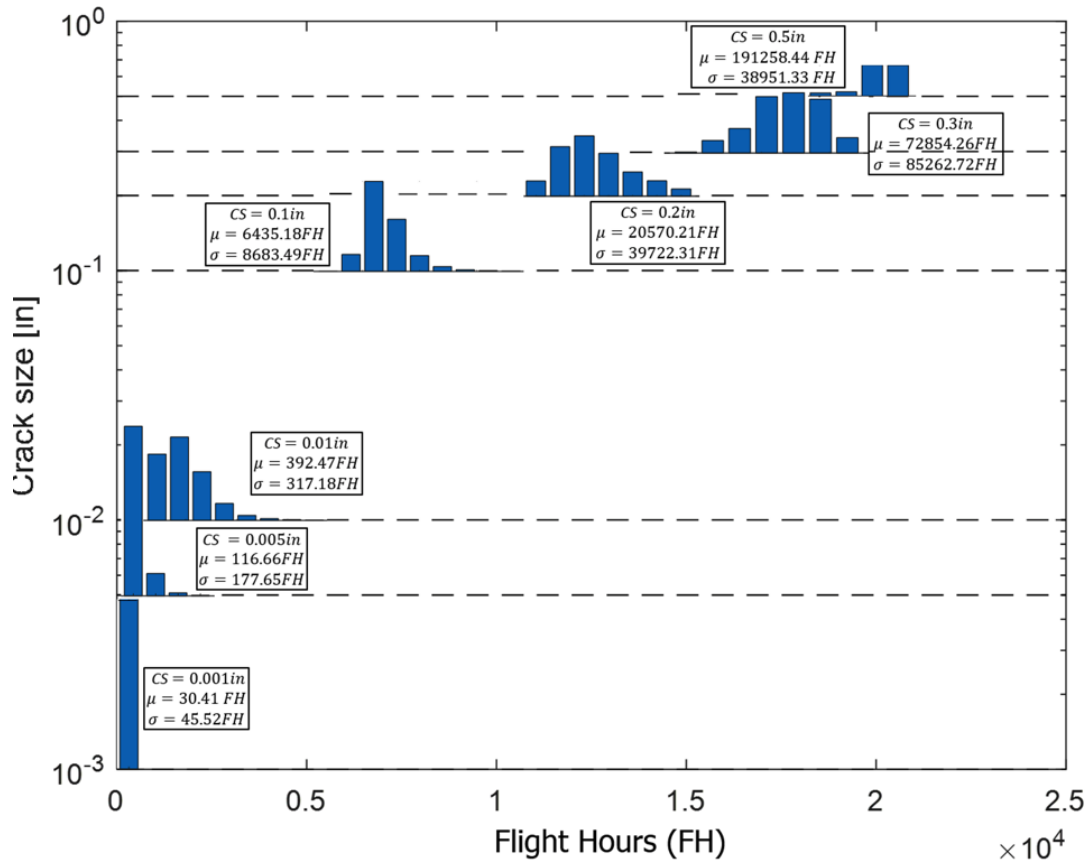


Figure 3-3: Example of the Remaining Useful Life Computed At Crack sizes (0.001, 0.005, 0.1, 0.2, 0.3, 0.5) in [Ocampo, et al., 2019]

### 3.4 Crack size distributions:

The crack size distributions are required for inspection simulations. The distributions are used to compare with the POD curves to assess if a crack is detected and for fusing with external data such as field findings using Bayesian updating methods. The distributions are straightforward to compute using standard Monte Carlo or other numerical methods.

#### 4.0 EXAMPLE PROBLEM

The geometry for this example was a corner crack growing from a fastener hole under remote tension. Crack growth was evaluated by NASGRO using a constant amplitude stress spectrum, the stress intensity factor solution “CC16”, and the Paris crack growth equation. Eight random variables were considered as shown in the table below. The SFPOF was computed using the Lincoln formulation. The inspection schedule was to inspect every 2000 flight hours starting at 7000.

The SFPOF results are shown in Figure 6 (top) and the coefficient of variation (COV) of the results are shown (bottom). The orange line is the SFPOF without inspections and repairs. The blue solid line is the result if detected cracks are removed then replaced with a repair crack size and allowed to grow during future times. The dotted line is the result if detected cracks are removed and not allowed to grow during future times – this option models “perfect” repair. Convergence was assured using a threshold of 20% COV (that is, the coefficient of variation of the SFPOF was less than 20% at all evaluation times). The total fracture mechanics samples was 8540 (4340 samples for SFPOF as without inspections and 4200 samples for computing the SFPOF including inspections and repairs) demonstrating the efficiency of the AMIS method.

Parameter	Value
Width	Deterministic (2.5 in)
Thickness	Deterministic (0.25 in)
Initial defect size	Lognormal (0.005, 0.002) in
Crack aspect ratio	Normal (1.5, 0.14)
Fracture toughness	Normal (34.8, 3.90) ksi sqrt(in)
Log Paris C	Normal(-8.777, 0.08)
Paris exponent	Deterministic 3.273
Hole diameter	Deterministic 0.1562 in
Edge distance	Normal (0.5, 0.05) in
Max stress per flight ( $\sigma_{EVD}$ )	Gumbel (16.74, 2.08)
Probability of detection	Lognormal (0.021, 0.028) in
Repair crack size	Lognormal (0.01, 0.004) in
Inspection & repair schedule	Every 2000 flight hours starting at 7000

Table 4-1: Example Problem Variables.

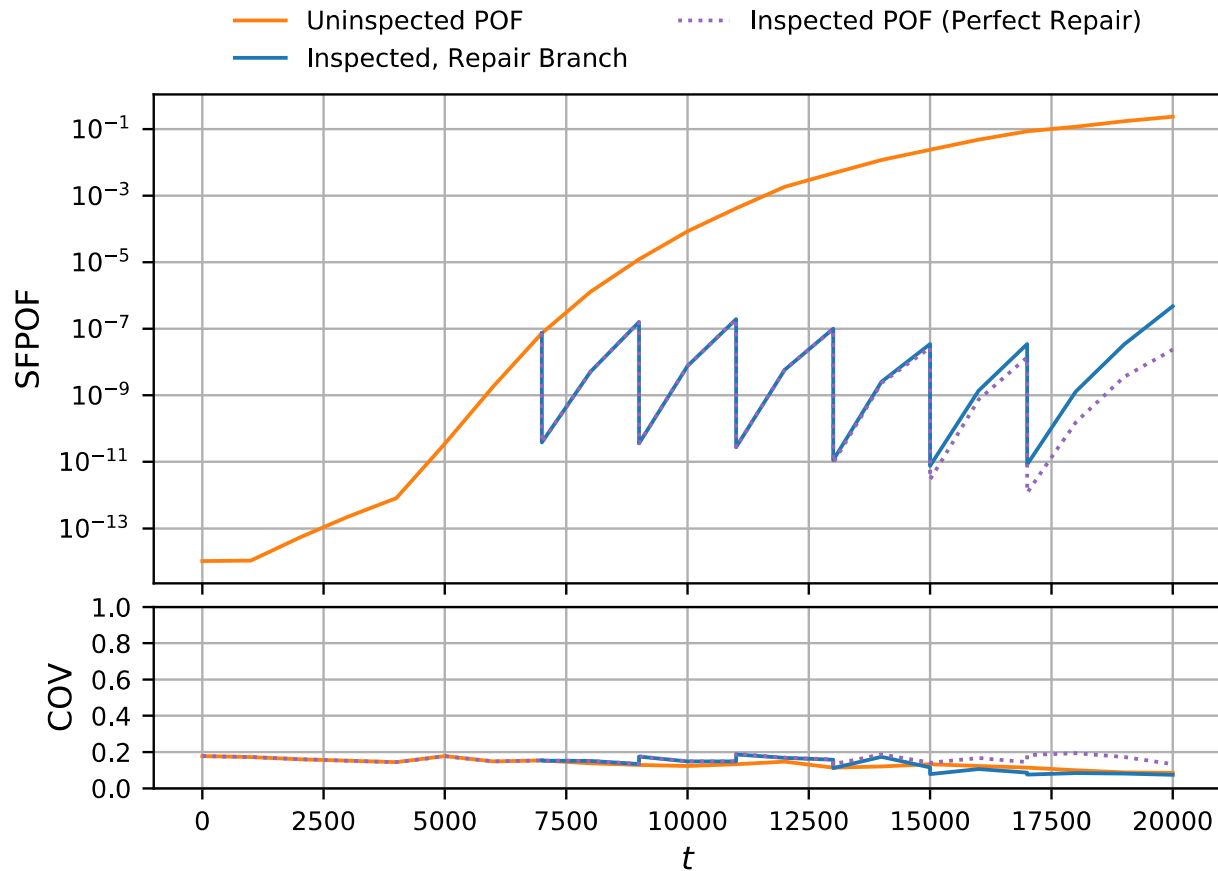


Figure 4-1: SFPOF Results Demonstrating Computed Using the Smart|DT AMIS Algorithm.

## 5.0 CHALLENGES AND FUTURE EFFORTS

A continuing challenge with ADT probabilistic calculations is the limitation with respect to the realism of the fracture mechanics models. That is, to date, relatively simple fracture solutions have been employed such as handbook solutions, in-plane crack growth, etc. whereas more sophisticated progressive crack growth analyses are desirable. Some examples include multisite damage with crack interactions, material degradation due to corrosion, non-planar crack growth such as with Franc3d [<http://fracanalysis.com>] or XFEM [Sukumar, et al., 2015]. To address this issue, we are developing efficient surrogate fracture models such that more realistic fracture scenarios can be considered during a risk assessment [Ananthasayanam, et al., 2017]. In the next few years Smart|DT will be enhanced to include surrogate models developed with machine learning algorithms.

A second challenge and research effort will involve the incorporation of Bayesian updating methods such that field findings and other sources of information can be used to update the probabilistic computations.

A third challenge is to enhance the existing risk assessment methodology to consider a more comprehensive set of random variables. In this regard AMIS is a major advance. The current technology is effective up to a range of approximately 20 random variables. Therefore, more sophisticated fracture models that include

## Probabilistic Methods for Risk Assessment of Airframe Digital Twin Structures

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microstructure, thermal effects, corrosion, geometry features, and other random variables can be addressed with the current technology.

### 6.0 CONCLUSIONS

The FAA-sponsored Smart|DT software is a publicly-available tool for simulating the essential probabilistic computations for an airframe digital twin. In particular, Smart|DT computes single flight probability-of-failure (SFPOF) with and without inspections, the remaining useful life, and the crack size distributions. The key ingredient of Smart|DT is the Adaptive Multiple Importance Sampling algorithm that computes the SFPOF with and without inspections in a highly efficient manner (typically 5 or 6 orders of magnitude faster than standard Monte Carlo). The Smart|DT ecosystem consists of the software, a probabilistic database (encompassing material property values, initial defect size, geometric parameters, and probability of detection curves), example problems, and training material.

Although focused on airframe components, the methodology is general and can be applied with modification to other structures and vehicles. The software, example problems, and training material can be obtained from the web site <https://smartdtsoftware.wixsite.com/smart>

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**Probabilistic Methods for Risk Assessment of Airframe Digital Twin Structures**

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