Probabilistic Damage Tolerance for Aviation Fleets Using a Kriging Surrogate Model

Nathan E. Crosby and Harry R. Millwater
The University of Texas at San Antonio

Juan D. Ocampo
St. Mary's University - San Antonio

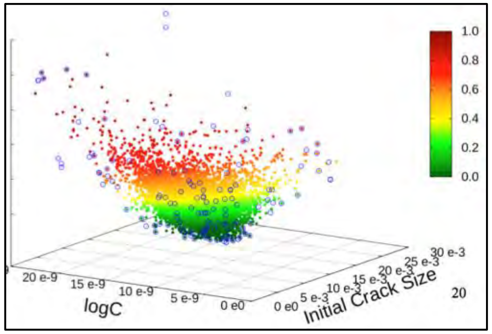
Elias L. Anagnostou, Stephen J. Engel, John S. Madsen, and Karin W. Engel
NG Next, Northrop Grumman Corporation

AIAA SciTech 2017, Dallas, Texas

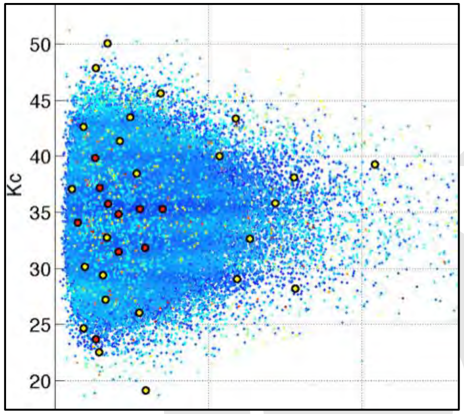
Outline

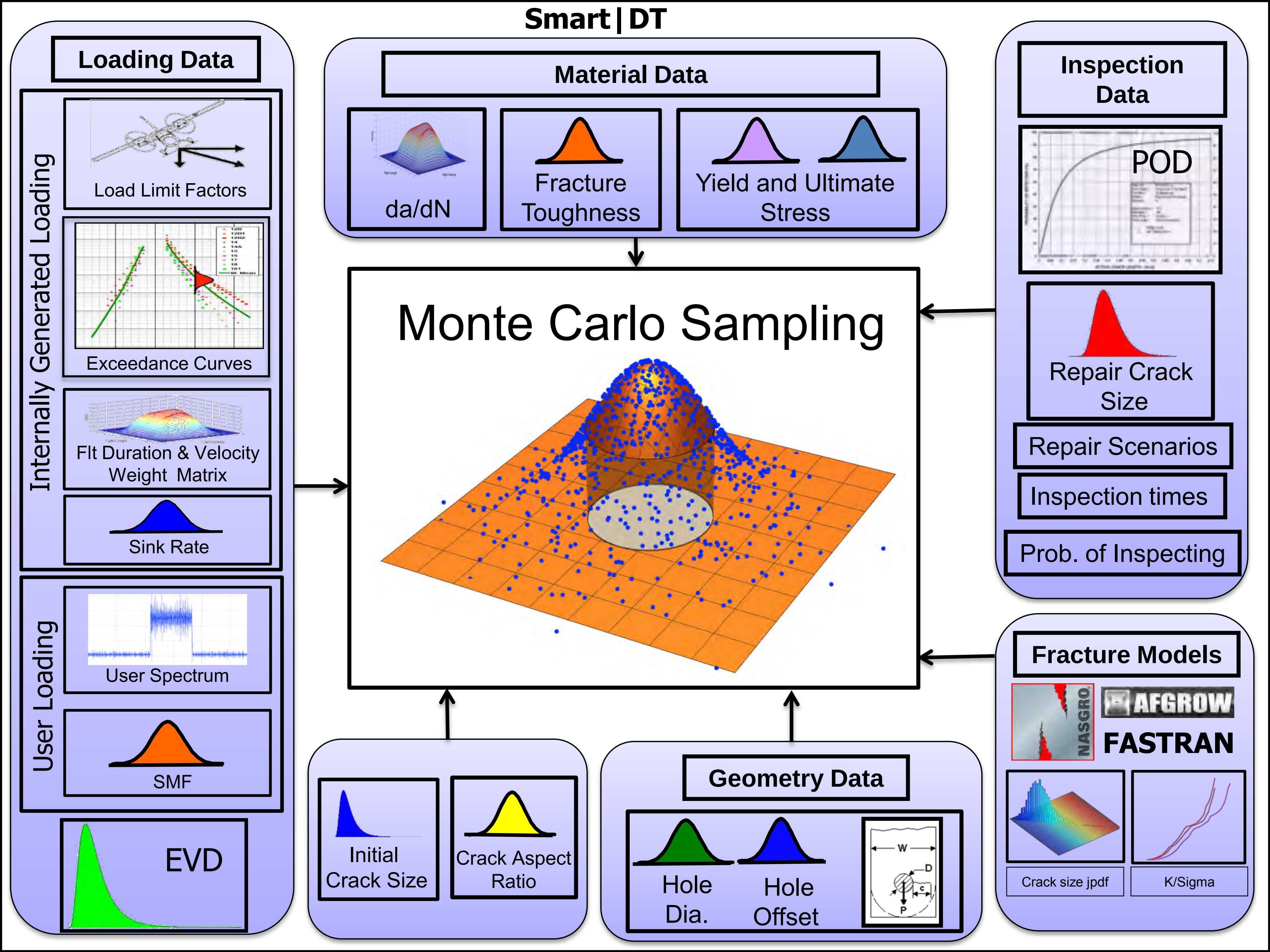
- SMART Flowchart
- Risk Assessment
- Master Curve Method
- Kriging Surrogate Model
- Parallel Computing
- Example Problems
- Conclusions

Surrogate Model

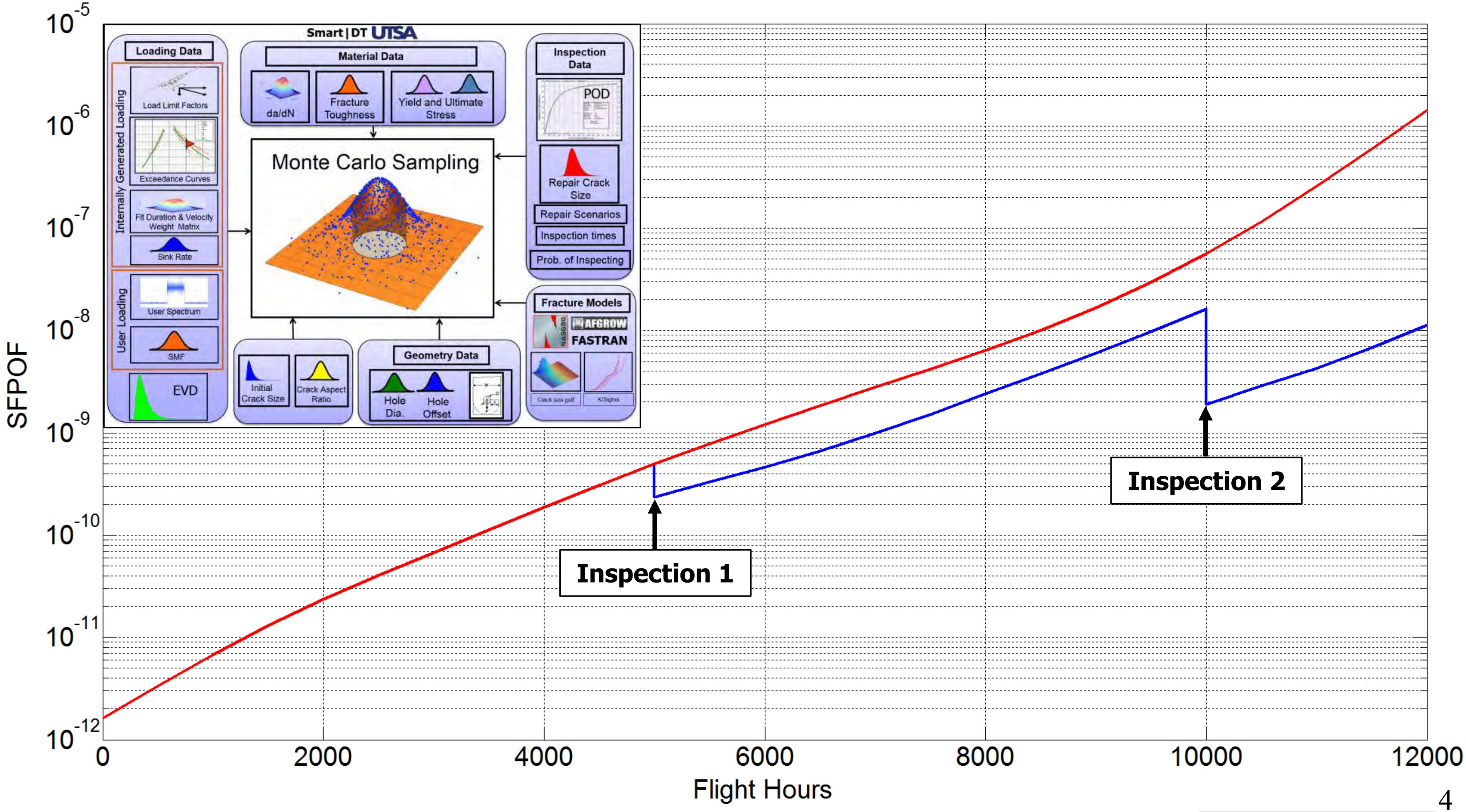


Monte Carlo




Smart | DT
Loading Data
Internally Generated Loading
Load Limit Factors
Exceedance Curves
Flt Duration & Velocity Weight Matrix
Sink Rate
User Loading
User Spectrum
SMF
EVD
Material Data
da/dN
Fracture Toughness
Yield and Ultimate Stress
Monte Carlo Sampling
Inspection Data
POD
Repair Crack Size
Repair Scenarios
Inspection times
Prob. of Inspecting
Fracture Models
NASGRO
AFGROW
FASTRAN
Crack size jpdf
K/Sigma
Initial Crack Size
Crack Aspect Ratio
Geometry Data
Hole Dia.
Hole Offset
W
D
c
P

Risk Assessment



Probability Equations

The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength σ_{RS} of the aircraft component

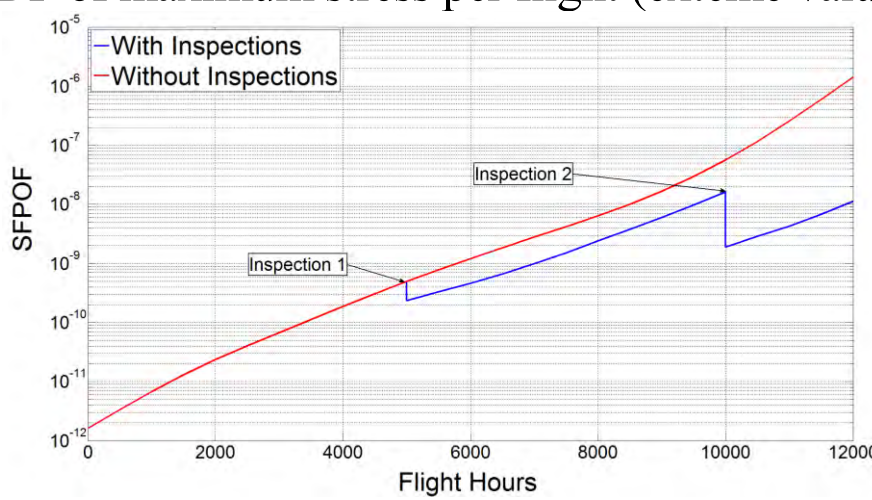
$$POF(t) = P\left[\sigma_{Max} > \sigma_{RS}(t)\right] = \int \left[1 - F_{EVD}\left(\sigma_{RS}(t)\right)\right] f_{\mathbf{x}}(\mathbf{x})\, d\mathbf{x}$$

$$CTPOF(t) = \int \left[1 - \prod_{i=1}^{t} F_{EVD}\left(\sigma_{RS}(t_i)\right) \right] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

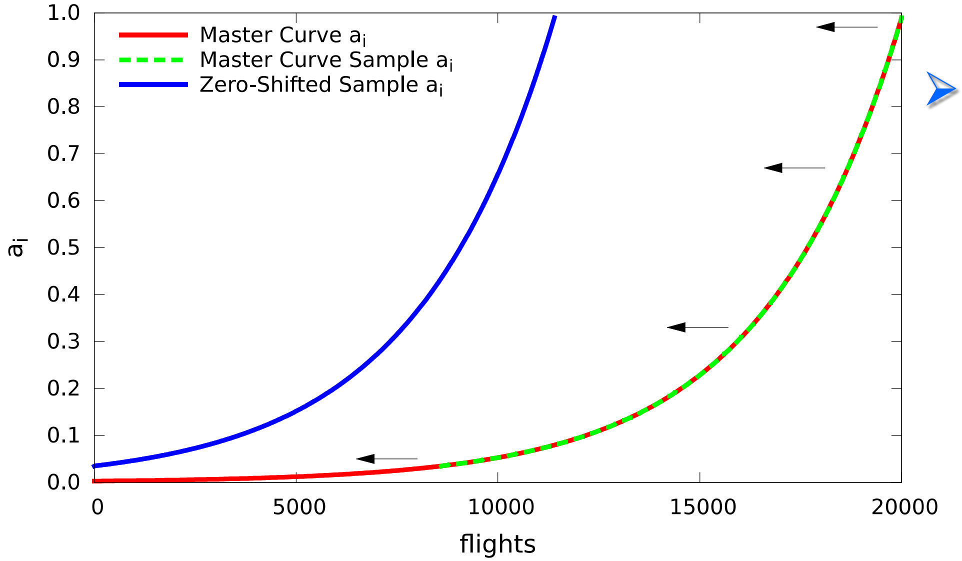
$$SFPOF(t) = \int \left[\prod_{i=1}^{t-1} F_{EVD}\left(\sigma_{RS}(t_i)\right) \right] \left[1 - F_{EVD}\left(\sigma_{RS}(t)\right)\right] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

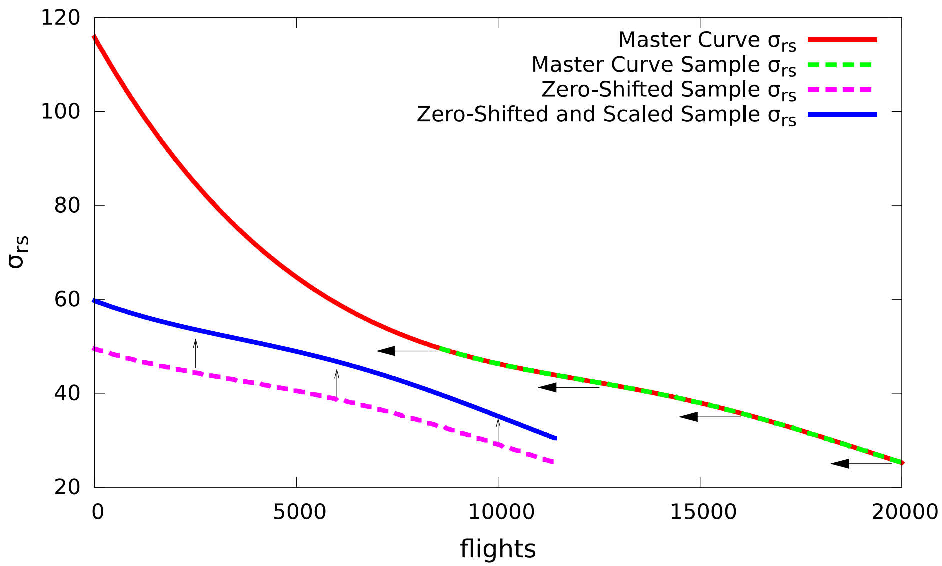
$$Hz(t) = \frac{SFPOF(t)}{1 - CTPOF(t)}$$

F_{EVD} = CDF of maximum stress per flight (exteme value distribution).



Master Curve Interpolation





- One crack growth curve for the whole simulation
  - **<u>Only Kc, a_i and EVD can be random</u>**
    - the structure has the same crack growth properties throughout the entire simulation.
  - One spectrum (representative) is used for the entire simulation.

Random Variables for Comprehensive PDTA

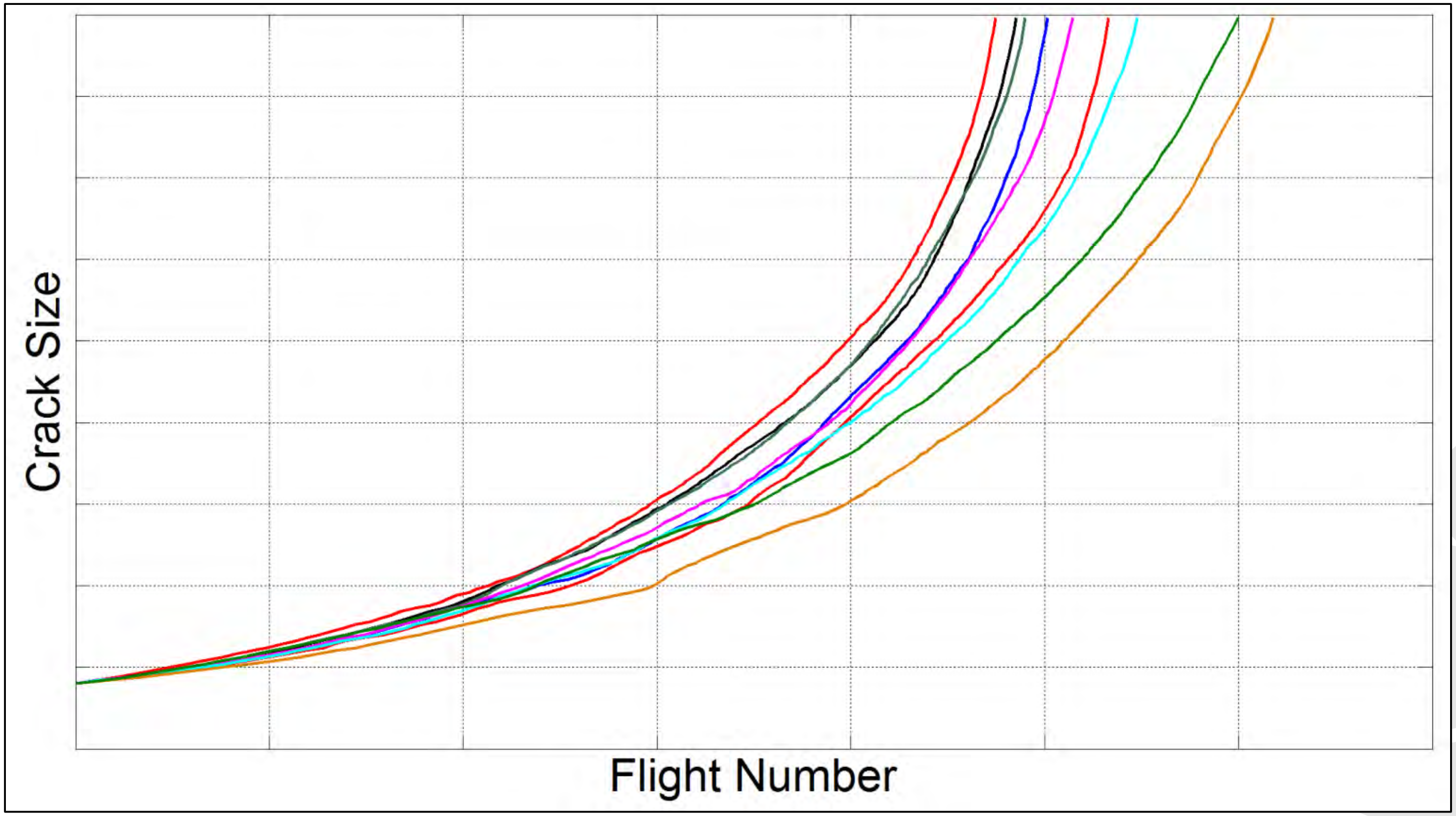
Random Variable	Options
Initial Crack Size	Lognormal, Weibull, Tabular, Tabular joint a and c
Fracture Toughness	Normal
Extreme Load per Flight	Gumbel, Weibull, Frechet
da/dN Parameters	Correlated normal
Crack Aspect Ratio	Normal, Tabular
Hole Diameter	Normal
Hole Offset	Normal
Yield Stress	Normal
Ultimate Stress	Normal
Peak Stress	Uniform
Random Variables and Distribution options ***expandable***	

Master Curve

Surrogate Model/Brute Monte Carlo

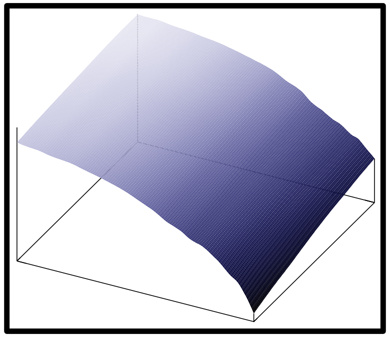
Getting Past the 3 Random Variable Limitation

- Random da/dn, material properties, and component dimensions require a crack growth analysis for each sample



Computational Workload Comparison

Master Curve Interpolation



Surrogate Model

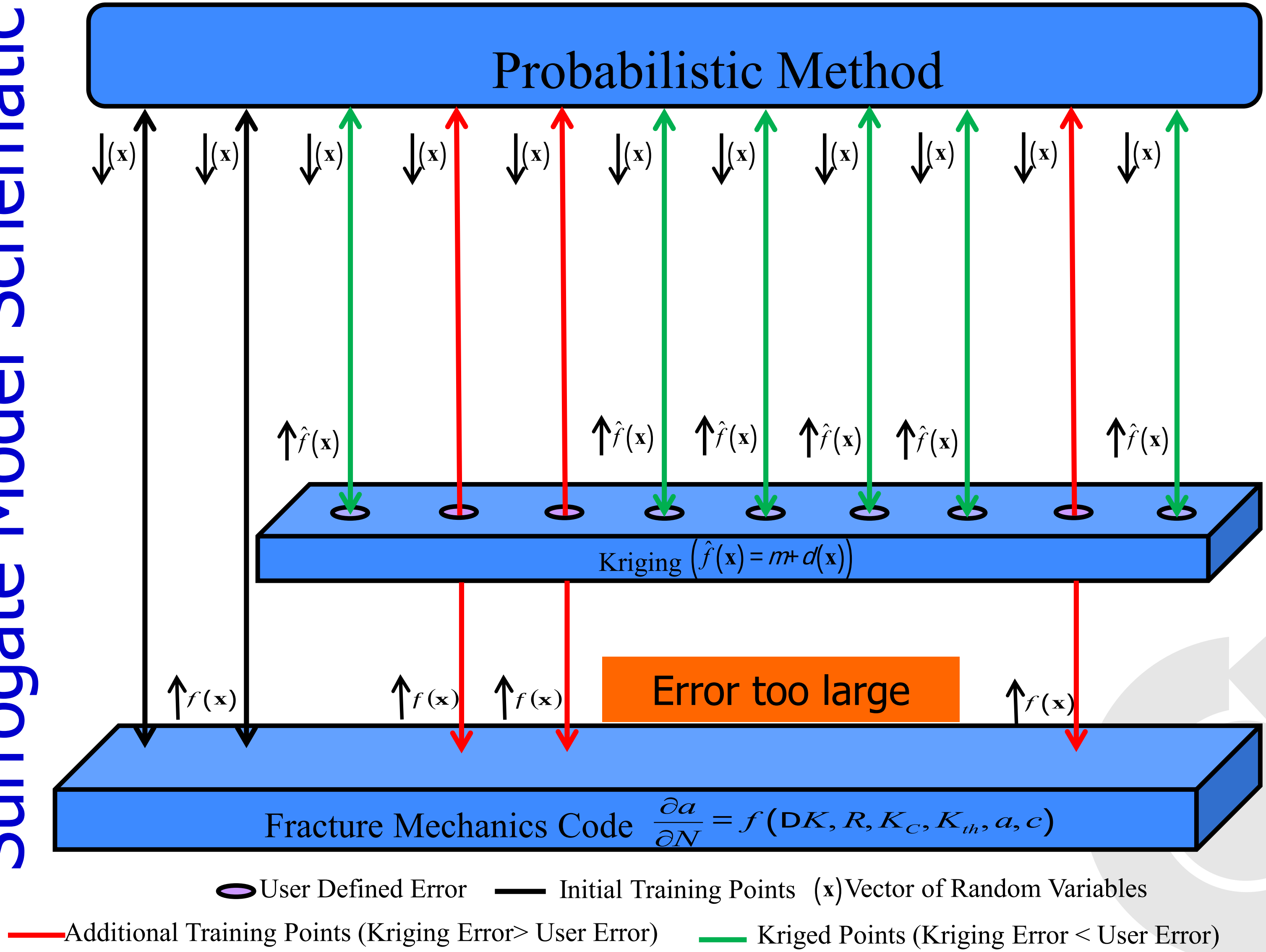
Fracture Mechanics Crack Growth Models

Options for computation time reduction

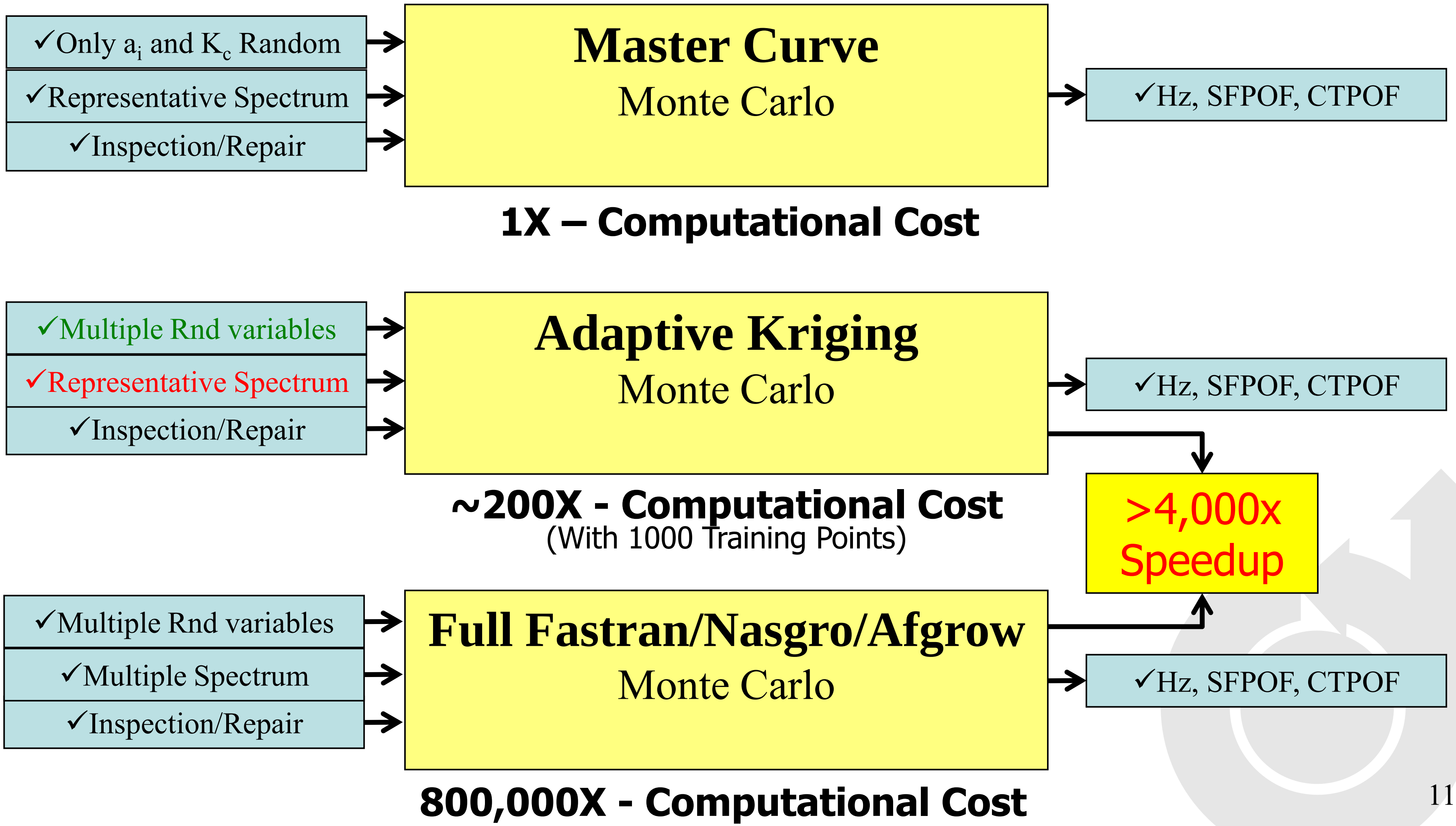
- Surrogate model
- Parallel computing

Surrogate Model Approach

Surrogate Model Schematic



Computational Expense



Kriging Surrogate Model

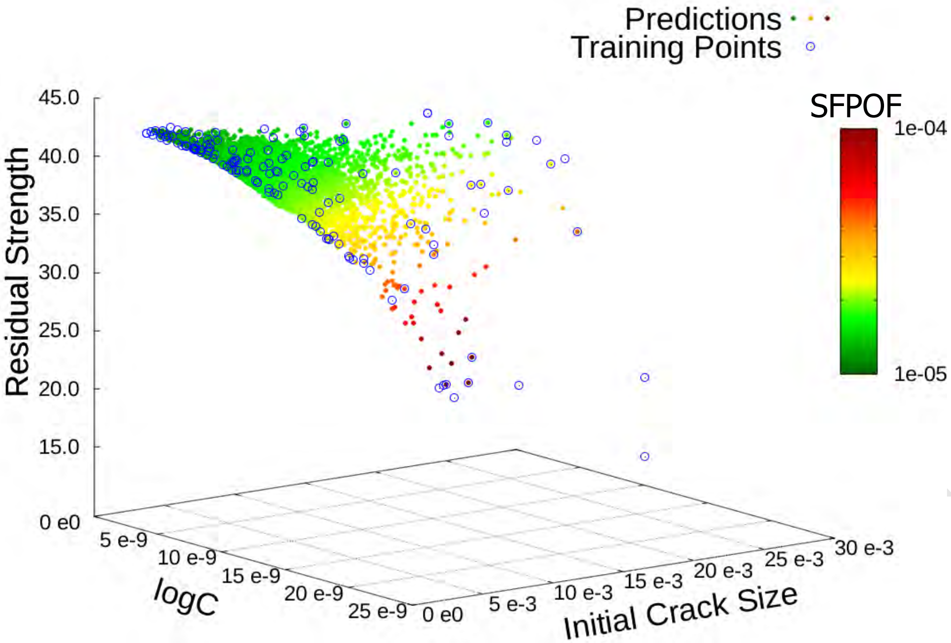
- Efficient Method to compute Crack Size (a) and Residual Strength (RS).
  - Train surface with crack growth analyses.

$$\frac{\partial a}{\partial N} = f(\Delta K, a, c)$$

$$\frac{\partial c}{\partial N} = f(\Delta K, a, c)$$

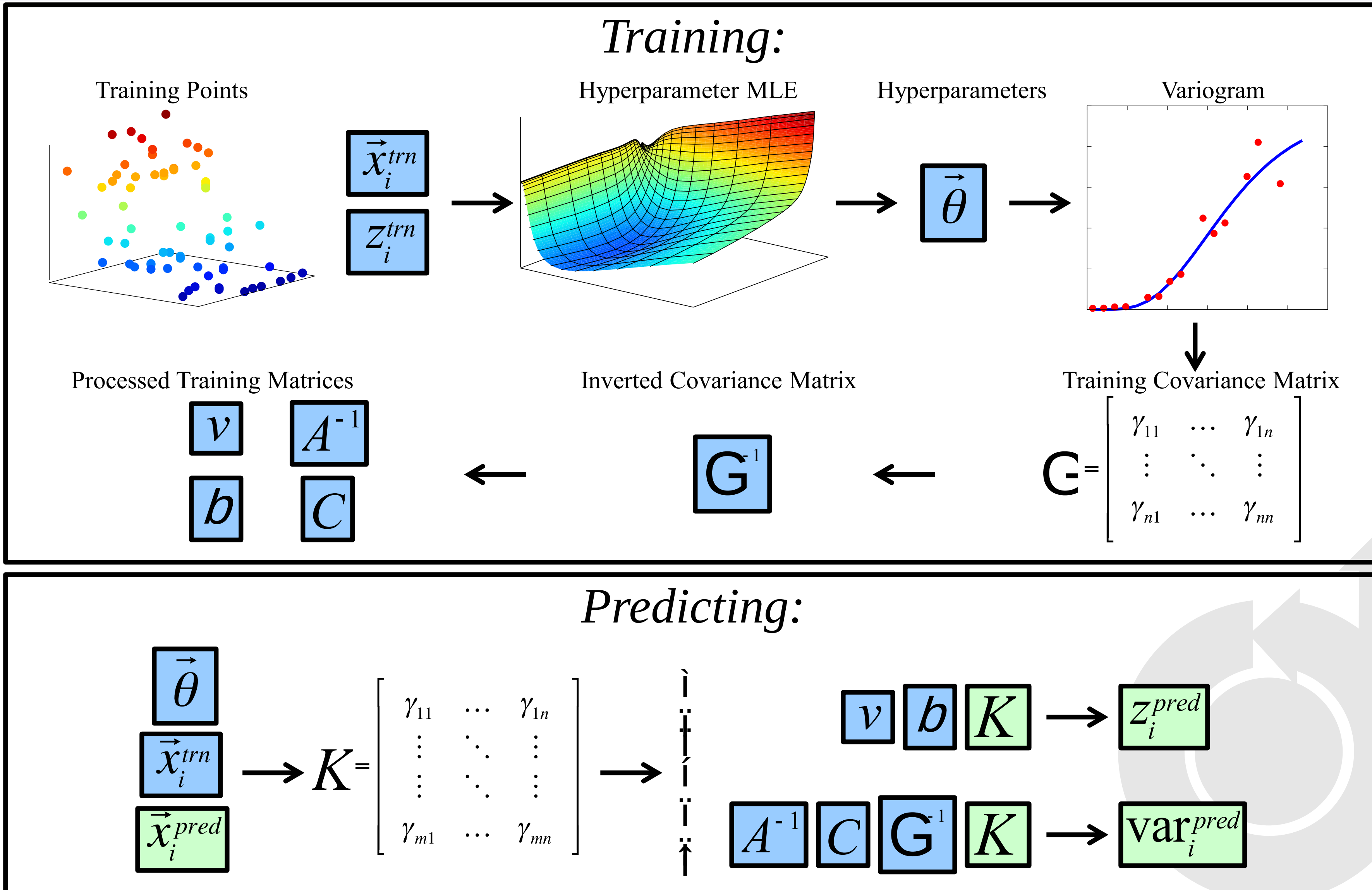
Kriging

$$f(x) + Z(x)$$



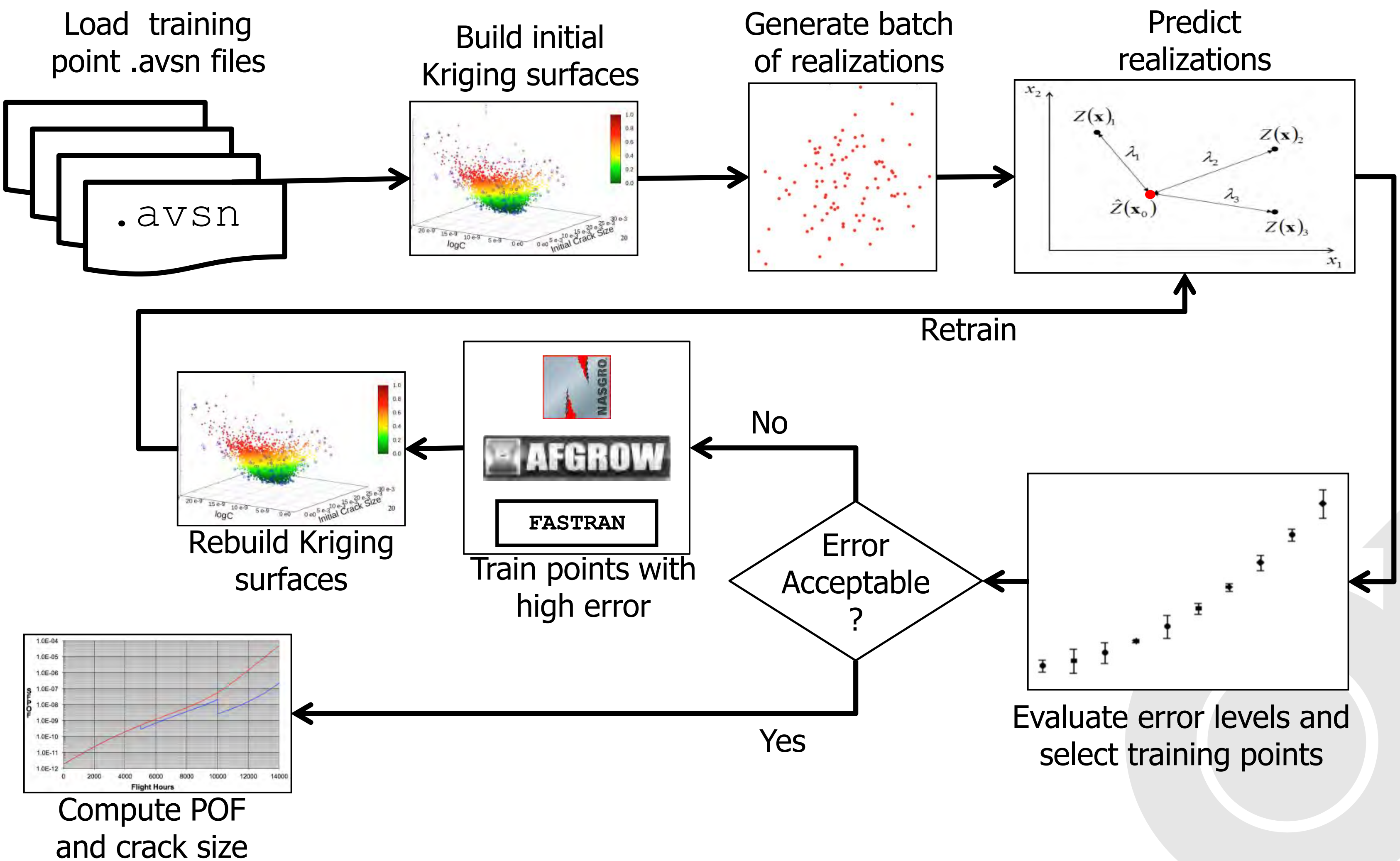
  - After building the Kriging surface predict "a" and "RS".

Kriging

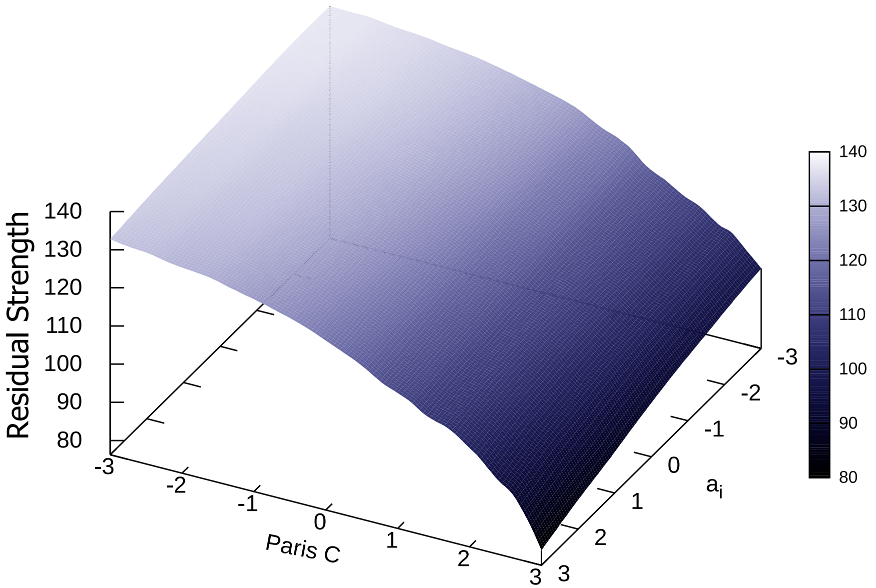


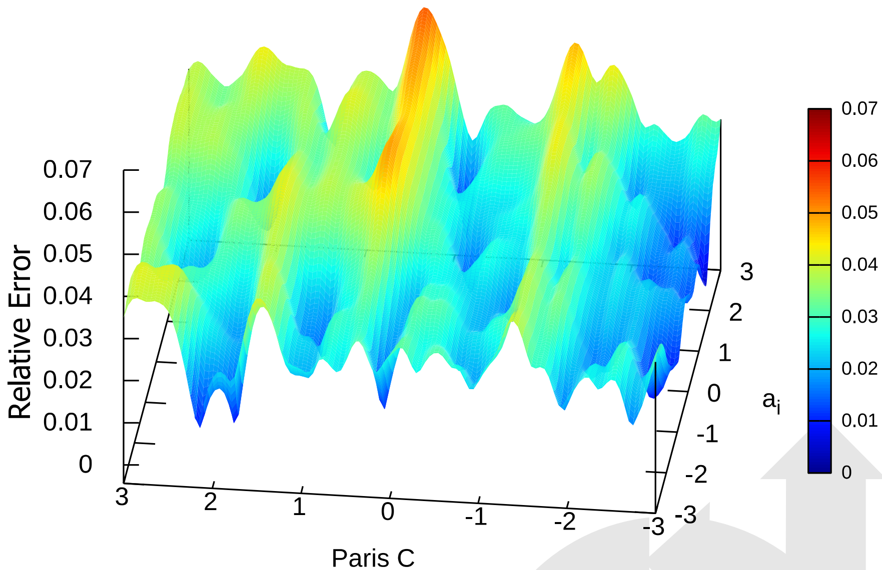
Training:
Training Points
Hyperparameter MLE
Hyperparameters
Variogram
$\vec{x}_i^{trn}$
z_i^{trn}
$\vec{\theta}$
Processed Training Matrices
Inverted Covariance Matrix
Training Covariance Matrix
v
A^{-1}
b
C
G^{-1}
$G = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix}$
Predicting:
$\vec{\theta}$
$\vec{x}_i^{trn}$
$\vec{x}_i^{pred}$
$K = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{m1} & \cdots & \gamma_{mn} \end{bmatrix}$
v b K → z_i^{pred}
A^{-1} C G^{-1} K → var_i^{pred}

Adaptive Surrogate Model Error Reduction



Kriging Response Surface Example

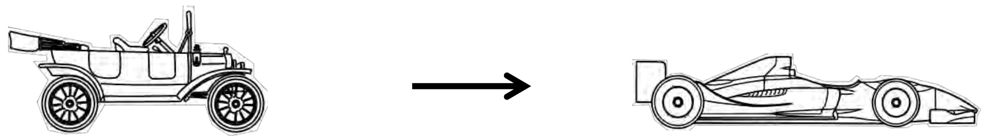




Residual strength surface at 7000 hours

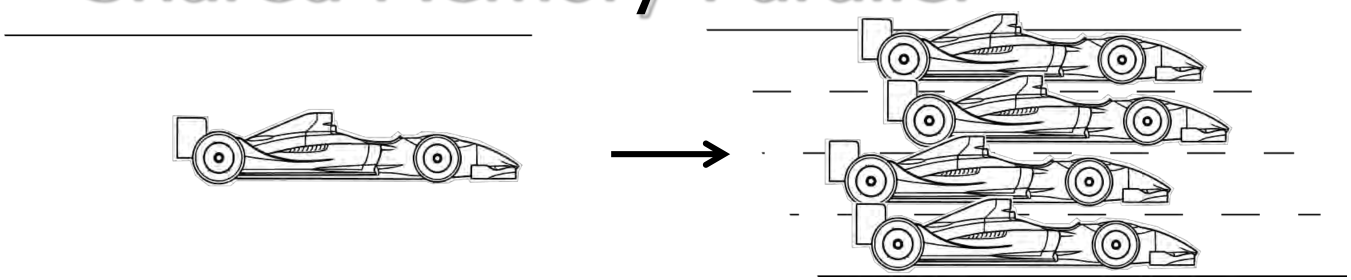
Parallel Processing

- ✓ Code Vectorization & Optimization



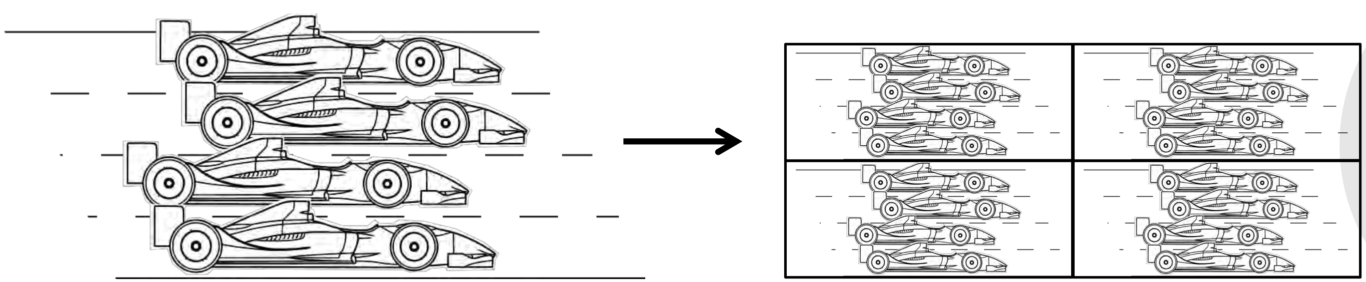
10x to 100x

- ✓ Shared Memory Parallel



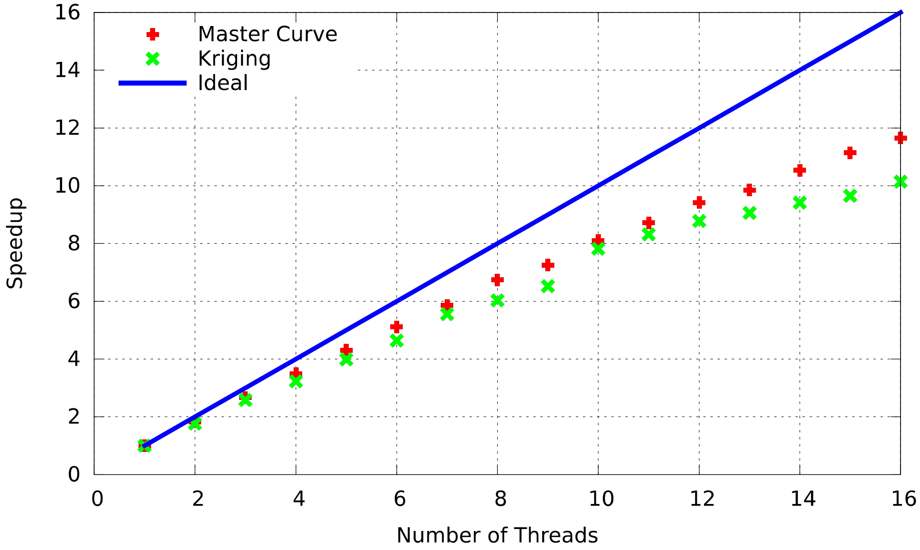
4x to 40x

- ✓ Grid Computing

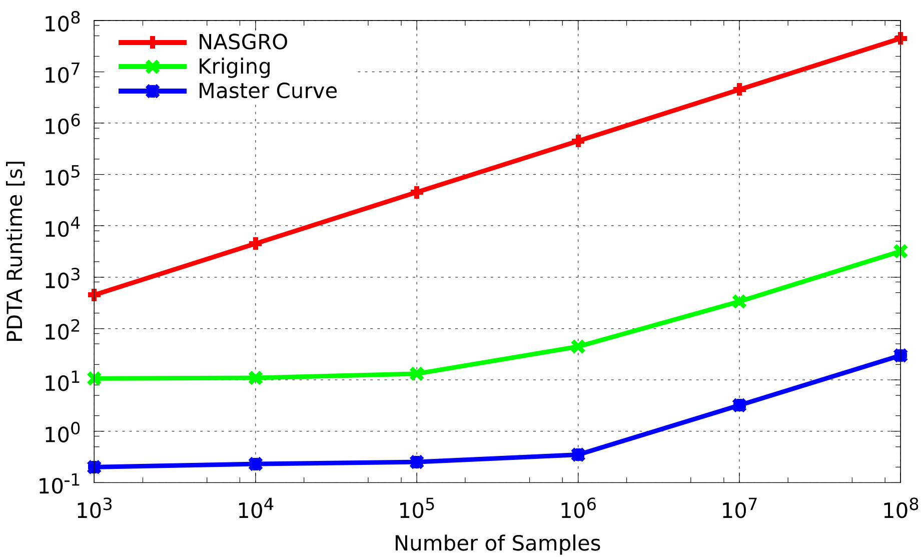


Machine dependent

SMART|DT Speedup

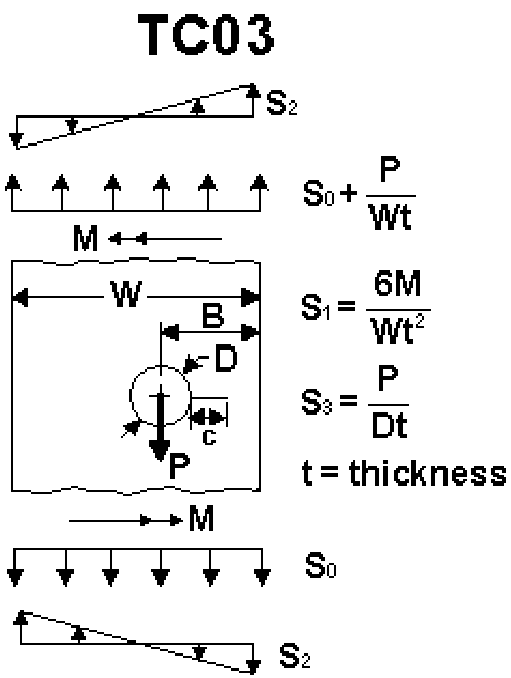


- ✓ Runtimes measured on a 16 core compute node
- ✓ Parallel performance reaches 10x to 12x speedup
- ✓ Speedup from vectorization and optimization (not shown) is compounded by parallel speedup



- ✓ Runtimes for 16 threads running on a 16 core node
- ✓ Surrogate model reduces runtime to 2 orders of magnitude above master curve

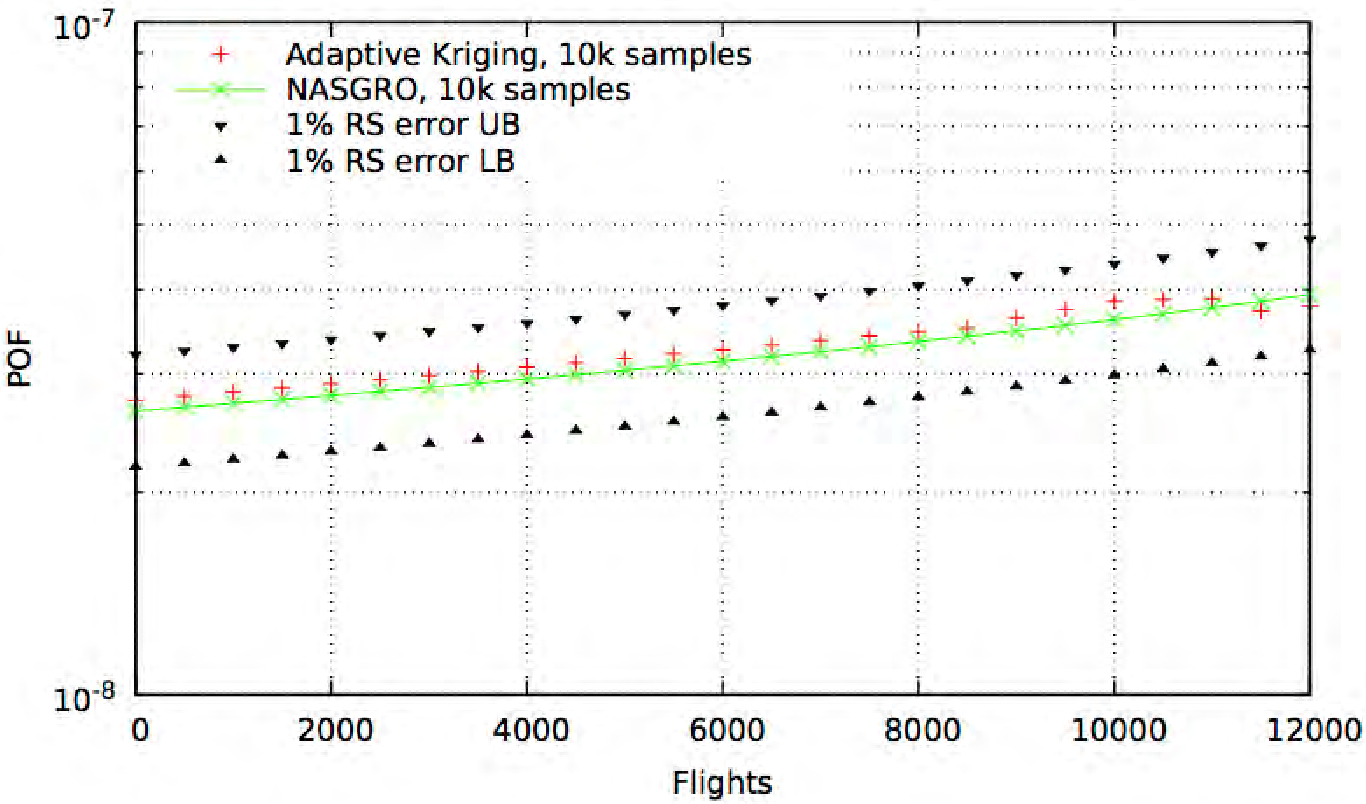
Example Problem 1



Quantity	Distribution	Parameters
Initial Crack Size	Lognormal	λ = -3.00 ($m = 0.05$ in) ζ = 0.0998 ($s.d. = 0.005$ in)
Fracture Toughness	Normal	μ = 30.0 in σ = 3.0 in
Log(ParisC)	Normal	$\mu = -8.1$ $\sigma = 0.142$
Hole Diameter	Normal	μ = 0.15625 in σ = 0.0052 in
Edge Distance	Normal	μ = 2.5 in σ = 0.0625 in
Maximum Load	Fréchet	μ = 12.35 ksi σ = 1.66 ksi ξ = 0.023

Quantity	Definition
Nasgro Crack Growth Model	TC03 - Through crack in a hole
Material	Al-2024
Geometric Variables	width : 5 in thickness : 0.2 in
Deterministic Variables	yield stress : 50 ksi ultimate stress : 70 ksi Paris n : 2.7

Example Problem 1 Results



6 RVs: a_i , k_c, paris c, $ø_{hole}$, edge dist, $σ_{max}$

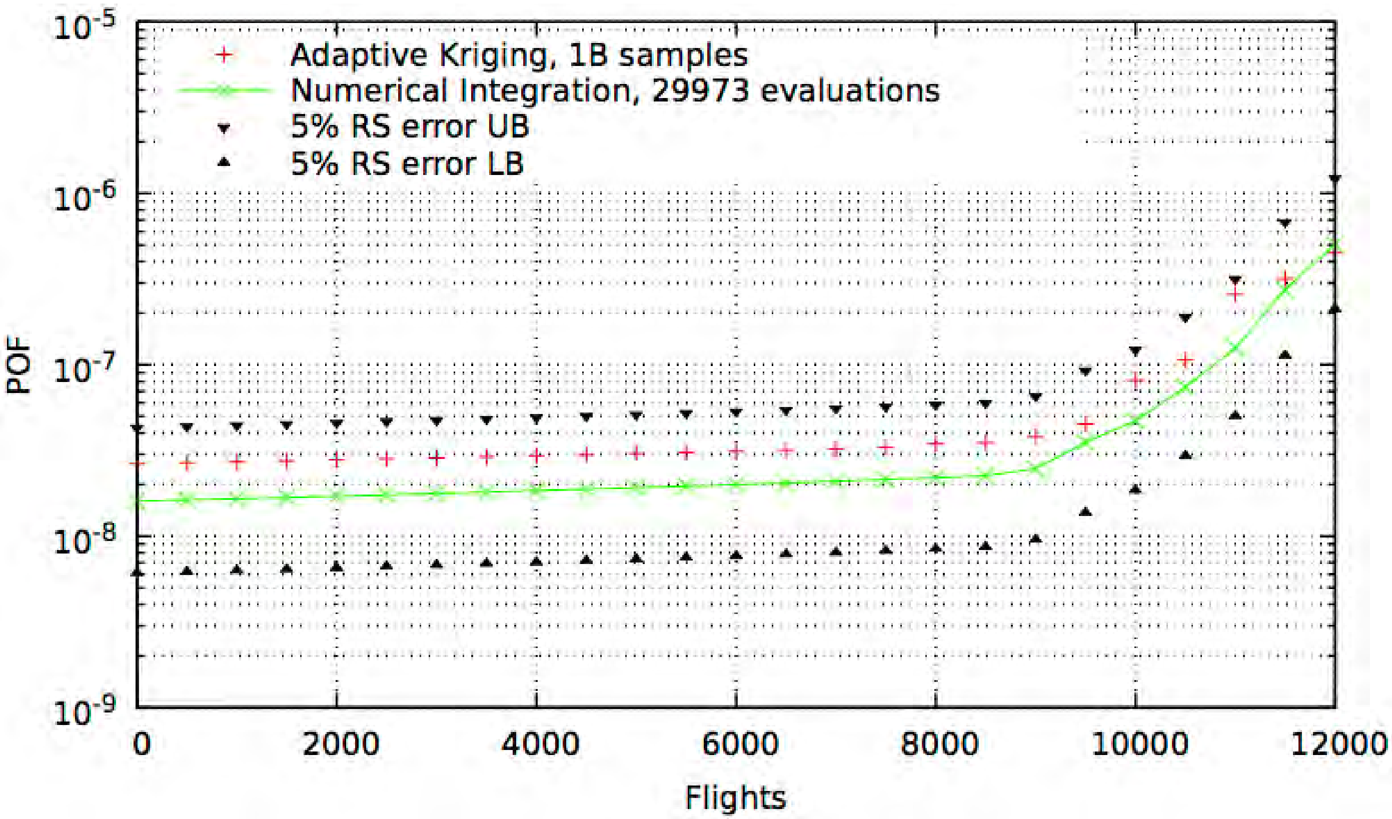
256 initial training points

93 additional training points

Runtime 281 seconds on 16 processors

- Kriging predictions are within 1% residual strength error bounds indicated
- NASGRO runtime for 10k TC03 evaluations is 4500 seconds (1 hour 15 minutes) using 16 processors

Example Problem 1 Results



Kriging predictions are within 5% residual strength error bounds indicated

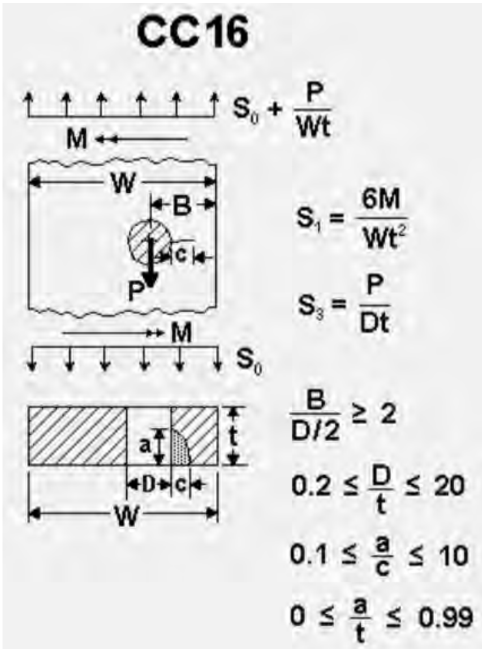
6 RVs: a_i , k_c, paris c, $ø_{hole}$, edge dist, $σ_{max}$

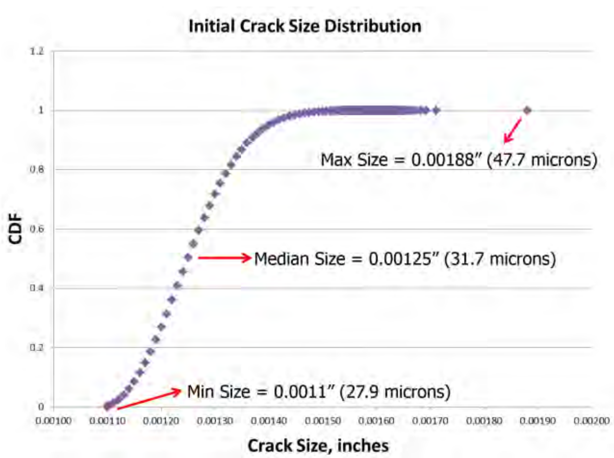
128 initial training points

921 additional training points

Runtime 8.5 hours on 16 processors

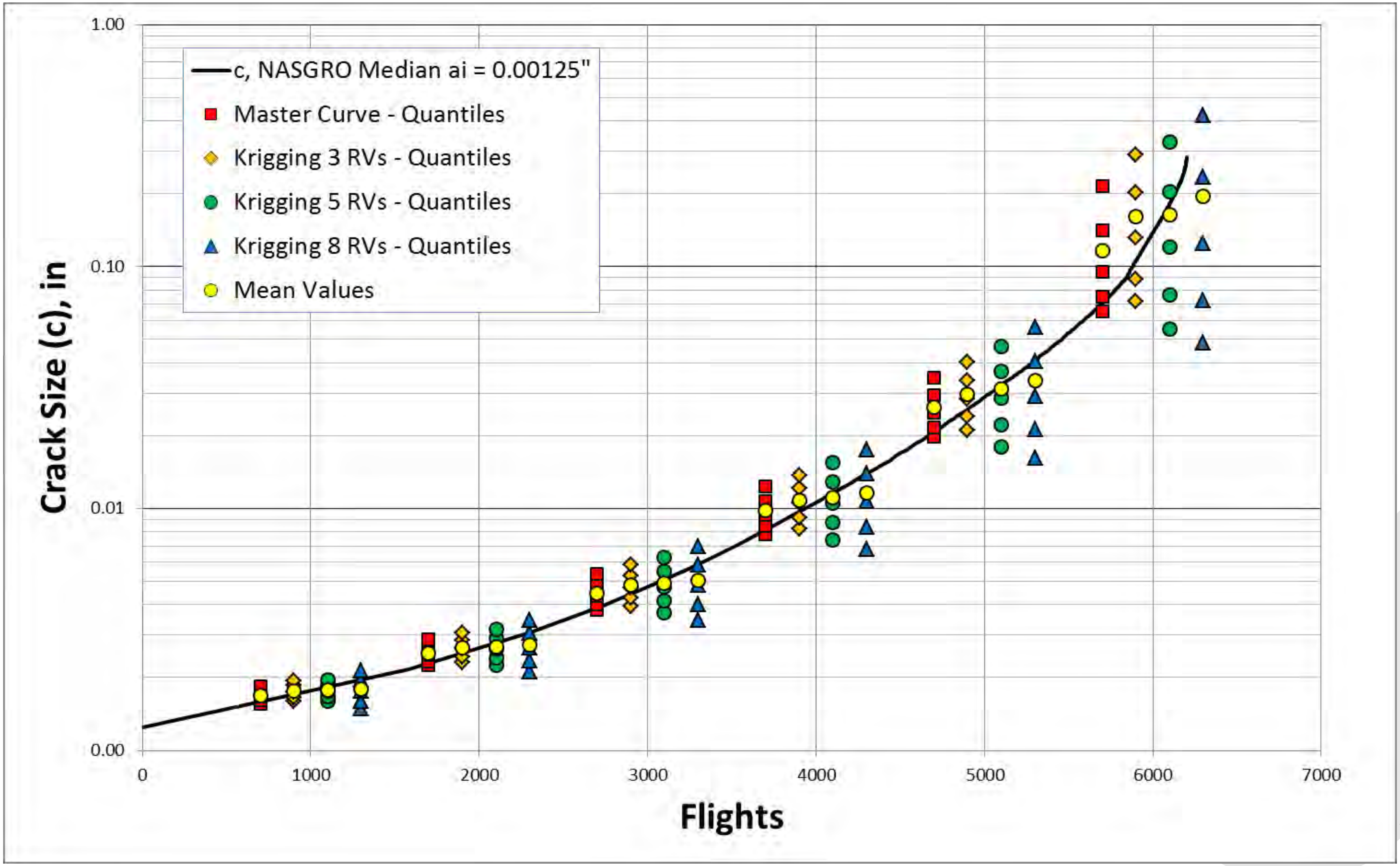
Example Problem 2



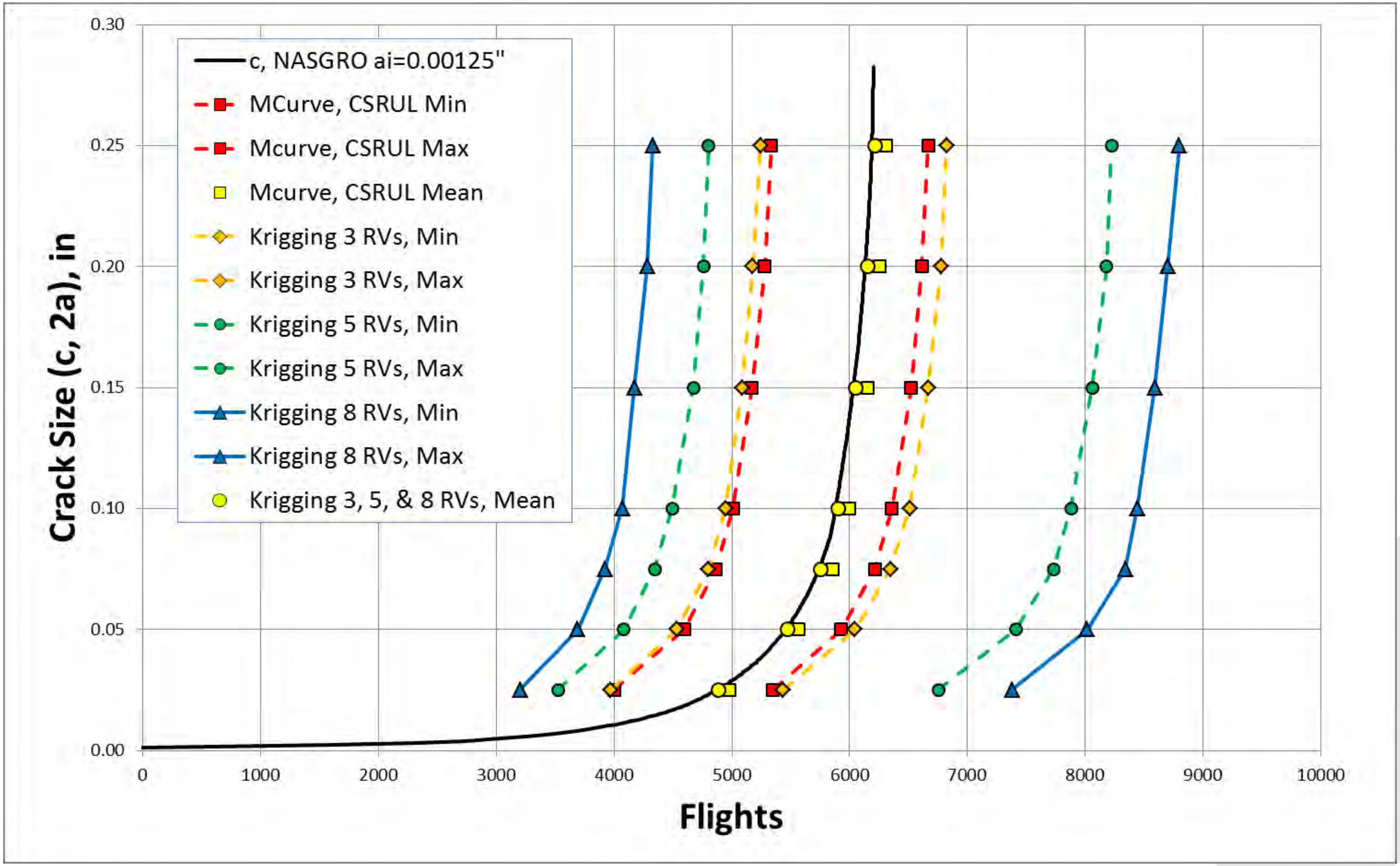


Random Variable	Distribution	Parameters
Initial Crack Size	Tabular	0.00125 in 0.0012 0.0013 0.0011 0.0497
Initial Crack Aspect Ratio	Uniform	Min: 0.75 Max: 1.25
Fracture Toughness	Normal	μ: 29 ksi √ in σ: 1.8
Log_{10}(Paris C)	Binormal	μ: -7.9 σ: 0.037
Paris m		μ: 3.405 σ: 0.0749
Yield Stress	Uniform	Min: 72 ksi Max: 79
Ultimate Stress	Uniform	Min: 79 ksi Max: 88
Maximum Load Stress	Gumbel	μ: 12.19 ksi σ: 1.18 ξ: 0.0

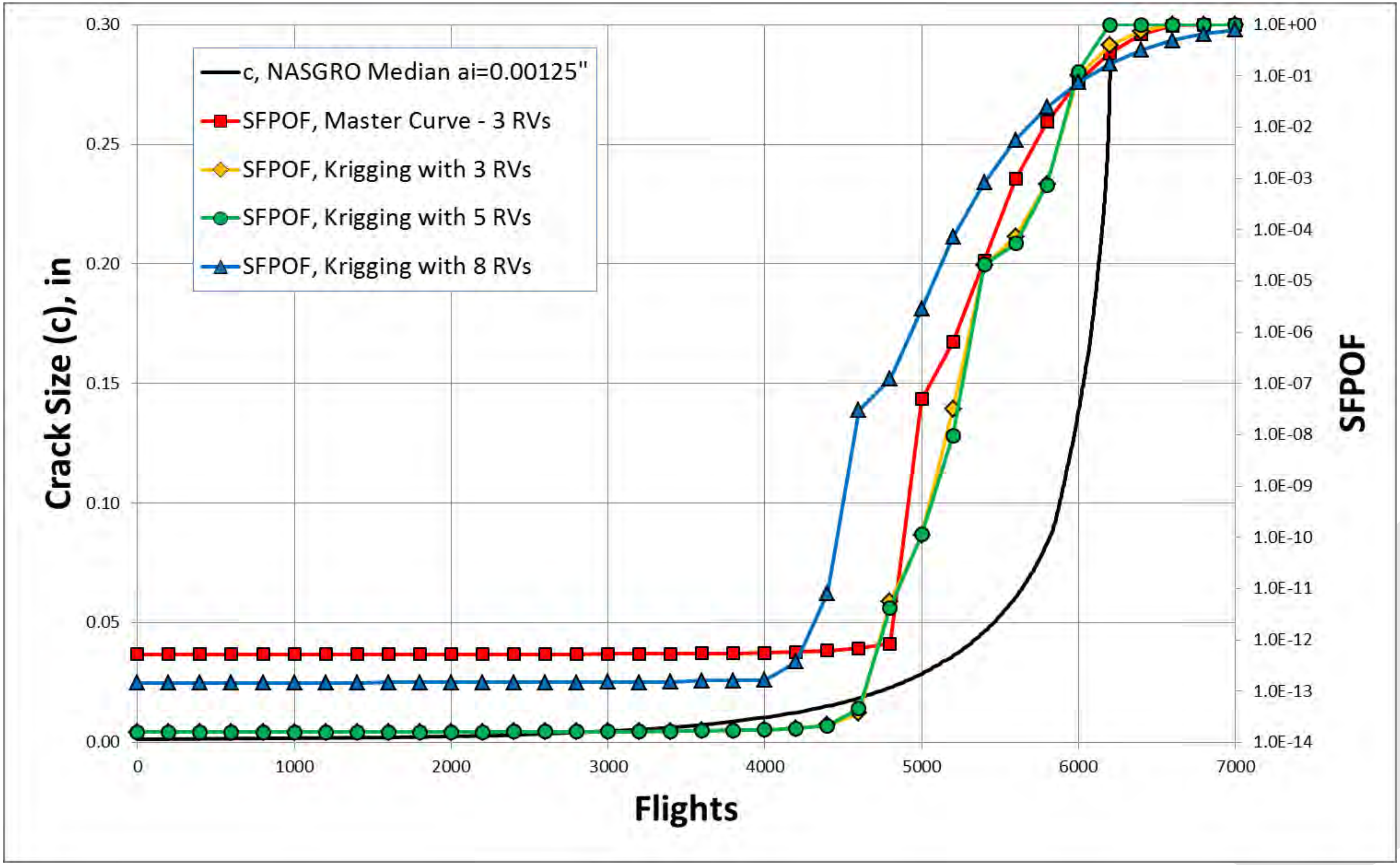
Example Problem 2 Results



Example Problem 2 Results



Example Problem 2 Results



Conclusions

- Master curve PDTA provides a good 1st order approximation of risk
- Comprehensive PDTA provides additional variability information for risk analysis
- Surrogate model and parallel computation reduce comprehensive PDTA compute time to useable timeframe applicable to digital twin
- PDTA can provide probabilistic damage information such as crack size quantiles and remaining useful life in addition to POF

Acknowledgments

- The authors are grateful to the Federal Aviation Administration for grant number 16-G-005 and to the United States Air Force Research Lab for contract number FA8650-14-D-2413, which support this research.

Questions