



TEXTRON AVIATION



# Introduction to Probabilistic Methods with Applications to Probabilistic Damage Tolerance Analysis

## PDTA Case Study



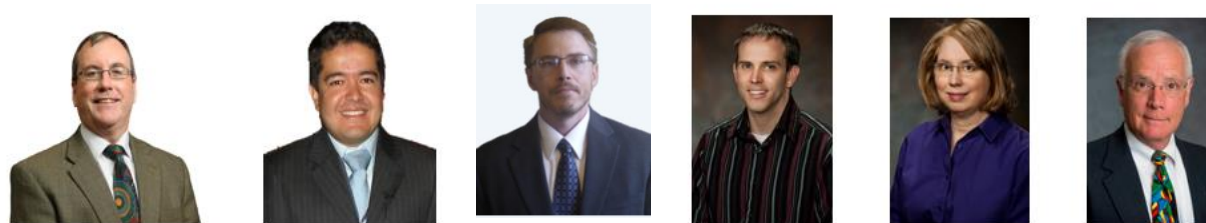
Harry Millwater - University of Texas at San Antonio,

Juan Ocampo, St. Mary's University,

Nathan Crosby, AeroMatter Inc.

Beth Gamble, Chris Hurst, Textron Aviation (Cessna)

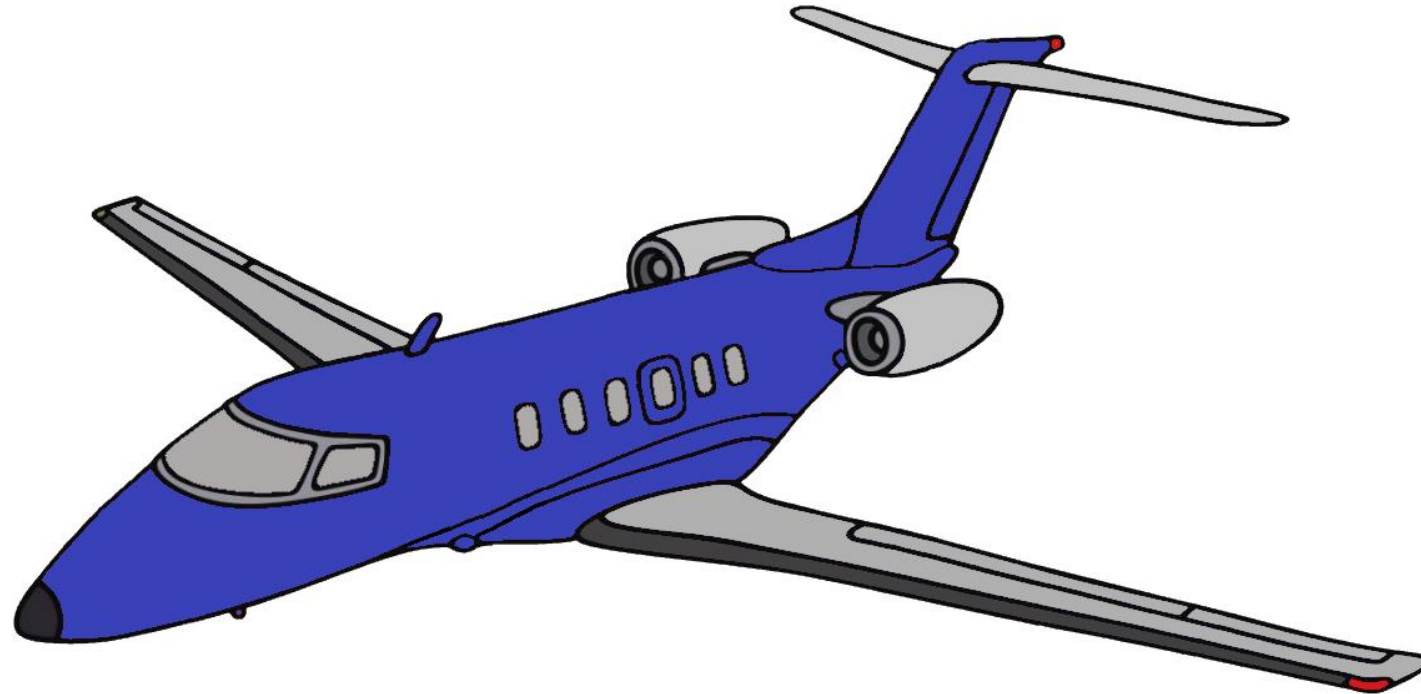
Marv Nuss, Nuss Sustainment Solutions



August 29, 2022



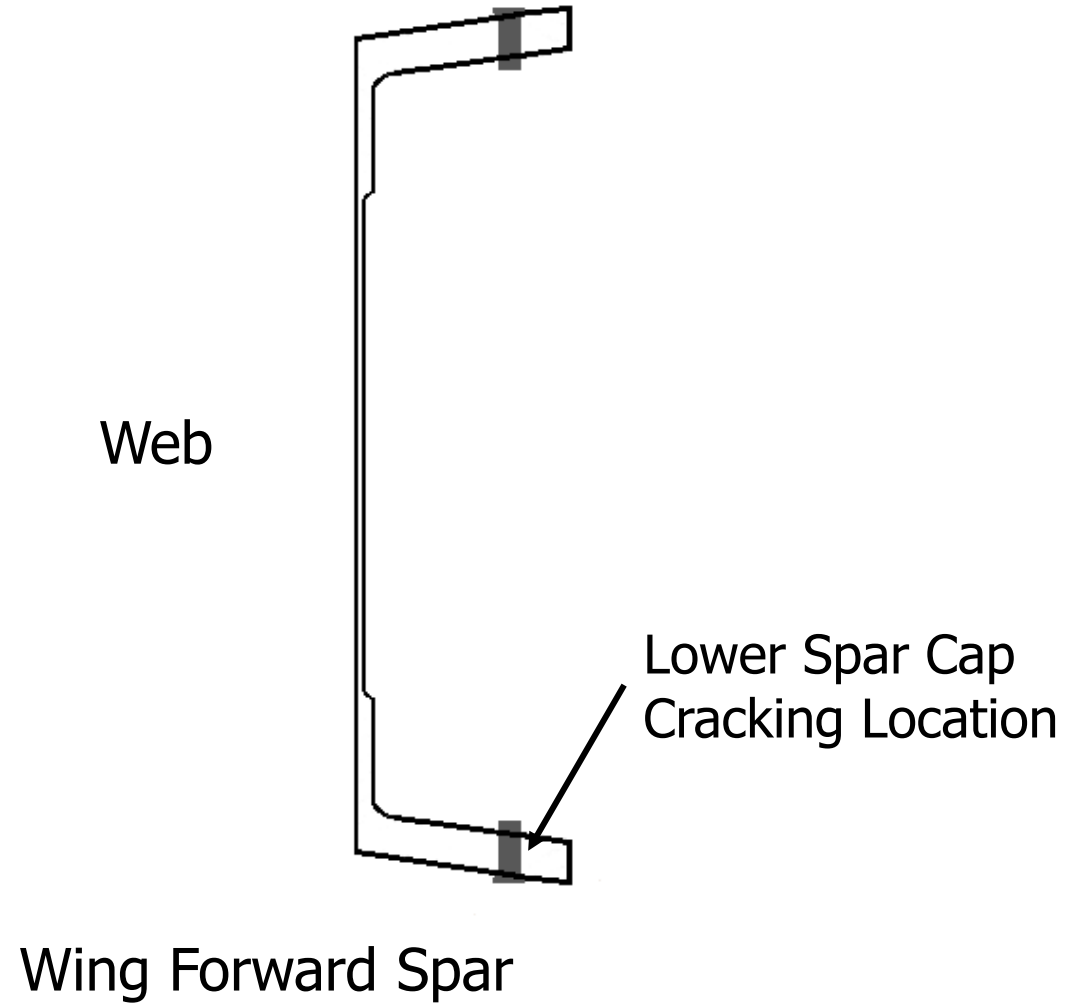
# Problem Overview



General Aviation Corporate Jet With  
Wing Forward Spar Cap Cracking



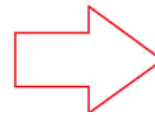
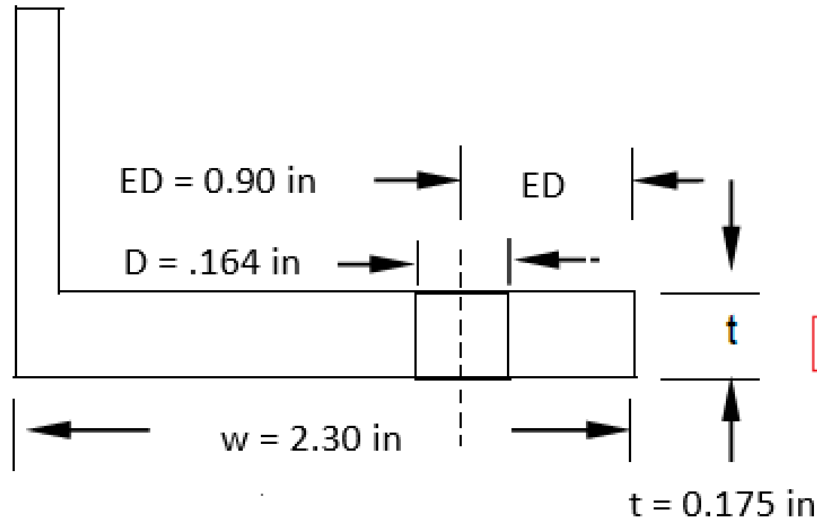
# Problem Overview



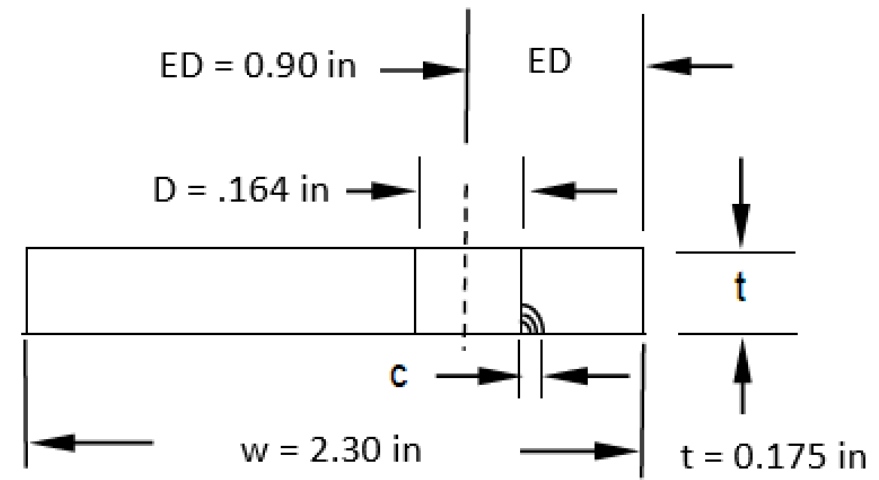


# Problem Overview

## Simplified Geometry



## Idealized Geometry



$$\frac{\sigma_{bearing}}{\sigma_{bypass}} = .66$$



# CLASSICAL CRACK GROWTH



# Classical Crack Growth

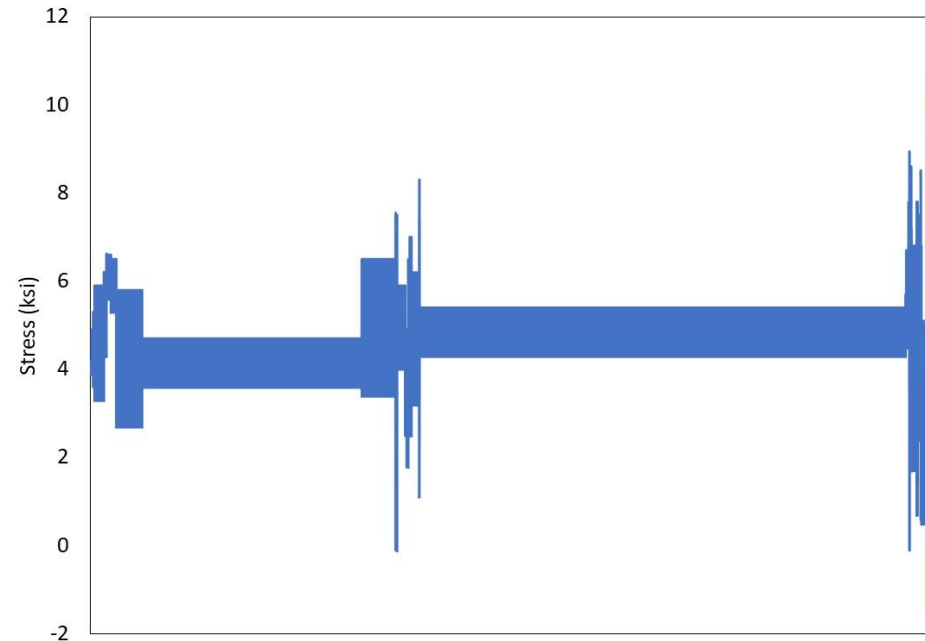
- Determine initial and recurring inspections using classical crack growth analysis from .05" flaw
  - AFGROW software used for crack growth
    - User defined spectrum
  - Detectable crack length = .20 in.
  - Inspections
    - Initial inspection =  $\frac{\text{Life}}{2}$
    - Repeat inspection =  $\frac{\text{Life} - \text{Life@detectable}}{2}$



# User Stress Spectrum

## - User defined spectra

- Gust cycles are based on PSD continuous gust criteria in 14 CFR 25 Appendix G
- Maneuver cycles are based on NASA measured data for business jets
- Spectra is in AFGROW format (1,000 flight hours, 1 flight = 1.65 Flt. Hrs.)
- Used SMART to create a simplified GAG spectrum with equivalent damage



50 different max-min stress pairs, 33,600 total pairs

# Classical Crack Growth



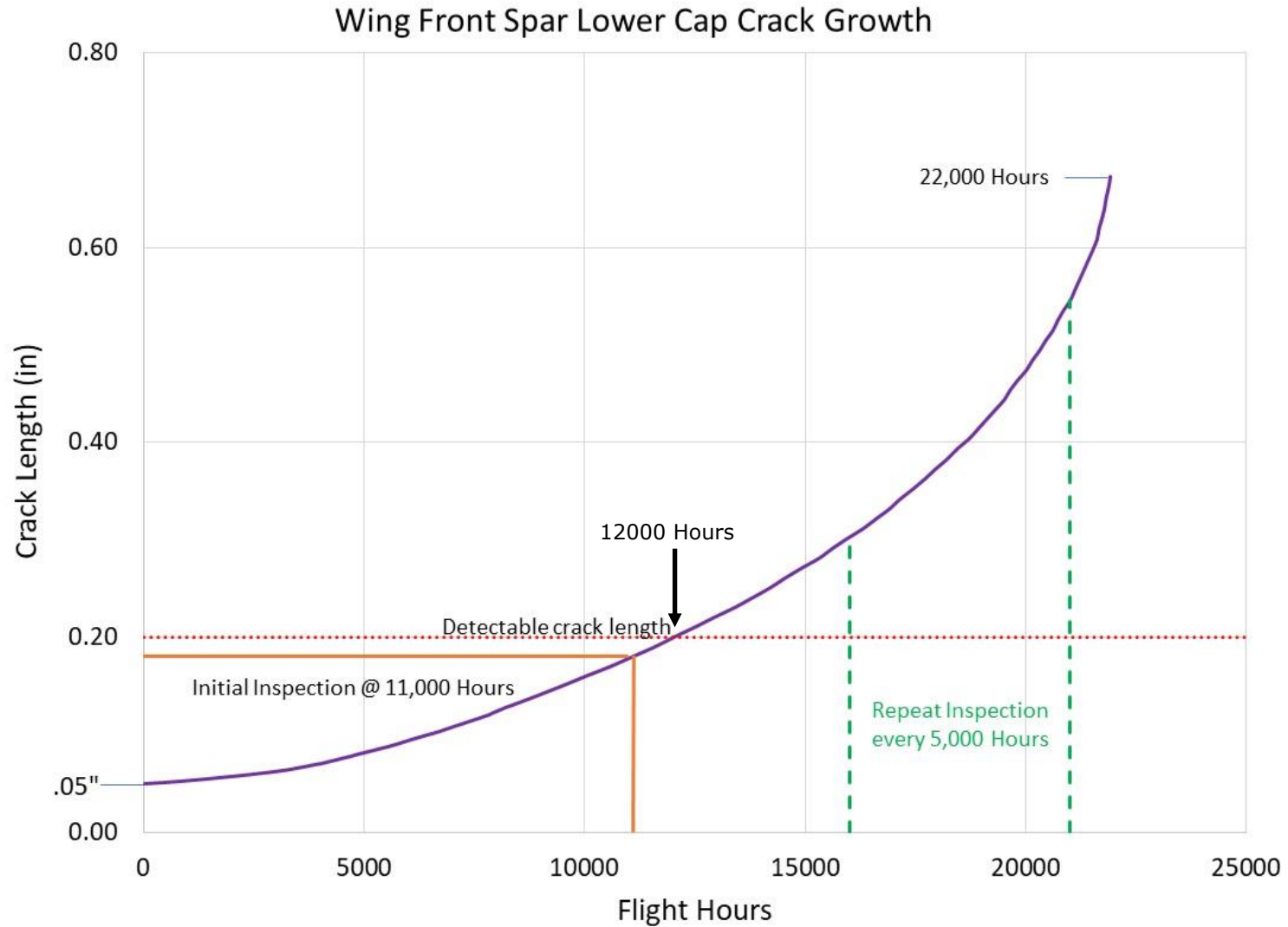
<u>Variable</u>	<u>Parameter</u>
Initial Crack Size	0.05 in
Fracture Toughness	37.0 ksi√in
Paris m	2.586
Paris c	1.29E-8
Walker exponent	.82
Ultimate Stress	69.0 ksi
Yield Stress	58.0 ksi
Hole Offset	0.90 in

AFGROW INPUT  
Mean only





# Classical Crack Growth





# HYPERGROW WITH AFGROW AND ADAPTIVE MULTIPLE IMPORTANCE SAMPLING

# Hypergrow and Adaptive Sampling



- Next, Hypergrow will be used for probabilistic analysis
- Hypergrow requires:
  - Probabilistic inputs for material properties
  - Probabilistic inputs for inspections
  - Beta solution generated by Afgrow and exported to .csv format
- Adaptive sampling used to evaluate POF with inspections
  - Adaptive sampling additional inputs:
    - Target coefficient of variation
    - Number of samples per iteration
- Example file provided: Capstone\_hypergrow.smdt

# Problem Overview



<u>Random Variable</u>	<u>Distribution</u>	<u>Parameters</u>
Initial Crack Size	Lognormal	Mean = 0.009055 in Standard Deviation = 0.001252 in
Fracture Toughness	Normal	Mean = 37.0 ksi√in Standard Deviation = 3.8 ksi√in
Paris m	Binormal	Mean = 2.586 Standard Deviation = 0.0
Paris c (log)	Binormal	Mean = -7.888 Standard Deviation = 0.04
Coefficient of Variance		0.0
Walker exponent		.82
Ultimate Stress	Normal	Mean = 69.0 ksi Standard Deviation = 0.0 ksi
Yield Stress	Normal	Mean = 58.0 ksi Standard Deviation = 0.0 ksi
Hole Offset	Normal	Mean = 0.9000 in Standard Deviation = 0.0 in

SMART INPUT  
Mean & StdDev

# SMART|DT – Project Information



SMART|DT

i  
 Information

⚙️ Analysis

🏗️ Material

📐 Geometry

📈 Loading

🔍 Inspections

▶️ Run

📊 Results

**Information**  
Provide information about the project.

**Project Summary** ⚙️

**NAME (REQUIRED)**

**DESCRIPTION**

**Aircraft Information** ✈️

**MAKE (OPTIONAL)**

**MODEL (OPTIONAL)**

**SERIAL NUMBER (OPTIONAL)**

**TYPE CERTIFICATE DATA SHEET - TCDS (OPTIONAL)**

This program was developed under sponsorship from the Federal Aviation Administration (grants 12-G-012 and 16-G-005) by the University of Texas at San Antonio (UTSA) and partners St. Mary's University, Textron Aviation, Nuss Sustainment Solutions, and Fieldstone Software. The responsible personnel are: Harry Millwater (PI - UTSA), Juan Ocampo (StMU), Beth Gamble (TA), Chris Hurst (TA), Marv Nuss (NSS), JR Lawhorne (Fieldstone), Nathan Crosby (UTSA PhD student), Daniel Ocampo (UTSA MS student), Sohrob Mattighi (Program Manager FAA), Mike Reyer (FAA Kansas City Office).

# SMART|DT – Analysis Information (Output Options)



SMART|DT

Information

**Analysis**

Material

Geometry

Loading

Inspections

Run

Results

**Analysis**

Output Options

Growth

Probabilistic

**Probability of Failure (POF)**

Flights	Maximum Flights Calculation	Flight Units
50	30000	Flights ▾

# SMART|DT – Analysis Information (Growth)



SMART|DT

Information

**Analysis**

Material

Geometry

Loading

Inspections

Run

Results

**Analysis**

Output Options

Growth

Probabilistic

**Model**

Crack Growth ▾

**Source**

HyperGROW ▾

**Crack Model**

Corner ▾

**HyperGROW**

**GEOMETRY FACTOR**

User Defined Beta Table ▾

**FILE**

betas\_FSpar\_CWS\_248- Browse

**WALKER EXPONENT**

0.82

**FAILURE CRITERIA**

Kc ▾

**Hours Per Flight**

1.65

15

# Afgrow Beta Solution File



! Afgrow Model 1030 / 2030 - 0.6619 Bearing Load transfer

! Width = 2.30 in

! Thk = 0.175 in

! H Dia = 0.164 in

! H Ofc = 0.90 in

!

c	beta
0.0050	3.59907
0.0100	3.20130
0.0150	2.89657
⋮	⋮
0.8100	3.86109
0.8150	6.21646
0.8200	27.93956

!

!beta c Crack Direction	0.0050	0.0125	0.0200	0.0275	0.0350	...	0.1475	0.1550	0.1625	0.1700	0.1750
0.0050	2.24485	3.19804	3.48930	3.62330	3.70123	...	3.99493	4.00368	4.01206	4.02010	4.02529
0.0125	1.00838	1.97282	2.45652	2.72711	2.89341	...	3.40218	3.41236	3.42193	3.43095	3.43671
0.0200	0.65282	1.21655	1.77304	2.07547	2.28183	...	2.97105	2.98348	2.99503	3.00581	3.01263
⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮	⋮	⋮	⋮
0.8000	0.02662	0.04209	0.05456	0.06696	0.08040	...	0.77079	0.90930	1.09491	1.36974	1.65756
0.8075	0.02630	0.04161	0.05398	0.06629	0.07965	...	0.80831	0.97114	1.20536	1.60190	2.12351
0.8150	0.02620	0.04147	0.05383	0.06615	0.07955	...	0.86325	1.06395	1.38532	2.08510	3.90826
0.8180	0.02616	0.04141	0.05377	0.06610	0.07952	...	0.88947	1.11091	1.48752	2.47098	2.17E+07

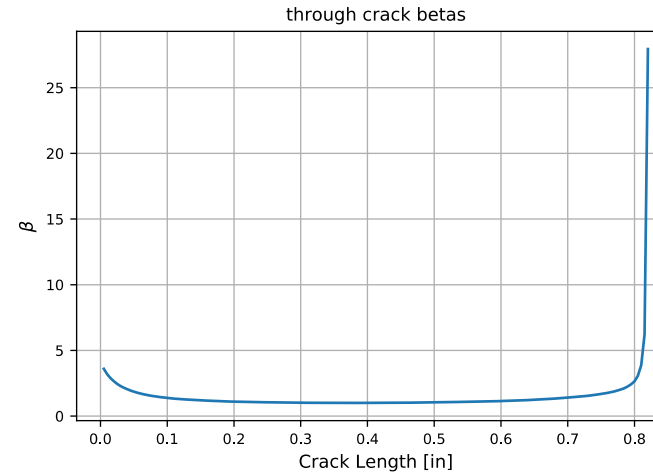
!

!beta a Crack Direction	0.0050	0.0125	0.0200	0.0275	0.0350	...	0.1475	0.1550	0.1625	0.1700	0.1750
0.0050	2.38094	1.40933	1.01682	0.81909	0.70182	...	0.32664	0.31896	0.31184	0.30520	0.30101
0.0125	2.85559	2.26857	1.78653	1.46485	1.24436	...	0.49365	0.48046	0.46834	0.45714	0.45014
0.0200	3.07986	2.35492	2.15685	1.84767	1.60820	...	0.61072	0.59219	0.57526	0.55972	0.55005
⋮	⋮	⋮	⋮	⋮	⋮	...	⋮	⋮	⋮	⋮	⋮
0.8075	1.15054	1.14212	1.15544	1.18763	1.23595	...	3.66367	4.14637	4.85539	6.09787	7.79295
0.8150	1.14916	1.14142	1.15570	1.18912	1.23898	...	3.94530	4.58217	5.63174	8.01747	14.50856
0.8180	1.14861	1.14116	1.15584	1.18977	1.24025	...	4.07878	4.80130	6.07016	9.54257	8.14E+07

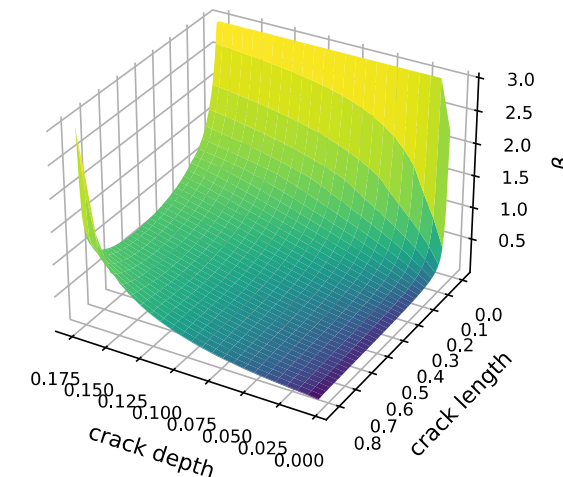
!

Thru crack  
Betas

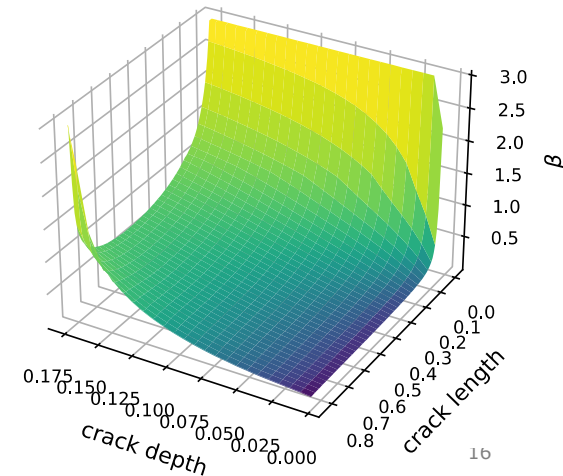
Part-thru crack  
Betas



c-direction (along surface) betas



a-direction (through thickness) betas





# SMART|DT – Analysis Information (Probabilistic)



**SMART|DT**

Information Analysis Material Geometry Loading Inspections Run Results

**Analysis**

Output Options  
Growth  
Probabilistic

**Method**

Adaptive Importance Sampling  
Monte Carlo  
Adaptive Importance Sampling

0.1

**Random Seed**

2394

**Samples Per Iteration**

100

**Maximum Iterations**

100

# SMART|DT – Material Information



SMART|DT

Information

Analysis

Material

Geometry

Loading

Inspections

Run

Results

Category	Group	Treatment	Form, Orientation	Summary
Custom	2014 Series	7475-T7351	Plate L	Length: Inches Stress: KSI  Category: Aluminum Group: 7475 Series Treatment: 7475-T7351 Form, Orientation: Plate TL
Aluminum	2024 Series	7475-T761	Plate LT	
Steel	2124 Series	7475-T7651	Plate TL	
Titanium	2224 Series			
	7050 Series			
	7075 Series			
	7150 Series			
	7175 Series			
	7475 Series			

**Published data for some of the material properties is unavailable.** Material property data is available for Fracture Toughness, Yield Strength and Ultimate Strength. User specified values for **Paris Constant** and **Paris Exponent** inputs are needed.

**FRACTURE TOUGHNESS**

*T = 1.3-4.0*

**DISTRIBUTION**

Normal

**MEAN**

**STDEV**

**YIELD STRENGTH**

**DISTRIBUTION**

Deterministic

**VALUE**

**ULTIMATE STRENGTH**

**DISTRIBUTION**

Deterministic

**VALUE**

**PARIS CONSTANT Log(C)**

**DISTRIBUTION**

Normal

**MEAN**

**STDEV**

**PARIS EXPONENT**

**DISTRIBUTION**

Deterministic

**VALUE**

18

# SMART|DT – Geometry Information



SMART|DT

Information

Analysis

Material

Geometry

Loading

Inspections

Run

Results

### Equivalent Initial Flaw Size (EIFS)

Category	Group	Data Set	Summary
Custom			
Commercial Transport			
Military Fighter			
Military Transport			

### Initial Crack Size Distribution

**DISTRIBUTION**

LogNormal

**MEAN**       **STDEV**

**Aspect Ratio**

**DISTRIBUTION**

Deterministic

**VALUE**

The EIFS is traditionally determined through the process of growing in-service or tear-down cracks backwards to time zero. As such, the results are dependent upon the aircraft location, assumed material parameters, and loading history. As a result, it is not recommended to use an EIFS distribution for a different application than for which it was derived. The EIFS values are provided here as a guide and care should be taken to select the distribution that best matches the aircraft mission, joint geometry and manufacturing methods, or ensure that the distribution is appropriately conservative.

# SMART|DT – Loading Information



**SMART|DT**

Information Analysis Material Geometry **Loading** Inspections Run Results

**Extreme Value Distribution (EVD) Method**  
Ultimate Load

**Limit / Ultimate Load**  
19.5

**Constant Amplitude Loading**

Maximum Stress	Cycles Per Flight
3.9985	55.5374

# Analysis Inspection Parameters



<u>Inspection Parameter</u>	<u>Value</u>
Number of Inspection Types	One- Single Repair
Inspection Type	Sliding Probe – Eddy Current
Inspection Schedule	14,000 Flights (23,100 Hr) Initial 600 Flights (1,000 Hr) Repeat
Probability of Inspection	90%
Probability of Detection	Deterministic
Detectable Crack Size	.20 inch
Repair Crack POD	.20 inch
Repair Crack Size	Mean = 0.009055 in Standard Deviation = 0.001252 in

# SMART|DT – Inspection Information



SMART|DT Capstone.smdt

File Help

**SMART|DT**

Information Analysis Material Geometry Loading **Inspections** Run Results

**Inspection Presets**

Name

Inspection 1

Delete

**Inspections**

Flights

Delete

Add Inspection Preset

**Add Inspection Preset**

Name

Inspection 1

Material	Inspection Type	Geometry	Equipment	Summary
Custom				
Aluminum				

**Probability of Detection**

DISTRIBUTION: Deterministic\_

LENGTH (c): 0.2      DEPTH (a): 0.0

**Probability of Inspection**

VALUE: 0.9

**Repaired Crack Size**

Same as Original

Custom

Perfect

DISTRIBUTION: LogNormal

MEAN: 0.009055      STANDARD DEVIATION: 0.001252

Cancel Add

Delete Edit Add

# SMART|DT – Inspection Information



SMART|DT Capstone.smdt

File Help

**SMART|DT**

Information Analysis Material Geometry Loading **Inspections** Run Results

### Inspection Presets

Name	Type	Inspection Prob.	Detection Prob.	Repaired Crack
Inspection 1		0.9	2c: 0.2 a: 0.0	* $\mu$ 0.009055 $\sigma$ 0.001252 LN

Delete Edit Add

### Inspections

Flights	Preset	Type	Inspection Prob.	Detection Prob.	Repaired Crack
14000	Inspection 1		0.9	2c: 0.2 a: 0.0	* $\mu$ 0.009055 $\sigma$ 0.001252

Add Inspections

**Add Inspections**

**Preset**  
Inspection 1

**Quantity**  
Multiple

**Minimum Flights** 14000 **Maximum Flights** 30300 **Frequency (Flights)** 600

Delete Cancel Add

# SMART|DT – Run



SMART|DT

Information
 Analysis
 Material
 Geometry
 Loading
 Inspections
 Run
 Results

Ready to Start
Start Analysis
✕

**DAT File**

```

!-----
!   AIRCRAFT INFORMATION
!-----
TITLE = Capstone
AC_MAKE = Acme
AC_MODEL = Sky Runner
AC_SERIAL_NUM = All
AC_TCDS = TCDSSR1
!-----
!   METHOD
!-----
HOURS_PER_FLIGHT = 1.65
INTEGRATION_METHOD = AIS 2394
AIS_TARGET_COV = 0.1
AIS_NSAMPLES = 100
AIS_MAXITER = 100
DOE_MAX_INC = 20000.00
          
```

**Analysis Details**

Show/Export



# Hypergrow Input



```

! -----
!           AIRCRAFT INFORMATION
! -----
TITLE = Capstone
AC_MAKE = Acme
AC_MODEL = Sky Runner
AC_SERIAL_NUM = All
AC_TCDS = TCDSSR1
! -----
!           METHOD
! -----
HOURS_PER_FLIGHT = 1.65
INTEGRATION_METHOD = MC 1000000000 2394
POF_MAX_INC = 30300 50
ANALYSIS_TIME_UNITS = FLIGHTS
! -----
!           FRACTURE MECHANICS
! -----
CRACK_GROWTH_CODE = HYPERGROW
INITIAL_CRACK_SIZE = LOGNORMAL 0.009055 0.001252
FRACTURE_TOUGHNESS = NORMAL 37.0 3.8
PARIS_M_AND_LOGC = BINORMAL 2.856534 0.0 -7.8888 0.04 0.0
CRACK_ASPECT_RATIO = DETERMINISTIC 1.0
YIELD_STRENGTH = DETERMINISTIC 57.0

```



Probabilistic Variables

# Hypergrow Input



```

! -----
!                               HYPERGROW
! -----
FAILURE_CRITERIA = KC
WALKER_EXPONENT = 0.82
CRACKTYPE = CORNER
GEOMETRY = betas_FSpar_CWS_248-87.csv
EQUIVALENT_STRESS_CYCLESPERFLIGHT = 3.9985 55.5374
! -----
!                               INSPECTIONS
! -----
INSPECTIONS = 14000 14600 15200 15800 16400 17000 17600 18200 18800 19400
              20600 21200 21800 22400 23000 23600 24200 24800 25400 26000
              27200 27800 28400 29000 29600 30200
INSPECTION_TYPE = 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
! -----
!                               Inspection Parameters
INSPECTION_ID = 1
PROB_OF_INSPECTION = DETERMINISTIC 0.9
POD = DETERMINISTIC 0.2 0.0
REPAIR_CRACK_SIZE = LOGNORMAL 0.009055 0.001252
! -----
!                               LOADING AND EVD PARAMETERS
! -----
EVD_TYPE = LIMIT 19.5
! -----
!                               DESCRIPTION
! -----
! Front Spar Analysis

```

# SMART|DT – Run



SMART|DT

Information
 Analysis
 Material
 Geometry
 Loading
 Inspections
 Run
 Results

0% complete.
Start Analysis
✕

**DAT File**

```

!-----!
!   AIRCRAFT INFORMATION   !
!-----!
TITLE = Capstone
AC_MAKE = Acme
AC_MODEL = Sky Runner
AC_SERIAL_NUM = All
AC_TCDS = TCDSSR1
!-----!
!   METHOD                   !
!-----!
HOURS_PER_FLIGHT = 1.65
INTEGRATION_METHOD = AIS 2394
AIS_TARGET_COV = 0.1
AIS_NSAMPLES = 100
AIS_MAXITER = 100
DOE_MAX_INC = 20000.00
            
```

**Analysis Details**

```

40 % complete.
50 % complete.
60 % complete.
70 % complete.
80 % complete.
90 % complete.
100 % complete.

*****
***** PDTA analysis complete *****
*****

Total CPU time =    45.334 secs
Total wall time =     7.131 secs
            
```

Total Runtime: 7 seconds

Show/Export

# SMART|DT - Result



## SMART|DT

Information  
  Analysis  
  Material  
  Geometry  
  Loading  
  Inspections  
  Run  
  Results

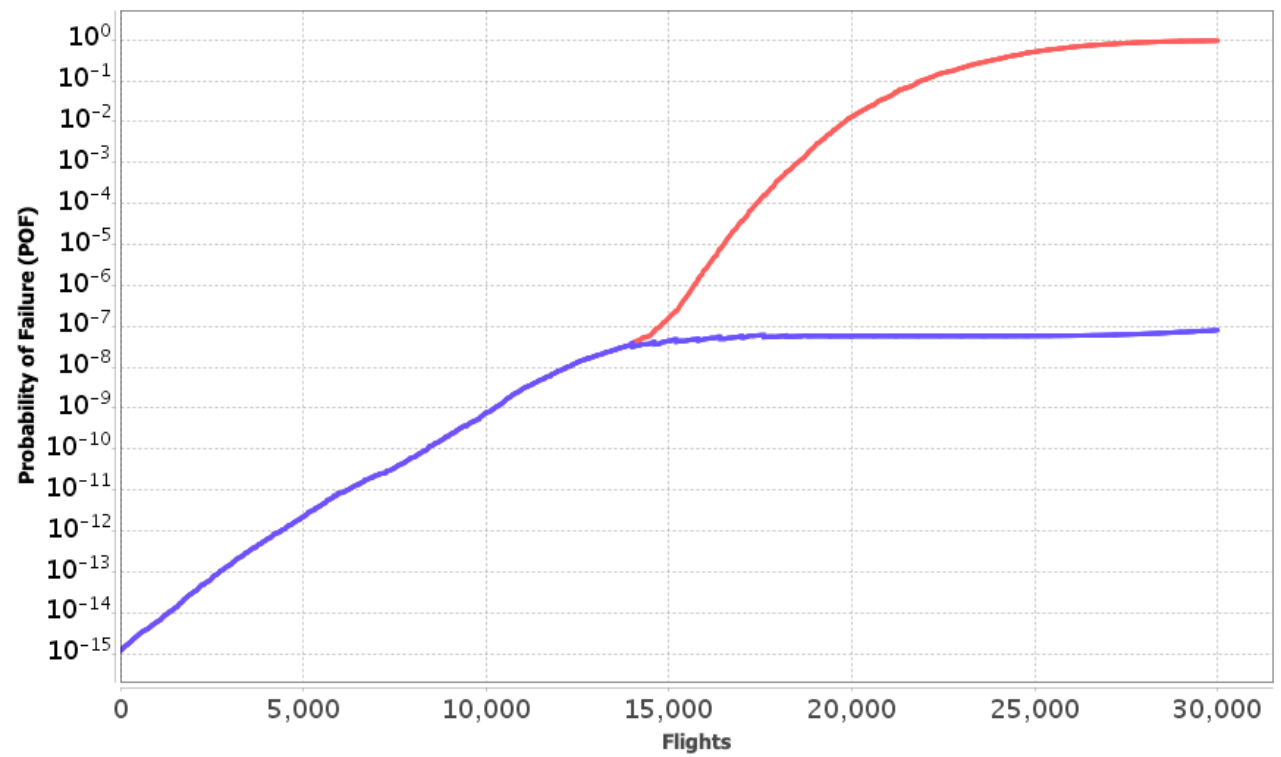
### Results

Probability of Failure

Load External POF

POF  
  Cumulative  
 Flights  
  Hours

Probability of Failure (POF) vs. Flights



Vertical Grid  
  Horizontal Grid

Initial Flaw Size  
 Lognormal  
 $\mu = .009055$   
 $\sigma = .001252$

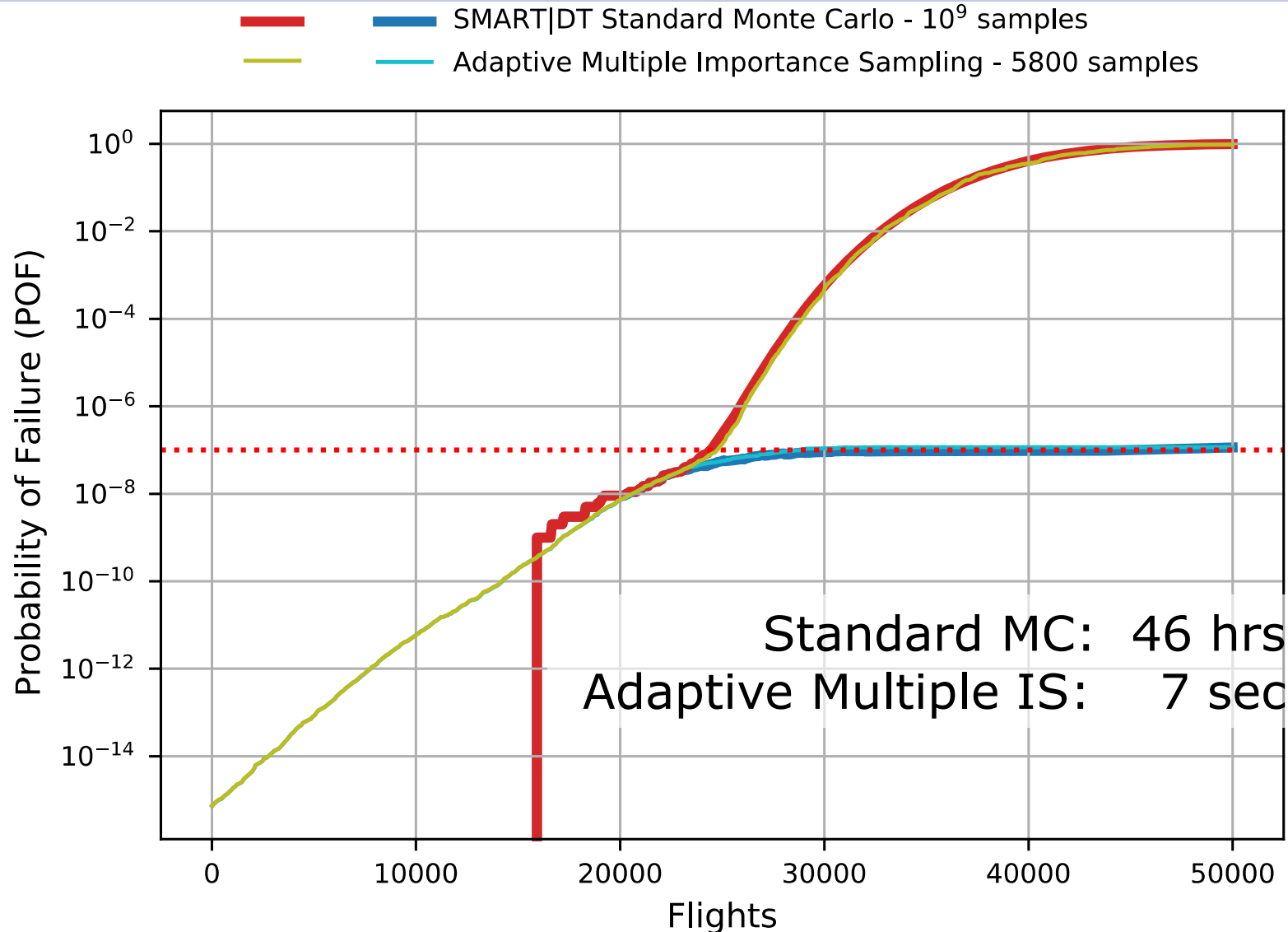
Fracture Toughness  
 $\mu = 37.0$   
 $\sigma = 3.8$

da/dN Paris m  
 $\mu = 2.586$   
 $\sigma = 0.0$

da/dN Log C  
 $\mu = -7.8888$   
 $\sigma = 0.04$

Initial Inspection  
 23,100 Hours  
 Repeat Inspection  
 1,000 Hours

# Compare AMIS with Standard Monte Carlo



Initial Flaw Size  
 Lognormal  
 $\mu = .009055$   
 $\sigma = .001252$

Fracture Toughness  
 $\mu = 37.0$   
 $\sigma = 3.8$

da/dN Paris m  
 $\mu = 2.586$   
 $\sigma = 0.0$

da/dN Log C  
 $\mu = -7.8888$   
 $\sigma = 0.04$

Initial Inspection  
 23,100 Hours  
 Repeat Inspection  
 1,000 Hours

# Questions

