Efficient Probabilistic Damage Tolerance Analysis Using Adaptive Multiple Importance Sampling

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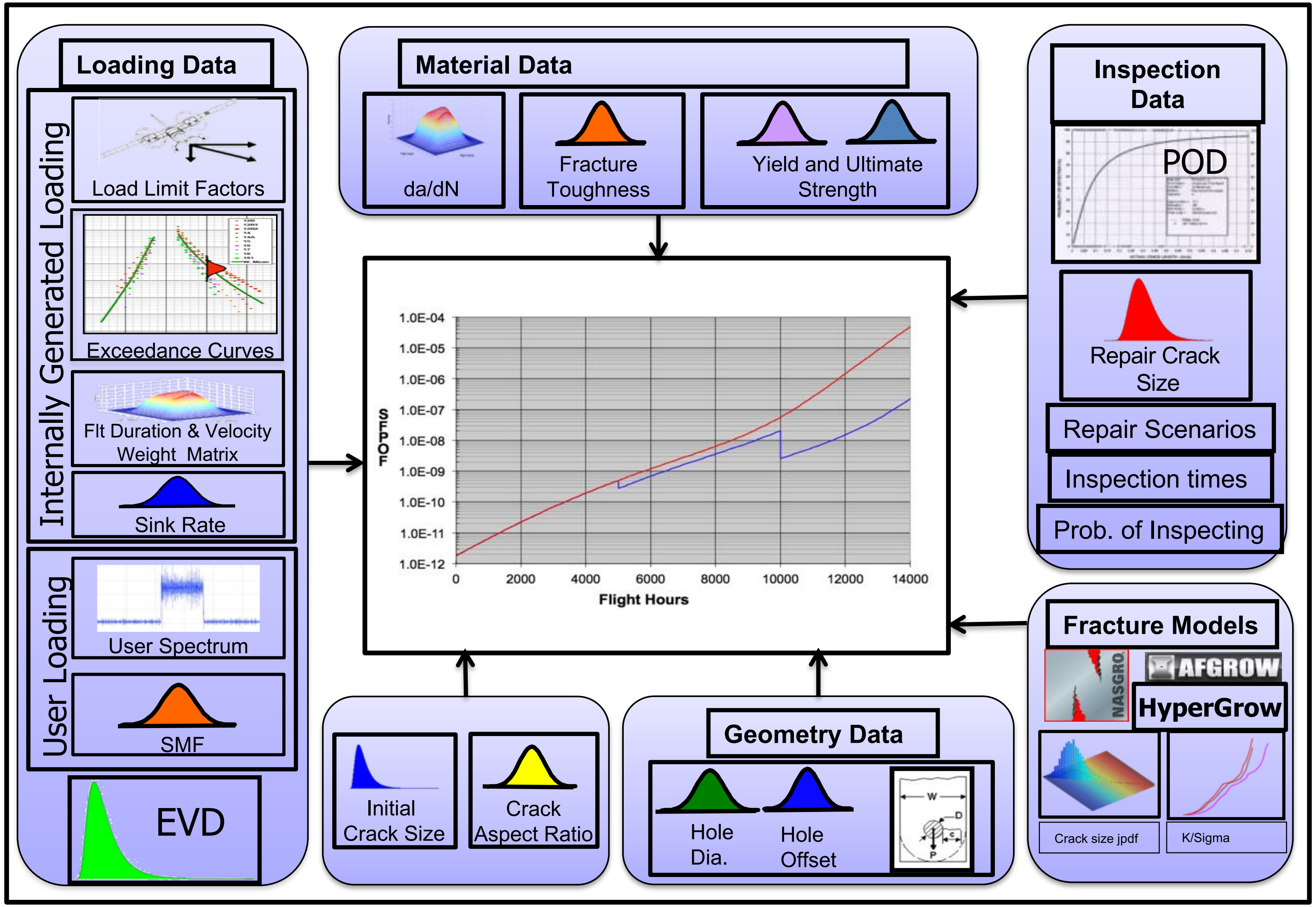
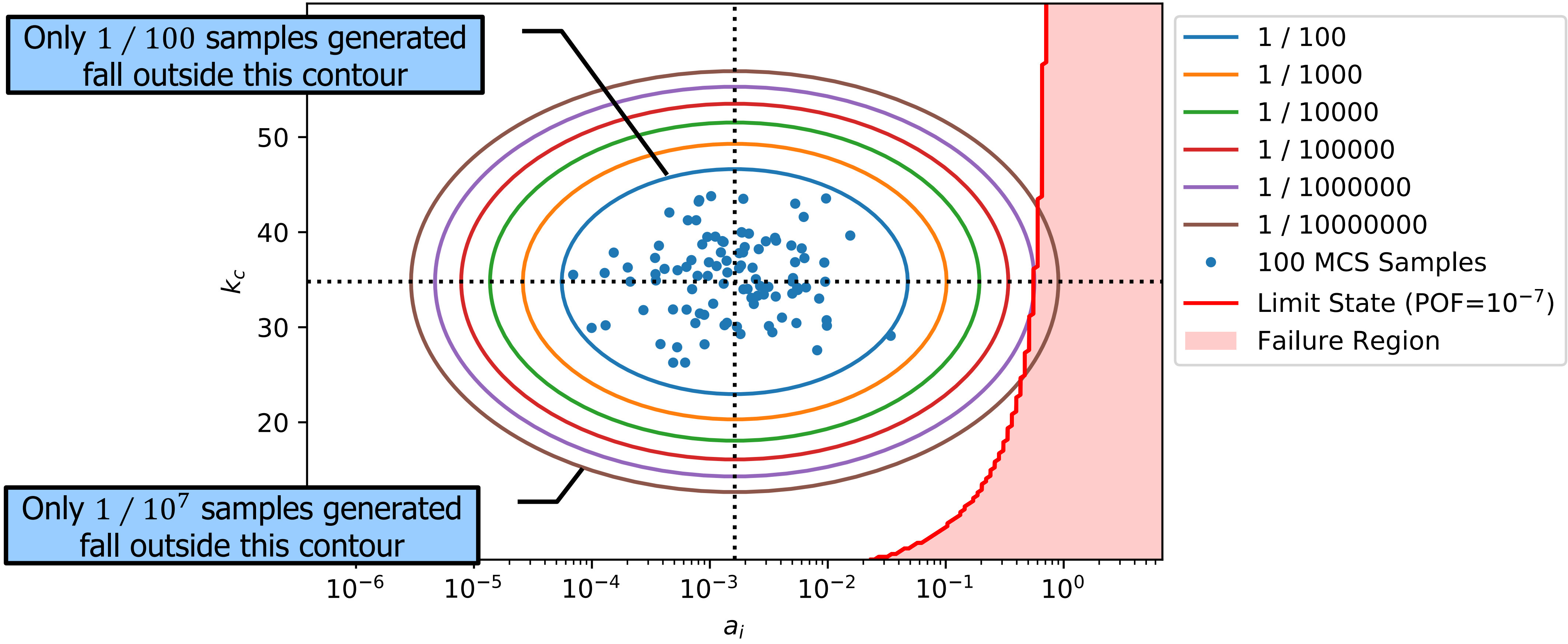
SMART|DT



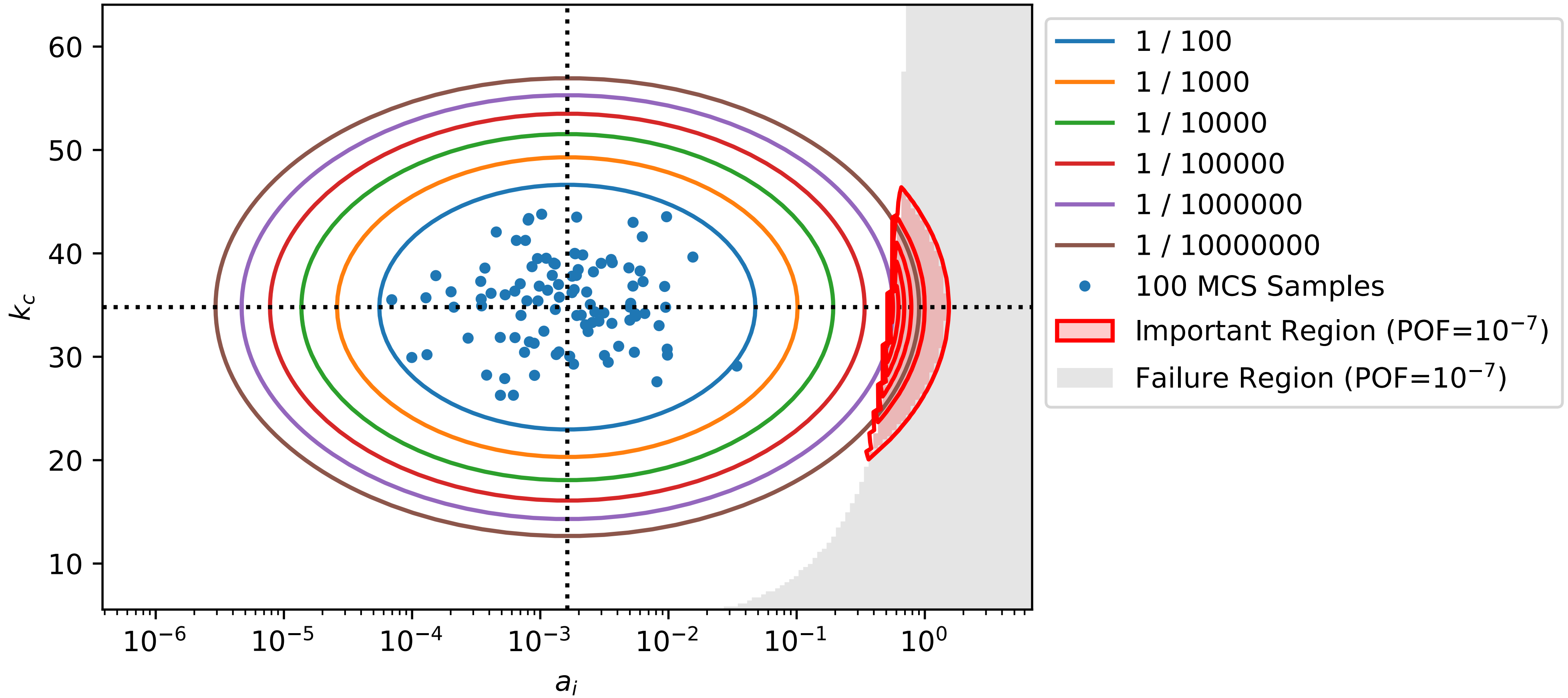
Loading Data
Internally Generated Loading
Load Limit Factors
Exceedance Curves
Flt Duration & Velocity Weight Matrix
Sink Rate
User Loading
User Spectrum
SMF
EVD
Material Data
da/dN
Fracture Toughness
Yield and Ultimate Strength
Inspection Data
POD
Repair Crack Size
Repair Scenarios
Inspection times
Prob. of Inspecting
SFPOF
1.0E-04
1.0E-05
1.0E-06
1.0E-07
1.0E-08
1.0E-09
1.0E-10
1.0E-11
1.0E-12
0
2000
4000
6000
8000
10000
12000
14000
Flight Hours
Fracture Models
NASGRO
AFGROW
HyperGrow
Crack size jpdf
K/Sigma
Initial Crack Size
Crack Aspect Ratio
Geometry Data
Hole Dia.
Hole Offset

Monte Carlo Sampling – Limit State In 2-Dimensions



- Standard Monte Carlo (SMC) is simple and robust, but inefficient for estimating rare event probabilities
- $N = \left(1 - \overline{P}\right) / \left(\overline{P}\ \delta_{\overline{P}}^2\right)$, so for $\overline{P} = 10^{-7}$ and $\delta_{\overline{P}} = 0.1$, $N = 10^9$

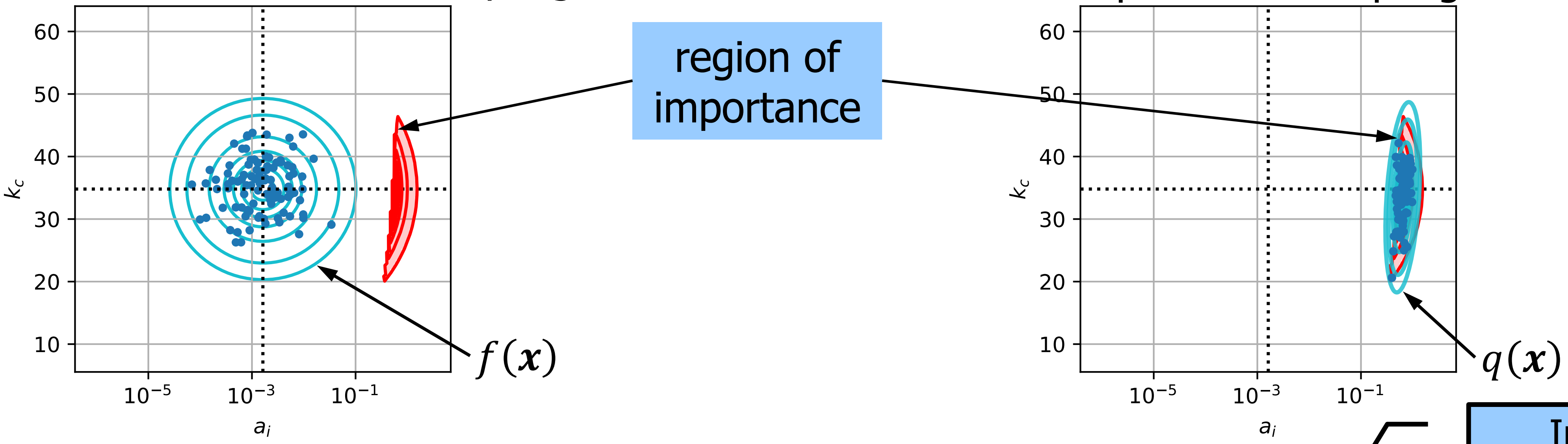
Optimal Sampling Region



- Only a small portion of the failure region is important for calculating POF
- The important region as drawn accounts for 99% of the POF

Importance Sampling

Standard Monte Carlo Sampling | Importance Sampling



Importance weight
$w(\boldsymbol{x}) = f(\boldsymbol{x}) \,/\, q(\boldsymbol{x})$

$$\mathbb{E}[H(\boldsymbol{x};t)] = \int H(\boldsymbol{x};t)\, f(\boldsymbol{x})\, \mathrm{d}\boldsymbol{x}$$

$$\mathbb{E}[H(\boldsymbol{x})] = \int H(\boldsymbol{x};t) \frac{f(\boldsymbol{x})}{q(\boldsymbol{x})}\, g(\boldsymbol{x})\, \mathrm{d}\boldsymbol{x}$$

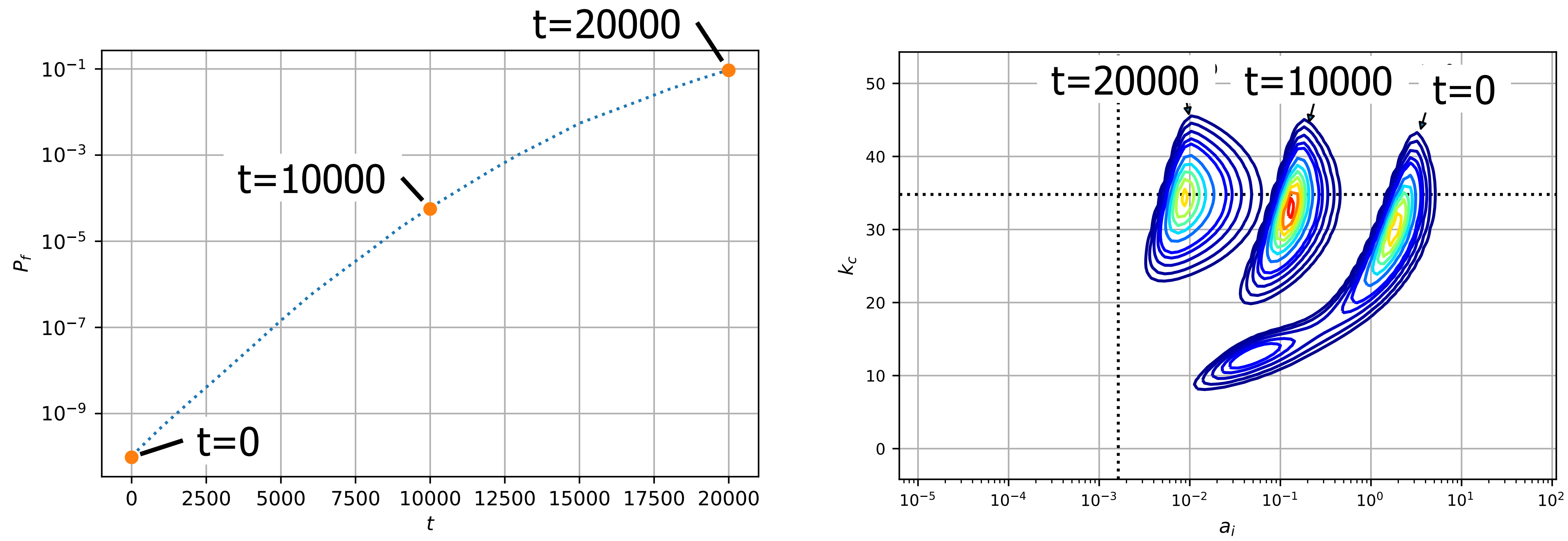
$$\hat{\mu} \approx \frac{1}{N} \sum_i H(\boldsymbol{x}_i;t)$$

$$\hat{\mu} \approx \frac{1}{N} \sum_i H(\boldsymbol{x}_i;t) \frac{f(\boldsymbol{x}_i)}{q(\boldsymbol{x}_i)}$$

$$\mathrm{Var}(\hat{\mu}) \approx \frac{1}{N^2} \sum_i (H(\boldsymbol{x}_i;t) - \hat{\mu})^2$$

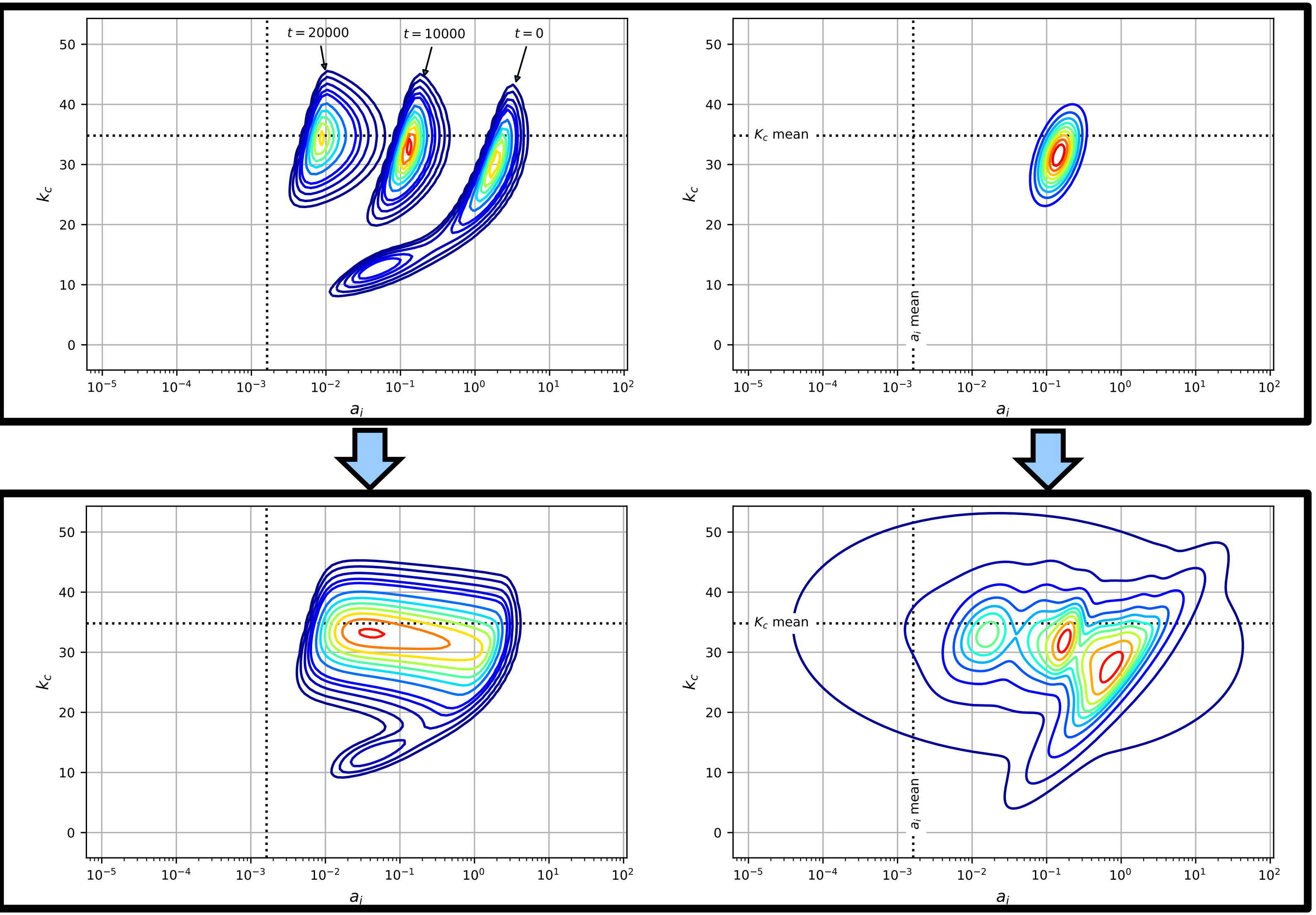
$$\mathrm{Var}(\hat{\mu}) \approx \frac{1}{N^2} \sum_i \left(H(\boldsymbol{x};t) \frac{f(\boldsymbol{x}_i)}{q(\boldsymbol{x}_i)} - \hat{\mu} \right)^2$$

Adaptive Importance Sampling for PDTA



- Adapt a sampling density to the important region for each evaluation time, t
  - Important region moves as t changes
  - Important region can be multimodal
- Adaptation process require several iterations to converge for each t using small sample sizes

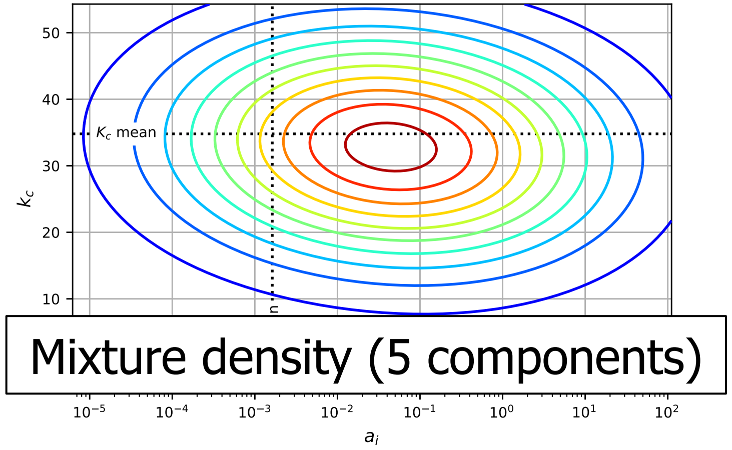
Multiple Importance Sampling Approach for PDTA



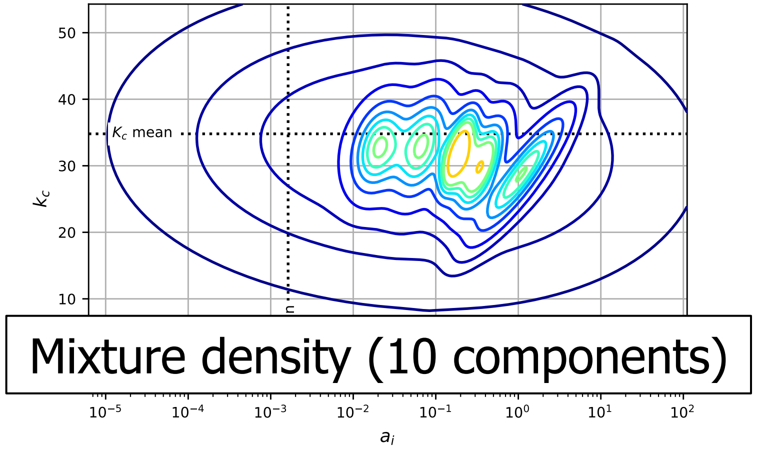
- Basic Importance sampling
  - Adapt single sampling densities for individual evaluation times

- Multiple Importance Sampling
  - Adapt a mixture density for a range of evaluation times

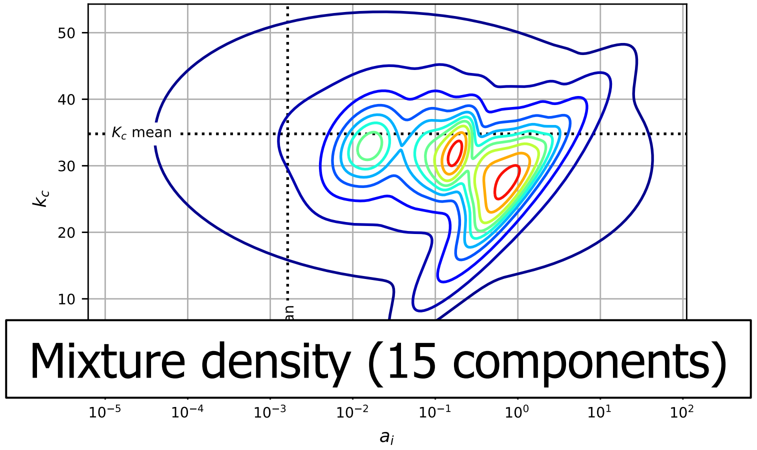
Adaptive Multiple Importance Sampling



Mixture density (5 components)

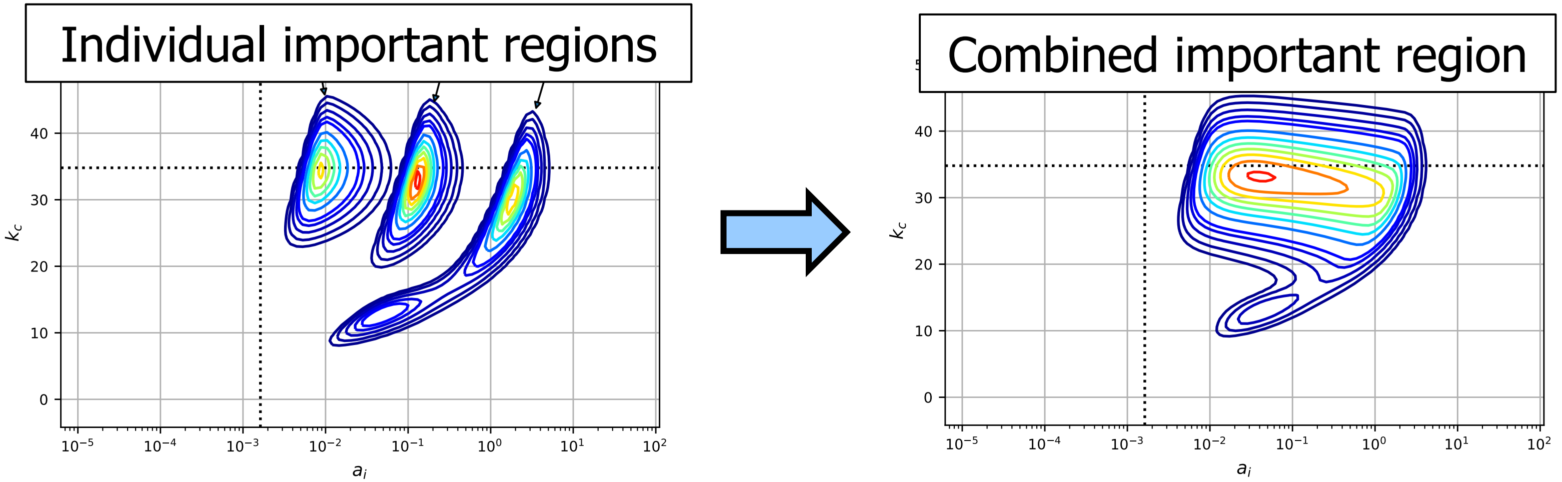


Mixture density (10 components)

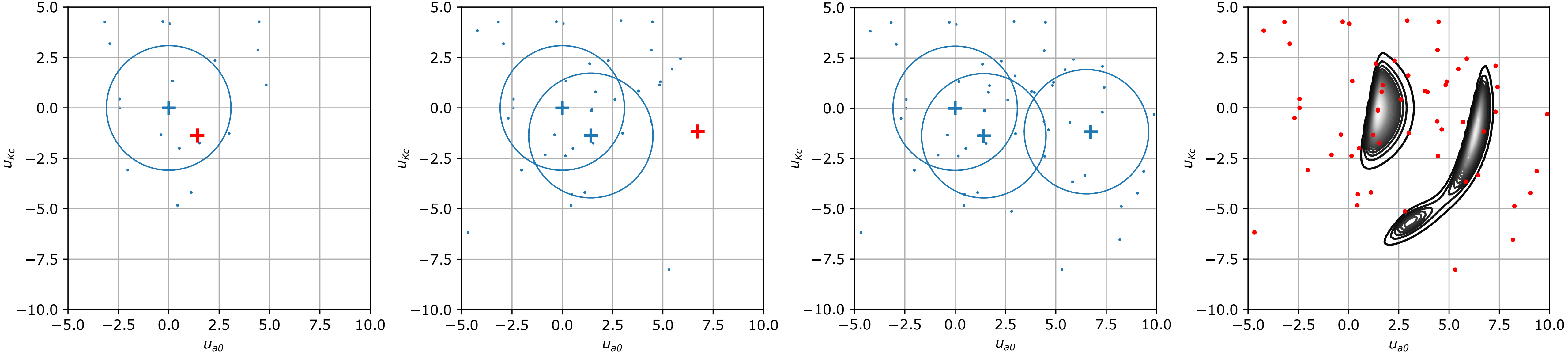


Mixture density (15 components)

- Mixture density adapted by add component densities where the POF estimate C.O.V. is highest
- Key advantage is that samples can be used for more than one important region where regions overlap

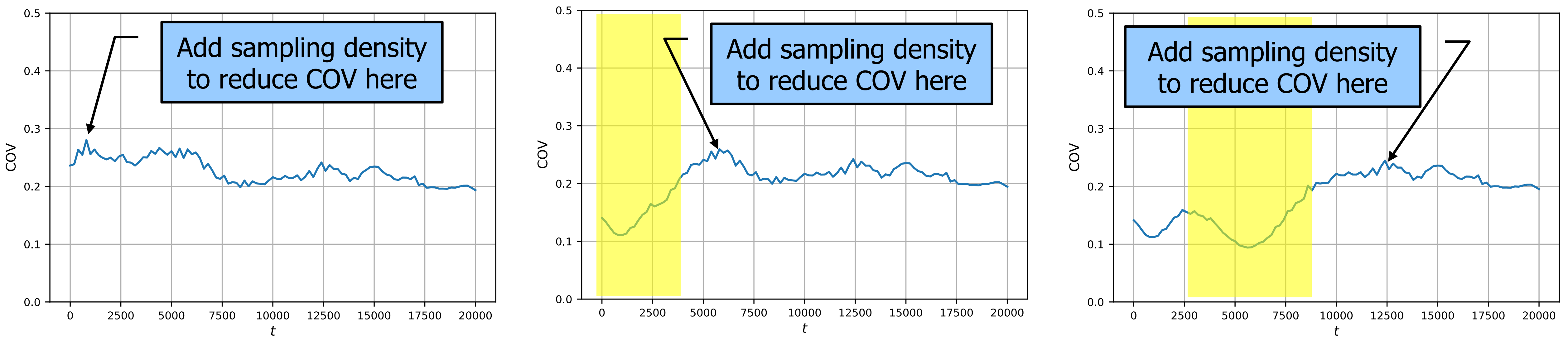


Initialization: Add Component Densities Near Important Regions for Each t



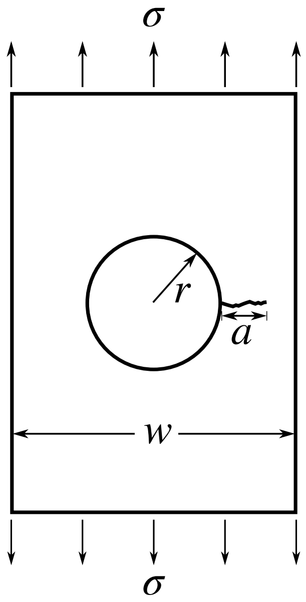
- Simplified case shows initialization with 2 important regions starting from a first component sampling density at the origin
- Performs exploration allowing the next stage, adaptation, to focus on reducing COV for each POF estimate

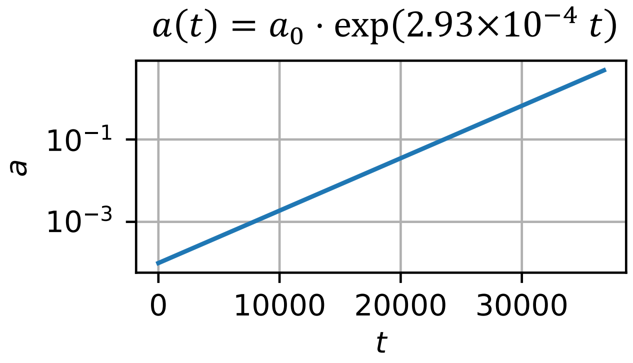
Adaptation: Add Component Density for t where COV is Highest

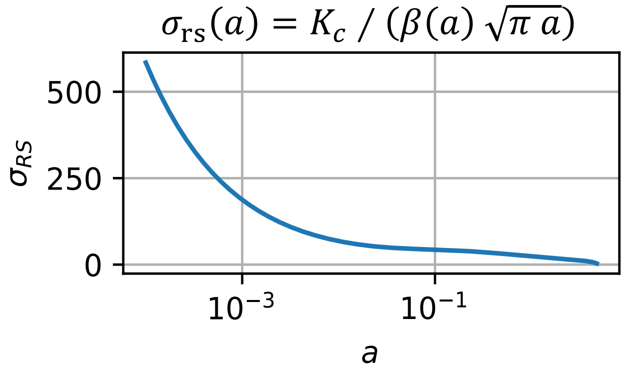


- Coefficient of Variation (COV) is a normalized error estimate
- Ensures COV across all evaluation times is below a user defined threshold

Risk Assessment Handbook Problem





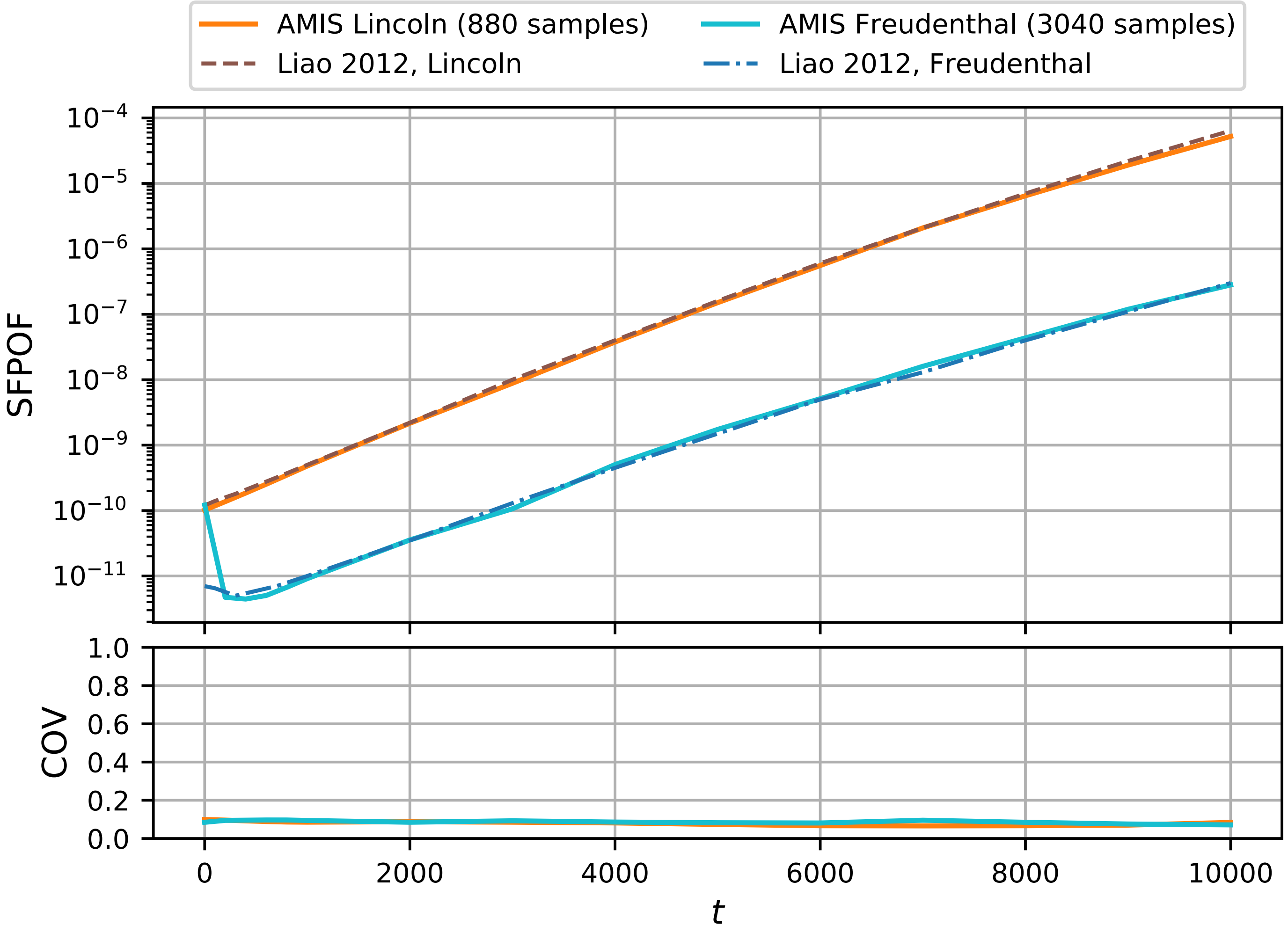


$$\beta(a) = \underbrace{\left(0.6762 + \frac{0.8734}{0.3254 + a \,/\, R}\right)}_{\beta_{\mathrm{hole}}} \cdot \underbrace{\sqrt{\sec\left(\frac{\pi\,(R+a)}{W}\right)}}_{\beta_{\mathrm{width}}}$$

Parameter	**Value**
Width	Deterministic 10 in
Radius	Deterministic 0.125 in
Initial Crack Size	$LN(0.0032, 0.0047)$ in
Fracture Toughness	$N(34.8, 3.90)$ ksi $\sqrt{\text{in}}$
Maximum Stress per Flight	$W(5.0, 10.0, 5.0)$ ksi

Tuegel et al., Aircraft structural reliability and risk analysis handbook volume 1: Basic analysis methods., Technical report, Air Force Research Lab, Wright-Patterson AFB, OH, Aerospace Systems Dir, 2013

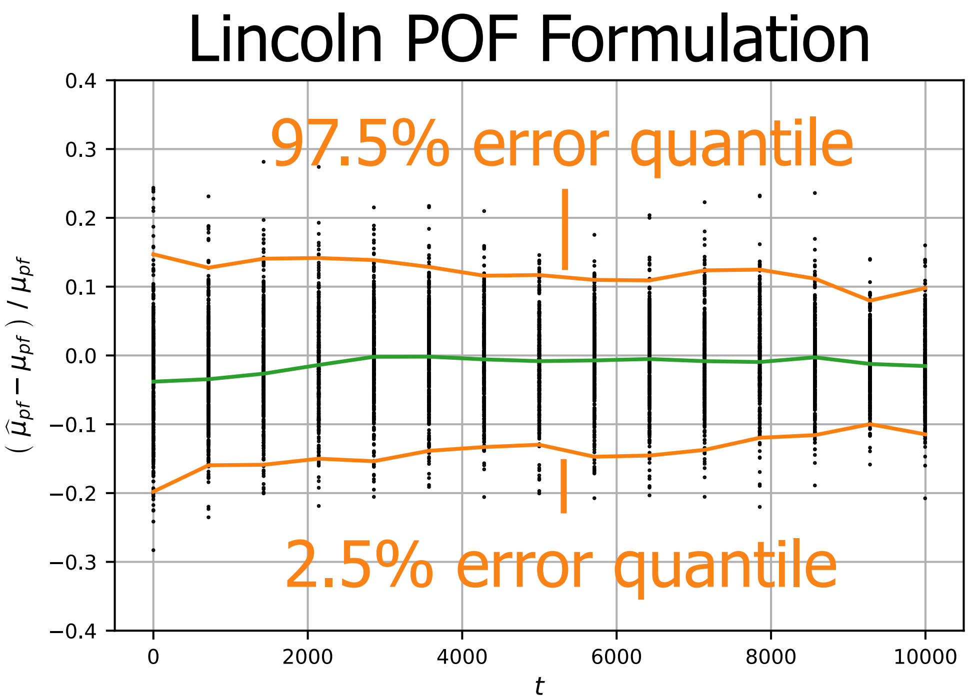
POF Results

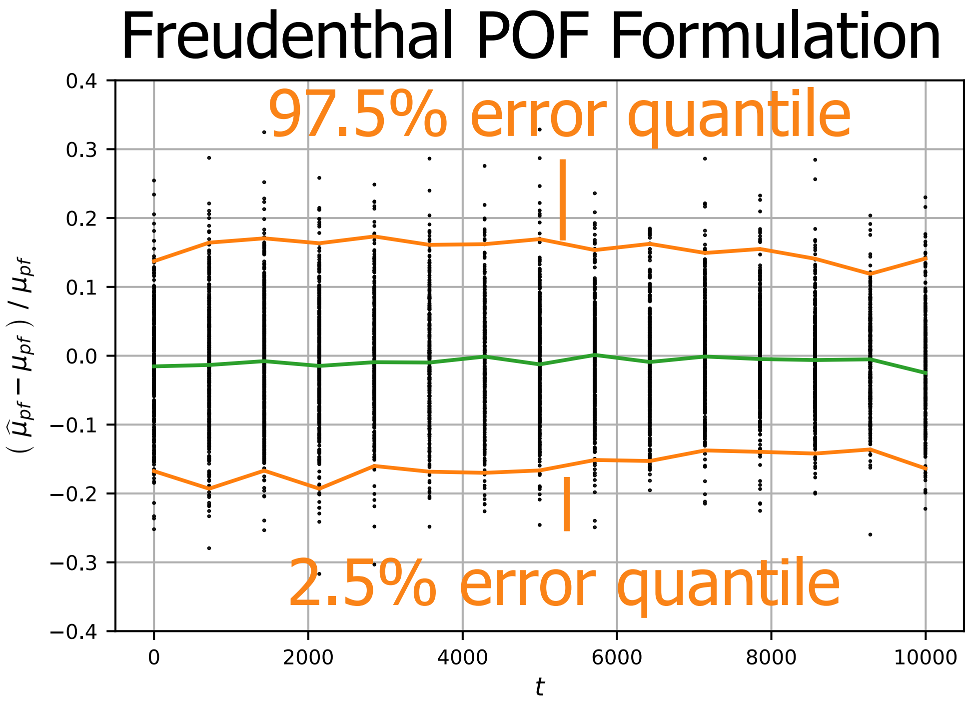


- 15 evaluation times
- COV threshold 0.1
- Lincoln Formulation
  - (assumes survival = 1 from flight 0 to flight t)
  - 80 samples per iteration
  - 11 iterations
  - 880 samples
- Freudenthal Formulation
  - (does not assume survival = 1 from flight 0 to flight t)
  - 160 samples per iteration
  - 19 iterations
  - 3040 samples

Liao M., Comparison of different single flight probability of failure (SFPOF) calculations for aircraft structural risk analysis. In Aircraft Airworthiness and Sustainment (AA&S) Conference, 2012

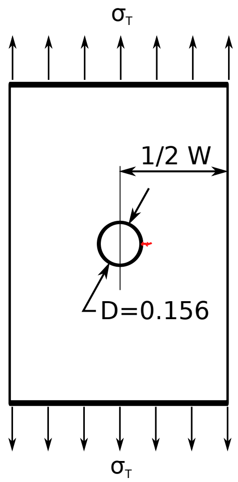
PDTA AMIS Accuracy of Error Estimates

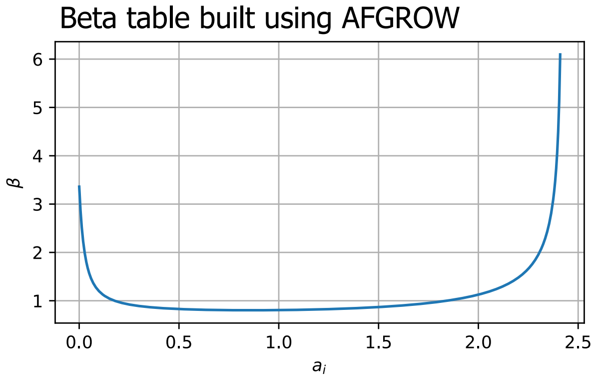


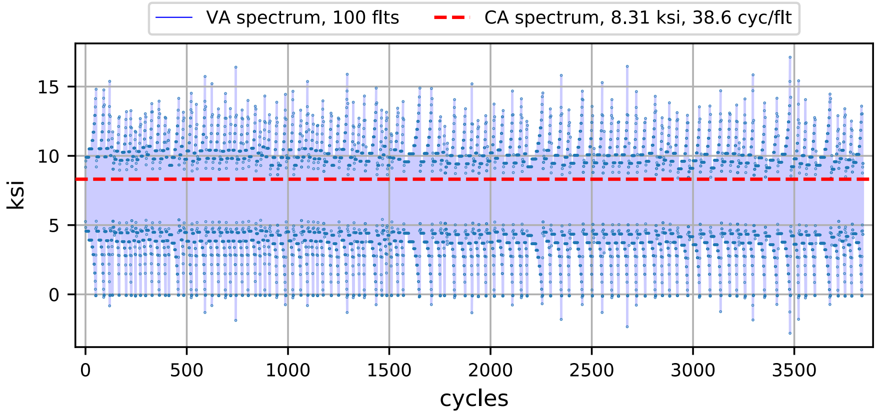


- Errors calculated for 400 PDTA AMIS runs
- From central limit theorem, $(\hat{\mu}_{pf} - \mu)/\mu = \pm 1.96\ COV = \pm 0.196$ for 95% confidence bounds and 0.1 COV.
- For both Lincoln and Freudenthal POF Formulations
  - PDTA AMIS estimates are within the expected error bands, showing the sampling variance gives a good indication of estimator error
  - PDTA AMIS median error is close to 0, showing the estimates are consistent

General Aviation Example Problem

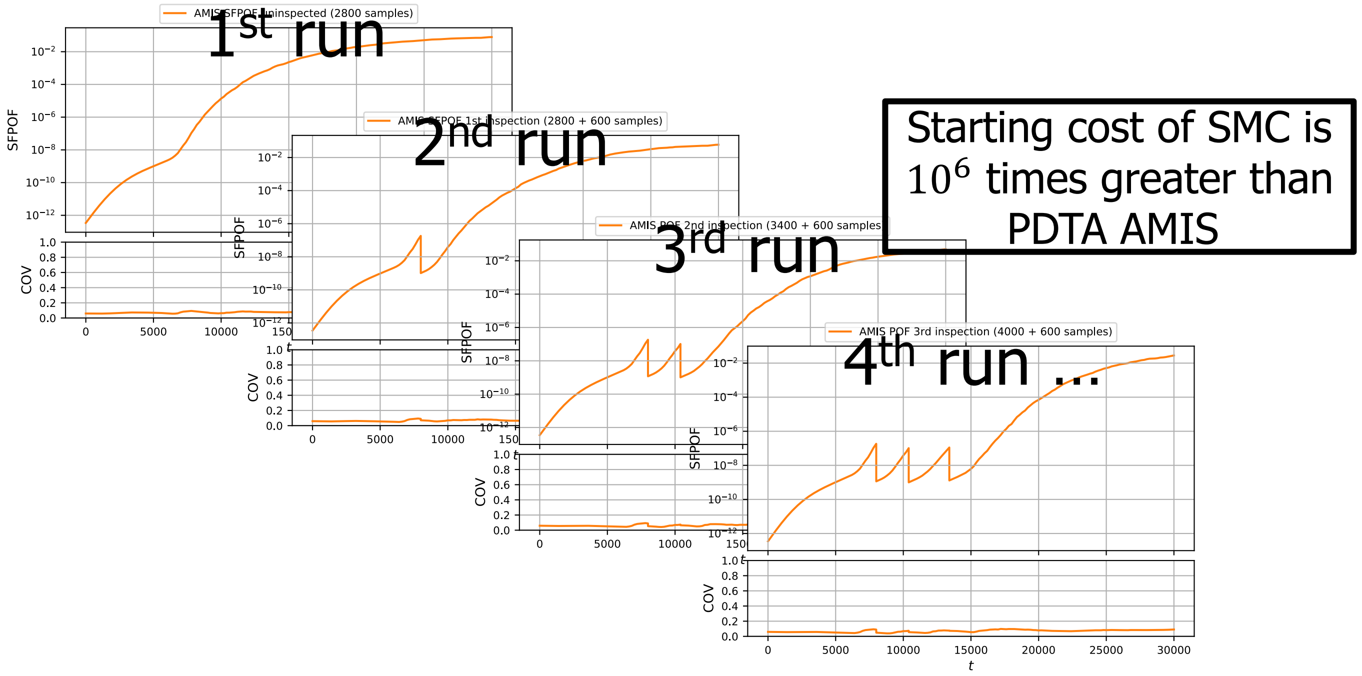


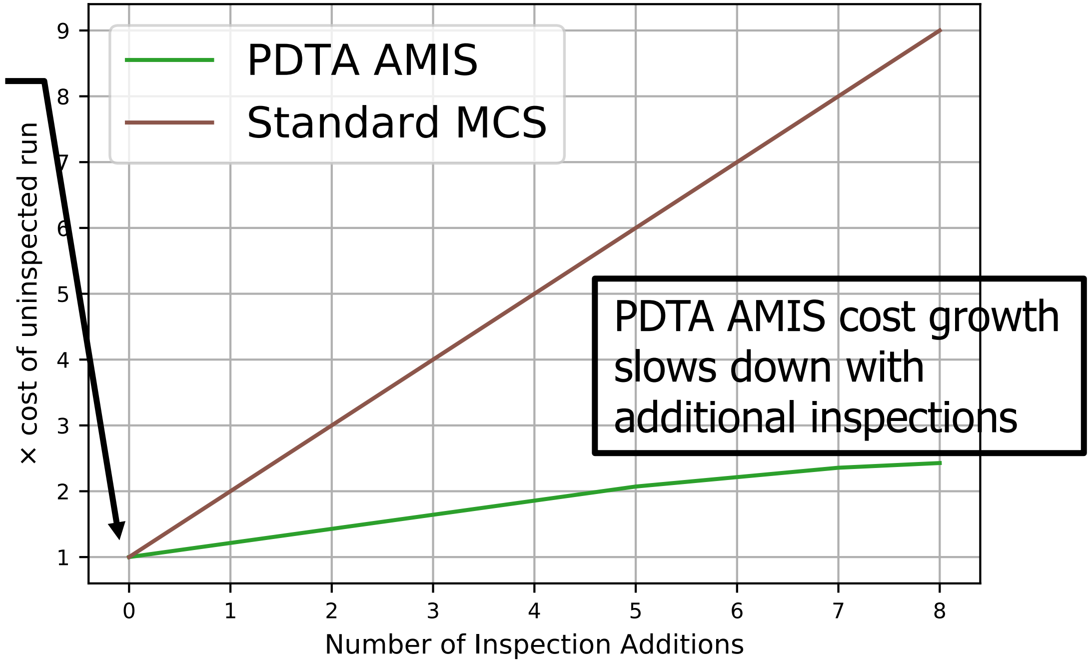




Parameter	**Values**
Width	Deterministic 5 in
Thickness	Deterministic 0.125 in
Log Paris Constant	$N(-9.0, 0.08)$
Paris Exponent	Deterministic 3.8
Initial Crack Size	$W(0.45, 4.17\times10^{-5})$ in
Fracture Toughness	$N(35.0, 3.5)$ ksi $\sqrt{\text{in}}$
Maximum Stress per Flight	$EVD(13.4, 1.3, 0.07)$ ksi
Probability of Detection	$LN(0.05, 0.065)$ in
Repair Quality (Crack Size)	Perfect

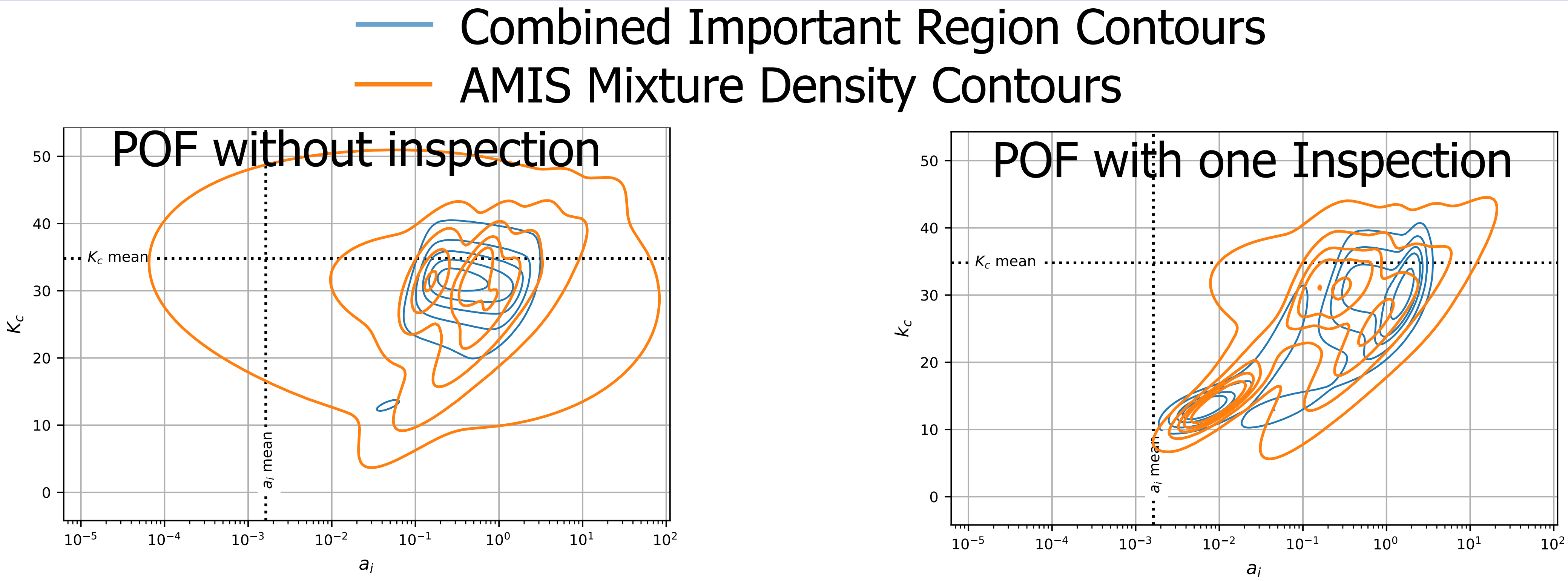
Adding Inspections One-at-a-Time





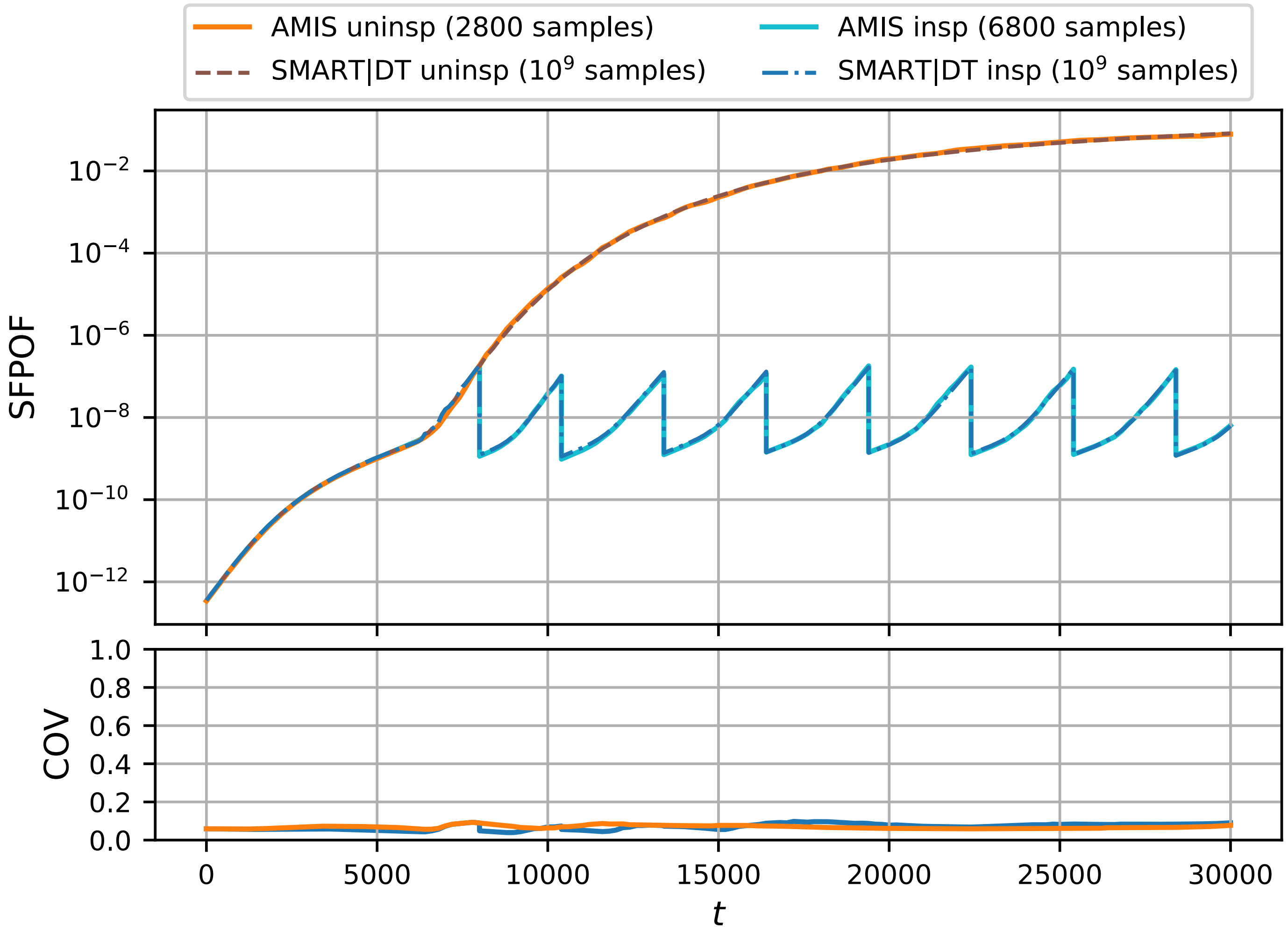
- PDTA AMIS only adds crack growth evaluations to adapt for the new inspection
  - POF for the existing crack growth results are re-evaluated with the new inspection schedule
- SMC must run a full analysis using 10^9 crack growth evaluations for each added inspection

PDTA AMIS Reuse



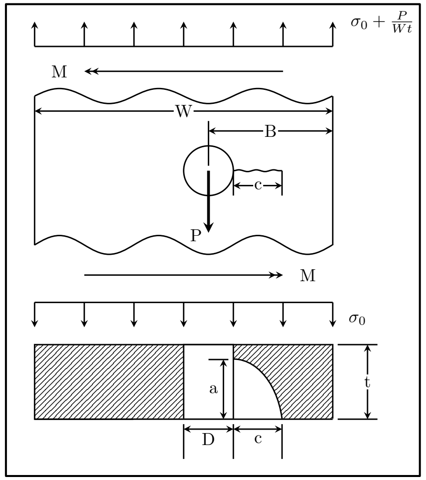
- Initial run (left plot) not including any inspections, completed with 880 samples
- POFs for the stored crack growth analyses were re-evaluated with the addition of an inspection
- After re-running the adaptation algorithm, the mixture density has been re-adapted using 320 additional samples to include the new important region near (0.01, 10)

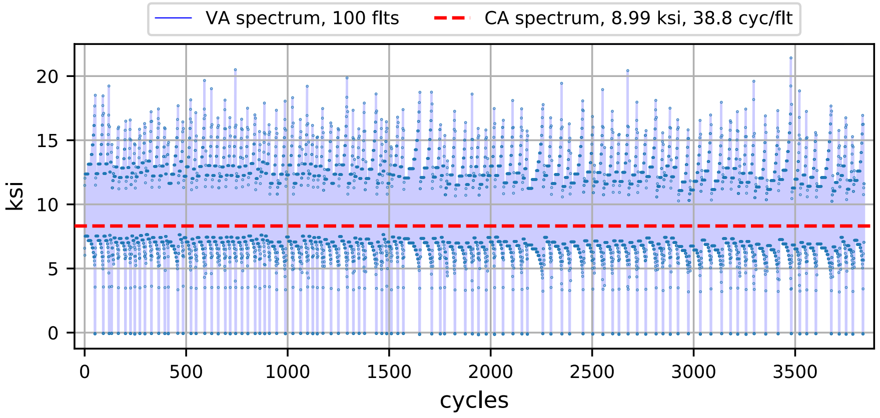
POF Results After Adding 8 Inspections



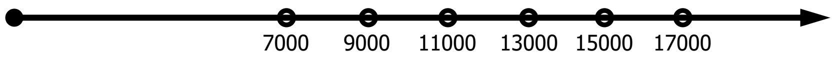
- PDTA AMIS
  - 2800 samples for uninspected POF
  - 6800 samples for inspected POF after adding 8 inspections one-at-a-time
- PDTA AMIS in excellent agreement with SMART|DT SMC using 10^9 samples

NASGRO Example with Inspections and Repairs





Inspection Schedule



Parameter	Value
Width	Deterministic 2.5 in
Thickness	Deterministic 0.25 in
Initial Crack Size	$LN(0.005, 0.002)$ in
Aspect Ratio (A/C) [1]	$N(1.5, 0.14)$
Fracture Toughness	$N(34.8, 3.90)$ ksi $\sqrt{\text{in}}$
Log Paris Constant	$N(-8.777, 0.08)$
Paris Exponent	Deterministic 3.273
Hole Diameter	Deterministic 0.1562 in
Hole Offset [2]	$N(0.5, 0.05)$ in
Maximum Stress per Flight	$EVD(16.74, 2.08, 0.0)$ ksi
Probability of Detection	$LN(0.021, 0.028)$ in
Repair Quality (crack size)	$LN(0.01, 0.004)$ in

[1] Random A/C values were clipped to Nasgro CC16 stress intensity factor limits

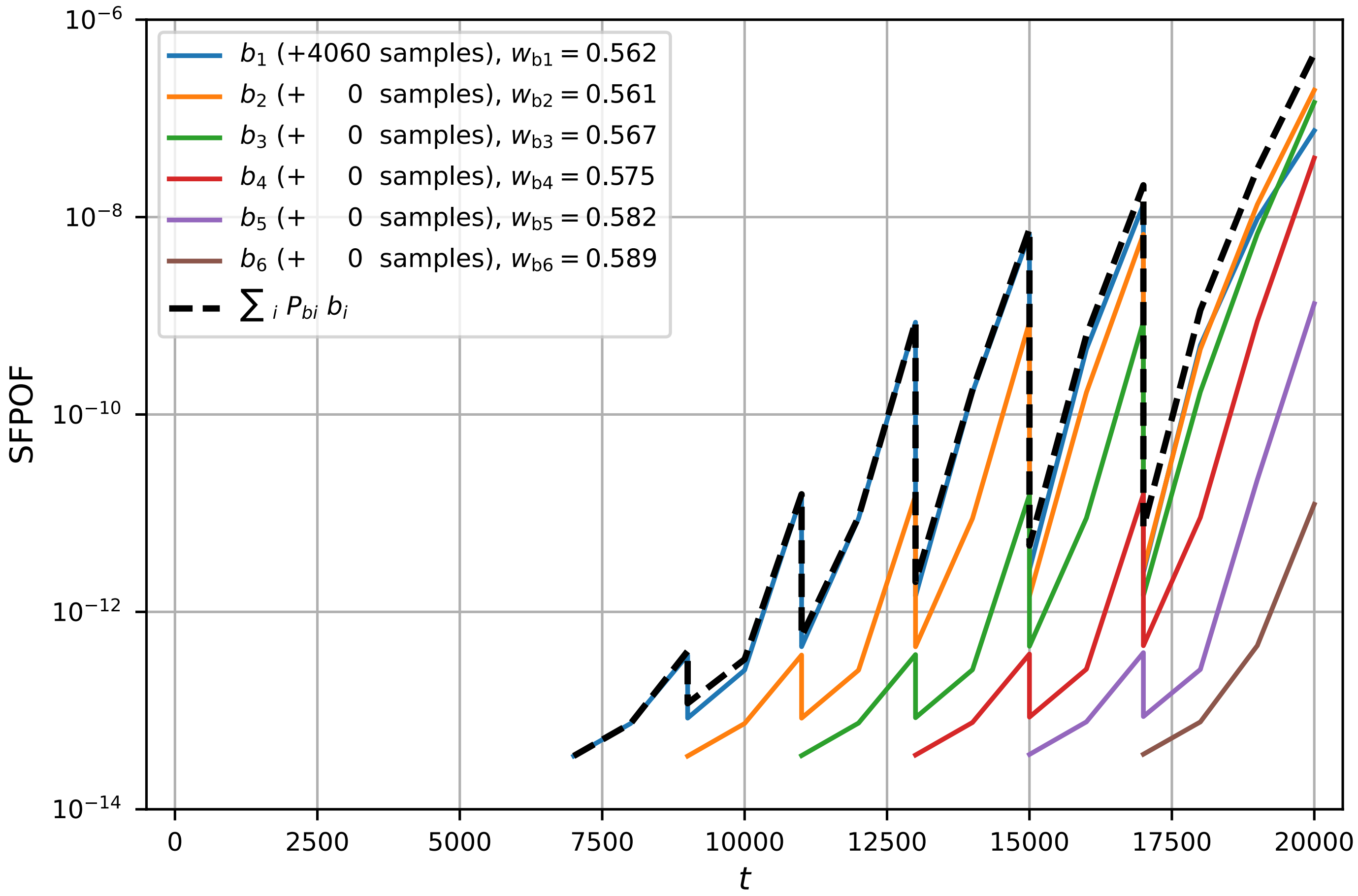
$$0.1 \leq A\,/\,C \leq 10$$

[2] Random Hole Offset values outside Nasgro CC16 stress intensity factor limit

$$\frac{D+C}{2\,B+C} \leq 0.7$$

were treated as immediate fracture

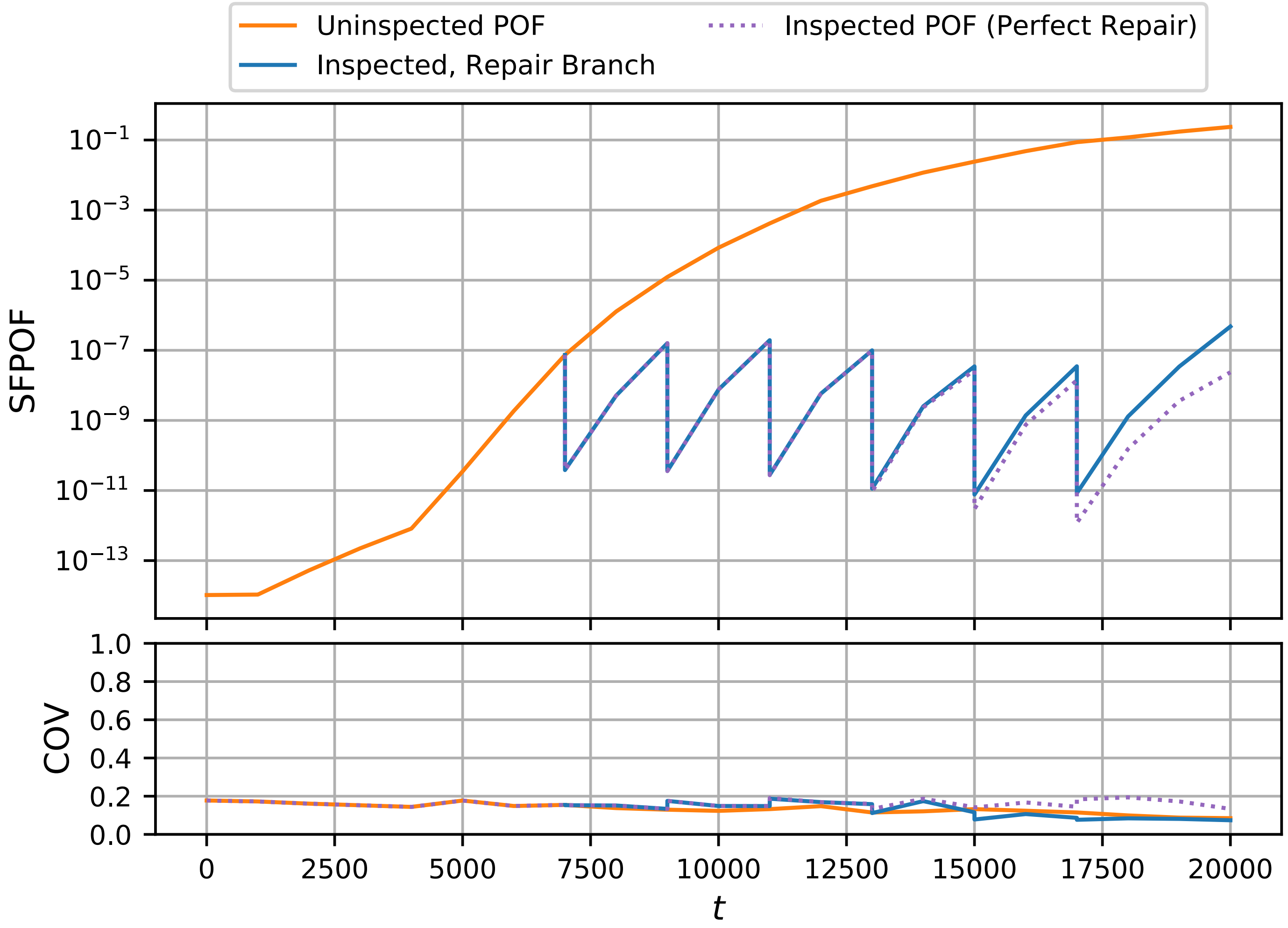
Repair Branch Analyses



	Percentage of Cracks Detected						
	7000	9000	11000	13000	15000	17000	w_{branch}
Trunk	0.562	0.268	0.106	0.035	0.010	0.003	**1.000**
B 1	-	0.521	0.301	0.135	0.047	0.013	**0.562**
B 2		-	0.521	0.301	0.135	0.047	**0.561**
B 3			-	0.521	0.301	0.135	**0.567**
B 4				-	0.521	0.301	**0.575**
B 5					-	0.521	**0.582**
B 6						-	**0.589**

- Branches are identical analyses except for the part from $t = 17000$ to $t = 20000$
- The PDTA AMIS algorithm is able to estimate POFs and PDETs for all branches from 1st branch samples

POF Results with Repairs



- PDTA AMIS
  - Main inspected POF: 4060 samples
  - Main uninspected POF: +0 samples
  - Main Percent Cracks Det: +140 samples
  - Repair POFs: 4060 samples
- 8260 total samples
- COV for the total POF including repairs decreases because the combined POF is increasing by an order of magnitude

Summary

- The PDTA AMIS algorithm estimates POF for PDTA using 6 orders of magnitude fewer samples compared to SMC for probabilities of 10^{-7} with COV of 0.1.

- The PDTA AMIS algorithm enables storing and reusing crack growth analyses for evaluation of multiple inspection schedules and evaluation of multiple repair branches.

- The PDTA AMIS algorithm accuracy was demonstrated by comparing analysis results with from an external source and with SMC using 10^{9} samples

Future Work

- Implementation in SMART|DT ongoing, release in late Fall 2021
- Explore multimodal adaptation methods for cases where the important region for a single evaluation time is multimodal.
- Additional work is needed for importance sampling to generate distribution-like outputs such as crack size at a given time.
- Investigate making the COV threshold dependent on the POF level to increase efficiency by reducing effort spent on insignificantly small POF values

Acknowledgments

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