# Probabilistic Fatigue and Damage Tolerance Analysis for General Aviation


Probabilistic fatigue and damage tolerance tool for the Federal Aviation Administration to perform risk analysis

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# Outline

- Smart Program Overview
- Smart|LD Capabilities
  - Methodology Review
  - Example Problem
- Smart|DT Capabilities
  - Methodology Review
    - Load Generation/EVD
    - Single Flight/Cumulative Total POF
    - Code Capabilities Flowchart
    - Crack Growth (Master Curve, Kriging, Software Direct Link)
    - Inspection and Repair
      - Recursive Probability Integration for Monte Carlo
      - Inspection and Repair for Numerical Integration
  - Example Problem
  - GUI
  - Current and Future Work

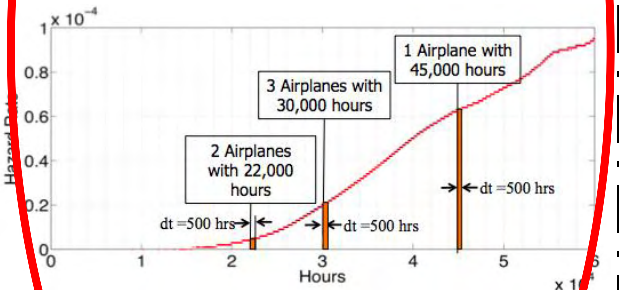
# Program Overview

2007-2011

Phase I

## Probabilistic Fatigue Analysis for Small Airplanes ($SMART_{LD}$)

Safe-life Approach

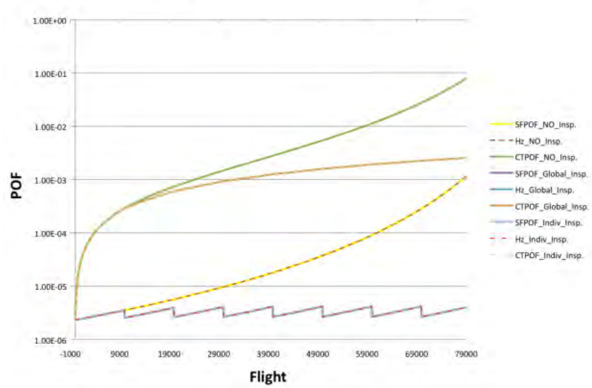



- Prob. Life distribution
- Hazard Rate
- Sensitivity Analysis

2009-2013

Phase II

## Probabilistic Damage Tolerance Analysis for Small Airplane ($SMART_{DT}$)




- SFPOF, Hz, CTPOF
- Inspection/Repair Effect
- Sensitivity Analysis

2012-2016

Phase III

## Probabilistic Fatigue Management Program for General Aviation

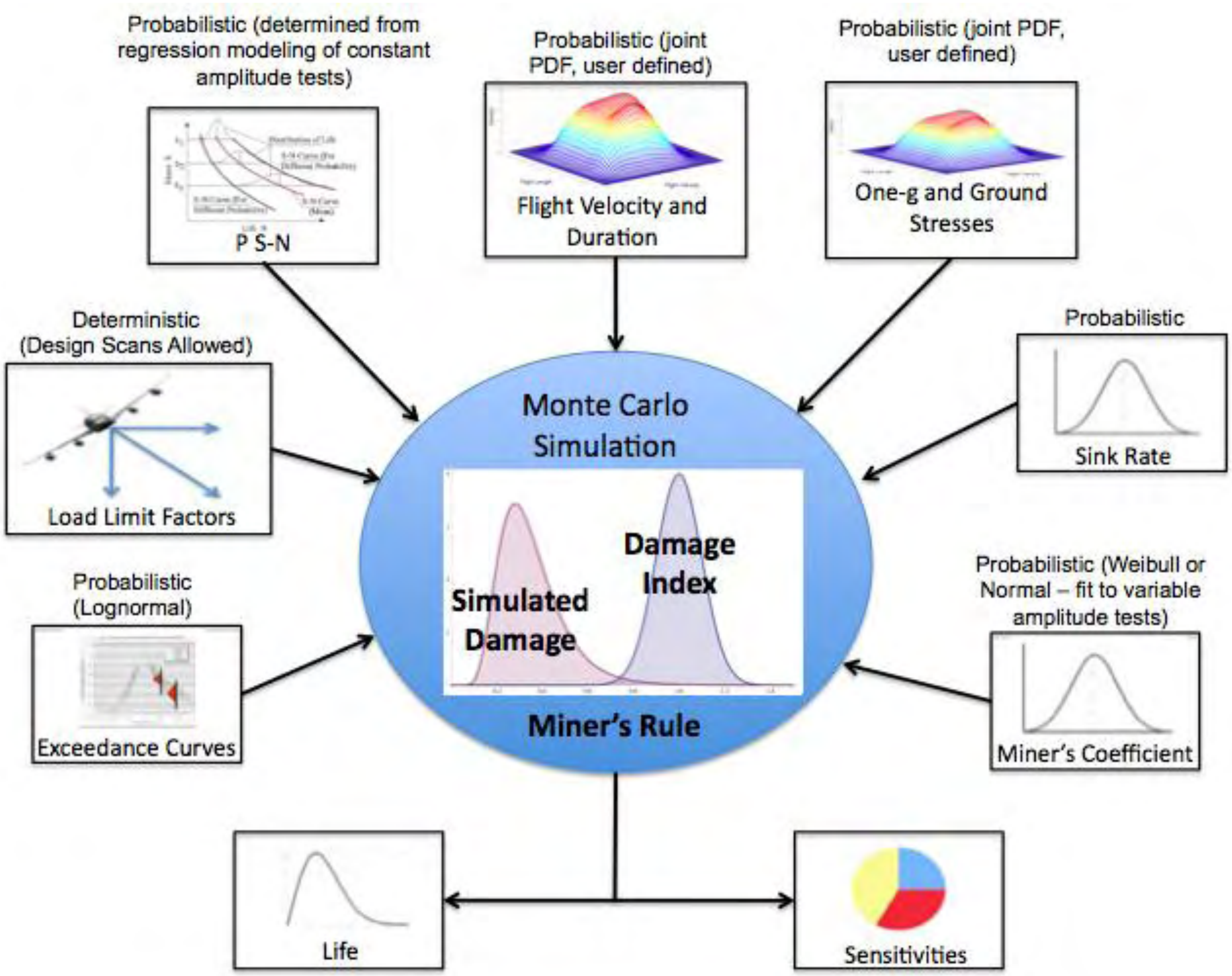

- Develop experience and familiarity with probabilistic approaches within engineering personnel that design, manufacture and maintain general aviation aircraft.
- Verification with in-service findings.
- Develop a Probabilistically-based fatigue management plan (PFMP) for general aviation

# Smart|LD Capabilities


- Loading Generation
  - Computed from exceedance curves (Internal library and user exceedance option) – Weighted usage available.
  - Flight Duration and Velocity/weight matrices, Design load limit factors, one-g stress, and ground stress as user input.
  - User spectra (Afgrow format)
- Damage accumulated using Miner's rule
  - Safe-Life calculations (in # of flights and # of hours) using Monte Carlo sampling
  - Accumulated damage calculation based on the user number of flight hours.
  - Probability of failure computed using MC sampling
- Multiple random variables
  - Library of exceedance curves (weighted mix ok) – Option for user input exceedance.
  - Flight duration, a/c velocity, one-g stress, and ground stress
  - PSN curve constructed from constant amplitude tests – Option for user input PSN
  - Sink Rate
  - Random damage coefficient.
  - Stress Severity Factor (SSF) option
- Text output files showing Monte Carlo results
- Sensitivities computed using correlation and scatter plots
- Life distribution and hazard rate calculation
- Standard Fortran 95/03, Unix and Windows
- GUI

# Risk Methodology

## Methodology Summary

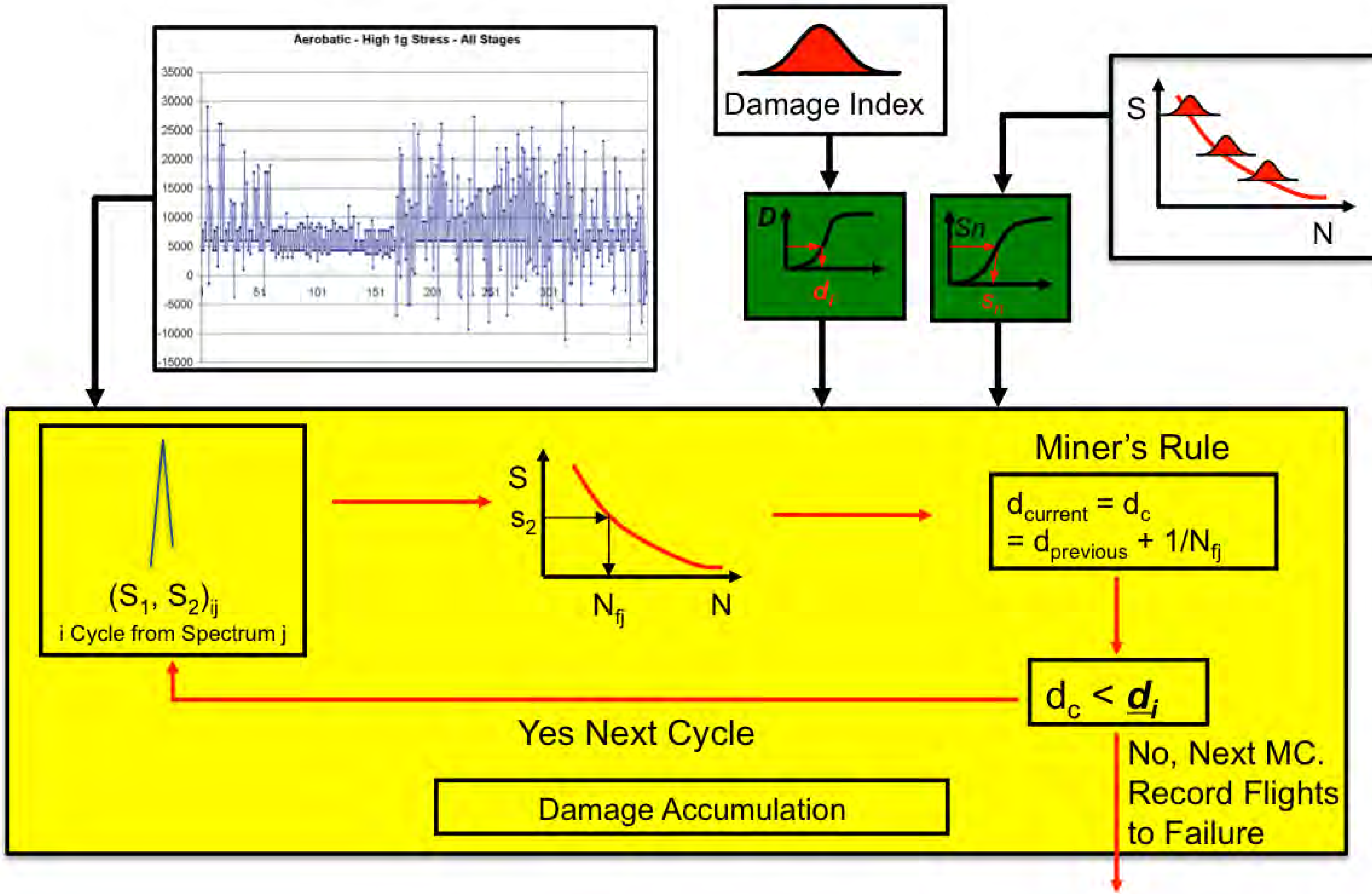


# Methodology

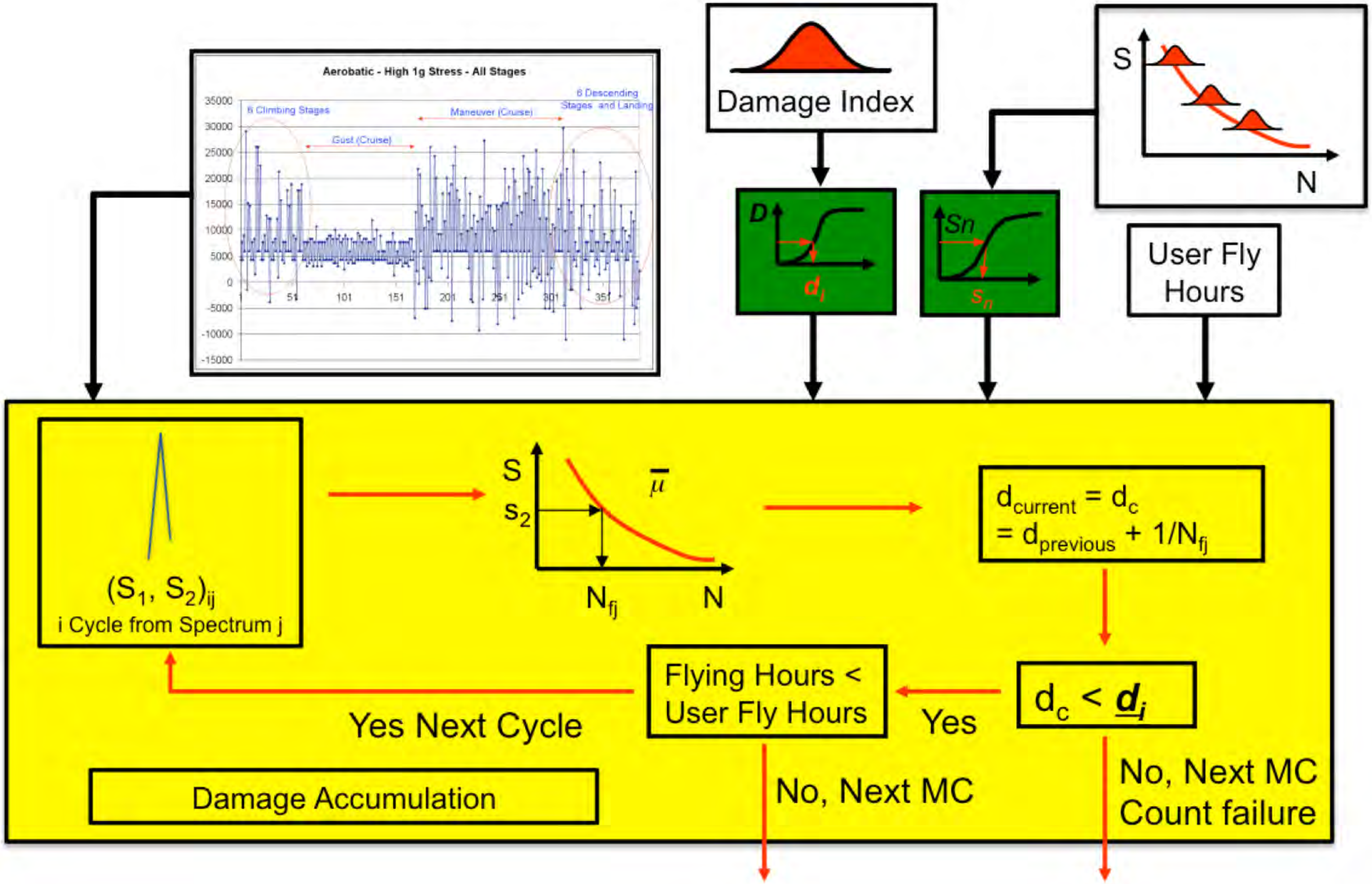
**SMART$_{LD}$**

**SMall Aircraft Risk Technology – Linear Damage Analysis**

# Damage Methodology (Safe Life)



# Hours Methodology (Current-Future Risk)



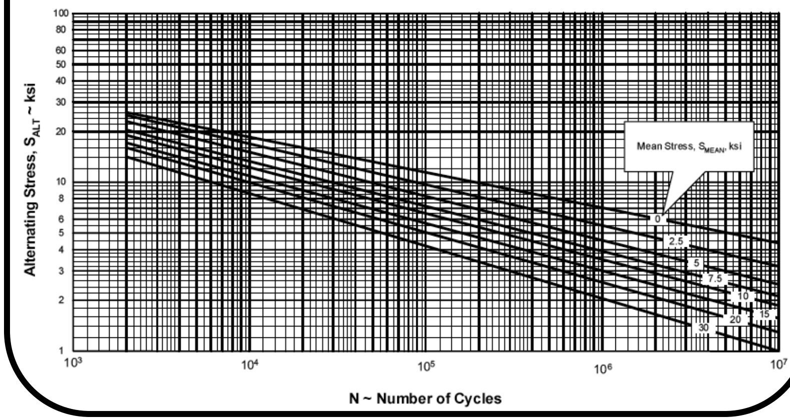
# Variables Classification

| Variable | Type |
|---|---|
| Gust/Maneuver Load Exceedances | Probabilistic: (Lognormal) |
| Aircraft Velocity and Flight Duration | Probabilistic: (Joint PDF with Correlated Variables) |
| Maneuver Load Limit Factors | Deterministic |
| Gust Load Limit Factors | Deterministic |
| Ground/One-g Stress and Flight Duration | Probabilistic: (Joint PDF with Correlated Variables) |
| Sink Rate | Probabilistic |
| P-S-N | Probabilistic (Determined from regression modeling of constant amplitude tests) |
| SSF | PSN Curves (Probabilistic)<br>User Input/ Direct Input (Deterministic) |
| Miner's Damage Index | Probabilistic (Weibull or Normal Distribution– fit to variable amplitude tests) |

# Stress Life Curves

## Risk Methodology

### FAA AC-23-13A



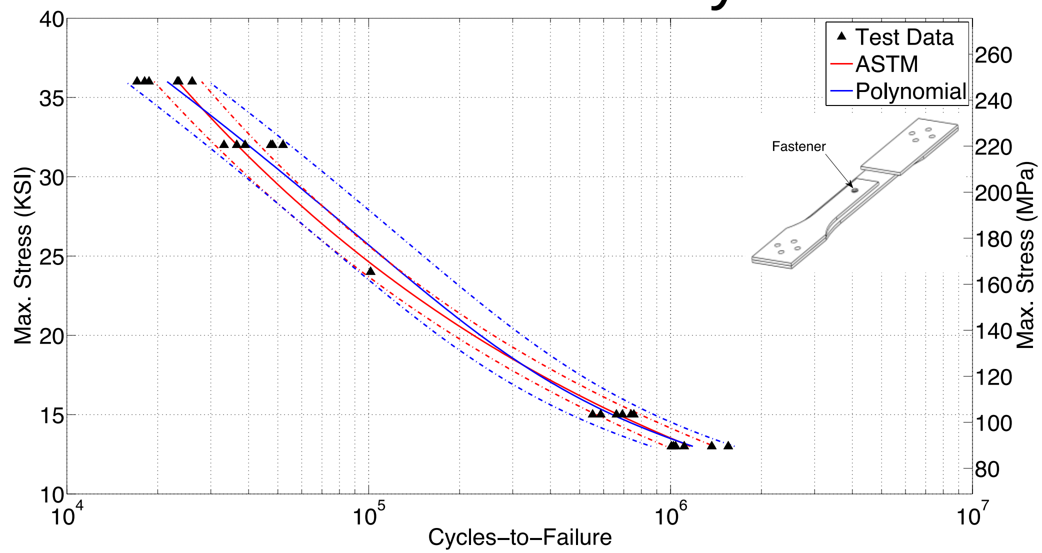

Al6061-T6.sn

```
! LOG(N) = A + B * LOG (Seq + C) + Z*Stdev
! Seq = Smax*(1-R)^D
! E = Endurance limit
! Z ~ N(0,1)

*** SN PARAMETERS ***
A = 11.3196
B = -5.4083
C = 0.0
D = 0.0
E = 0.0
Stdev = 0.5
```

User-defined PSN

### ASTM E739-91 & Polynomial

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Testing Data

Different Configurations

- ✓ Open Hole
- ✓ Filled Hole
- ✓ Load Transfer

# Example Problem


**SMART$_{LD}$**

**SMall Aircraft Risk Technology – Linear Damage Analysis**

# Example

High performance single-engine airplane
with 4,000 pounds of maximum take off


| Variable | Characteristics |
|---|---|
| Gust/Maneuver Load exceedances | Probabilistic exceedances curves for Single Engine Unpressurized Executive Usage |
| Sink Rate | Probabilistic sink rate |
| Design Maneuver Load Limit Factors | +3.41, -1.41 |
| Design Gust Load Limit Factors | 3.80, -1.52 |
| One g stress | +6,550 |
| Ground Stress | -1,987 |
| Aircraft Velocity | 153 |
| Damage Index | Normal distribution with mean 1.0 and standard deviation 0.1 |
| SN Curve | AC23, PSN ASTM |

# Example

High performance single-engine airplane
with 4,000 pounds of maximum take off


## Flight length and Velocity Matrix

| Dur/Vel | | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|
| 0.50: | 0.05 | 0.05 | 0.10 | 0.10 | 0.10 | 0.65 |
| 0.60: | 0.05 | 0.05 | 0.05 | 0.05 | 0.15 | 0.70 |
| 0.70: | 0.10 | 0.00 | 0.05 | 0.05 | 0.15 | 0.75 |
| 0.80: | 0.15 | 0.00 | 0.05 | 0.05 | 0.10 | 0.80 |
| 0.90: | 0.20 | 0.00 | 0.00 | 0.00 | 0.10 | 0.90 |
| 1.00: | 0.25 | 0.00 | 0.00 | 0.05 | 0.05 | 0.90 |
| 1.10: | 0.15 | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |
| 1.20: | 0.05 | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |

## Flight length and Weight Matrix

| Dur/Wei | | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|
| 0.50: | 0.05 | 0.00 | 0.00 | 0.00 | 0.20 | 0.80 |
| 0.60: | 0.05 | 0.00 | 0.00 | 0.00 | 0.20 | 0.80 |
| 0.70: | 0.10 | 0.00 | 0.00 | 0.00 | 0.15 | 0.85 |
| 0.80: | 0.15 | 0.00 | 0.00 | 0.00 | 0.15 | 0.85 |
| 0.90: | 0.20 | 0.00 | 0.00 | 0.00 | 0.10 | 0.90 |
| 1.00: | 0.25 | 0.00 | 0.00 | 0.00 | 0.10 | 0.90 |
| 1.10: | 0.15 | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |
| 1.20: | 0.05 | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |

# Detailed Output Info


Input Variables | Percent Damage | Hz. Fn

| Run | Flight Duration | A/C Velocity | Sink Rate | Damage Coefficient | Gust Factor | Man Factor | One-g Stress | Ground Stress | PSN |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.80 | 153.0 | 0.0867 | 0.9032 | -3.5319 | 0.9510 | 6450.00 | -1987.00 | -2.0287 |
| 2 | 0.90 | 153.0 | 1.0088 | 0.8401 | -3.3367 | 1.7079 | 6450.00 | -1987.00 | -1.9189 |
| 3 | 1.00 | 153.0 | 4.9786 | 0.9815 | -3.9665 | 0.7748 | 6450.00 | -1987.00 | -0.1591 |
| 4 | 0.70 | 137.7 | 4.8776 | 0.8983 | -3.0122 | 0.8723 | 6450.00 | -1987.00 | -2.7482 |
| 5 | 0.80 | 153.0 | 0.7149 | 0.8616 | -3.1957 | -1.3723 | 6450.00 | -1987.00 | -1.1429 |
| 6 | 1.10 | 153.0 | 0.0077 | 0.9052 | -3.0655 | -0.2338 | 6450.00 | -1987.00 | -1.7495 |
| 7 | 1.10 | 145.3 | 1.5870 | 1.0034 | -3.4075 | -0.7383 | 6450.00 | -1987.00 | -1.0805 |
| 8 | 0.90 | 153.0 | 0.1896 | 0.9334 | -2.9557 | -0.3078 | 6450.00 | -1987.00 | -1.6912 |
| 9 | 0.80 | 153.0 | 0.6404 | 0.9168 | -3.4898 | -0.3330 | 6127.50 | -1887.65 | -1.7717 |
| 10 | 0.90 | 153.0 | 1.7365 | 1.1311 | -3.6461 | 0.9379 | 6450.00 | -1987.00 | -0.1695 |
| 11 | 0.70 | 153.0 | 4.4751 | 0.8992 | -3.3696 | 0.3594 | 6450.00 | -1987.00 | 0.2918 |
| 12 | 1.00 | 153.0 | 1.1291 | 0.9252 | -3.0551 | -0.1995 | 6450.00 | -1987.00 | -0.7864 |
| 13 | 1.10 | 153.0 | 1.1373 | 0.9719 | -3.1162 | 0.1208 | 6450.00 | -1987.00 | -0.7990 |
| 14 | 0.60 | 153.0 | 3.0507 | 0.9877 | -2.8750 | -0.3939 | 6450.00 | -1987.00 | -1.4232 |
| 15 | 1.00 | 153.0 | 2.3413 | 1.2176 | -4.3748 | 0.5603 | 6450.00 | -1987.00 | 2.4868 |
| 16 | 1.00 | 153.0 | 0.1795 | 1.0153 | -2.6859 | -0.3496 | 6450.00 | -1987.00 | -1.8533 |
| 17 | 0.80 | 153.0 | 0.9126 | 0.9502 | -2.6171 | -0.5388 | 6450.00 | -1987.00 | -1.6914 |
| 18 | 0.90 | 153.0 | 0.9508 | 0.9375 | -2.3460 | -1.0720 | 6450.00 | -1987.00 | -2.3467 |
| 19 | 0.80 | 153.0 | 7.6019 | 0.9742 | -0.1849 | -0.5521 | 6127.50 | -1887.65 | 0.9565 |
| 20 | 0.70 | 153.0 | 0.8014 | 1.0769 | -3.3182 | 0.1026 | 6450.00 | -1987.00 | -0.1670 |
| 21 | 0.50 | 130.1 | 0.6784 | 0.9096 | -2.9564 | -0.9361 | 6450.00 | -1987.00 | -1.1854 |
| 22 | 0.90 | 153.0 | 1.3858 | 0.9690 | -3.0321 | -0.4130 | 6450.00 | -1987.00 | -0.4426 |
| 23 | 1.00 | 153.0 | 3.0495 | 0.8976 | -3.0681 | 1.2079 | 6450.00 | -1987.00 | 0.0086 |
| 24 | 1.10 | 153.0 | 1.1262 | 0.9034 | -2.0759 | 0.0009 | 6450.00 | -1987.00 | -2.6776 |
| 25 | 0.80 | 153.0 | 0.6629 | 1.0394 | -3.1585 | 0.6223 | 6450.00 | -1987.00 | -0.1940 |
| 26 | 1.10 | 153.0 | 0.2420 | 0.8657 | -2.3269 | 0.0721 | 6450.00 | -1987.00 | -1.7313 |
| 27 | 0.80 | 153.0 | 0.2055 | 0.9365 | -2.8561 | 2.4072 | 6450.00 | -1987.00 | -0.5645 |
| 28 | 0.70 | 153.0 | 0.7134 | 1.0149 | -3.0809 | 0.3561 | 6450.00 | -1987.00 | -0.1896 |
| 29 | 0.50 | 153.0 | 1.5763 | 1.0052 | -2.8593 | -0.9098 | 6450.00 | -1987.00 | -0.7027 |
| 30 | 0.80 | 153.0 | 3.0755 | 0.8226 | -2.1121 | -0.1530 | 6450.00 | -1987.00 | -1.8932 |

| Run | Percentage Gust Damage | Percentage Man Damage | Percentage Taxi Damage | Percentage Land & Reb Damage | Percentage GAG Damage | Flights to Failure | Hours to Failure | Hazard Function | Sample Usage |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.9551 | 0.0009 | 0.000000 | 0.0000 | 0.0440 | 5792 | 4633.60 | 0.000000 | SEUE |
| 2 | 0.9538 | 0.0006 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 3 | 0.9553 | 0.0008 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 4 | 0.9303 | 0.0014 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 5 | 0.9370 | 0.0110 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 6 | 0.9517 | 0.0039 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 7 | 0.9515 | 0.0049 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 8 | 0.9433 | 0.0044 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 9 | 0.9529 | 0.0026 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 10 | 0.9533 | 0.0009 | 0.000000 | 0.0000 | [illegible] | [illegible] | [illegible] | [illegible] | SEUE |
| 11 | 0.9370 | 0.0018 | 0.000000 | 0.0000 | 0.0612 | 12808 | 8965.60 | 0.000000 | SEUE |
| 12 | 0.9466 | 0.0038 | 0.000000 | 0.0000 | 0.0497 | 9064 | 9064.00 | 0.000000 | SEUE |
| 13 | 0.9517 | 0.0027 | 0.000000 | 0.0000 | 0.0456 | 8302 | 9132.20 | 0.000000 | SEUE |
| 14 | 0.9257 | 0.0050 | 0.000000 | 0.0000 | 0.0693 | 15378 | 9226.80 | 0.000000 | SEUE |
| 15 | 0.9611 | 0.0007 | 0.000000 | 0.0000 | 0.0382 | 9304 | 9304.00 | 0.000000 | SEUE |
| 16 | 0.9404 | 0.0056 | 0.000000 | 0.0000 | 0.0540 | 9902 | 9902.00 | 0.000001 | SEUE |
| 17 | 0.9316 | 0.0070 | 0.000000 | 0.0000 | 0.0615 | 12497 | 9997.60 | 0.000001 | SEUE |
| 18 | 0.9242 | 0.0141 | 0.000000 | 0.0000 | 0.0616 | 11113 | 10001.70 | 0.000001 | SEUE |
| 19 | 0.0671 | 0.0020 | 0.000000 | 0.0000 | 0.9309 | 12513 | 10010.40 | 0.000001 | SEUE |
| 20 | 0.9430 | 0.0023 | 0.000000 | 0.0000 | 0.0547 | 14368 | 10057.60 | 0.000001 | SEUE |
| 21 | 0.9317 | 0.0082 | 0.000000 | 0.0000 | 0.0601 | 20121 | 10060.50 | 0.000001 | SEUE |
| 22 | 0.9413 | 0.0047 | 0.000000 | 0.0000 | 0.0540 | 11581 | 10422.90 | 0.000001 | SEUE |
| 23 | 0.9457 | 0.0010 | 0.000000 | 0.0000 | 0.0533 | 10534 | 10534.00 | 0.000001 | SEUE |
| 24 | 0.9330 | 0.0058 | 0.000000 | 0.0000 | 0.0612 | 9695 | 10664.50 | 0.000001 | SEUE |
| 25 | 0.9443 | 0.0016 | 0.000000 | 0.0000 | 0.0541 | 13585 | 10868.00 | 0.000001 | SEUE |
| 26 | 0.9378 | 0.0047 | 0.000000 | 0.0000 | 0.0575 | 9995 | 10994.50 | 0.000001 | SEUE |
| 27 | 0.9413 | 0.0005 | 0.000000 | 0.0000 | 0.0582 | 13846 | 11076.80 | 0.000001 | SEUE |
| 28 | 0.9389 | 0.0022 | 0.000000 | 0.0000 | 0.0589 | 15960 | 11172.00 | 0.000001 | SEUE |
| 29 | 0.9252 | 0.0085 | 0.000000 | 0.0000 | 0.0663 | 22561 | 11280.50 | 0.000001 | SEUE |
| 30 | 0.9210 | 0.0066 | 0.000000 | 0.0000 | 0.0724 | 14133 | 11306.40 | 0.000001 | SEUE |

Hours/Flights-to-Failure

Run no.

Detailed output per MC run

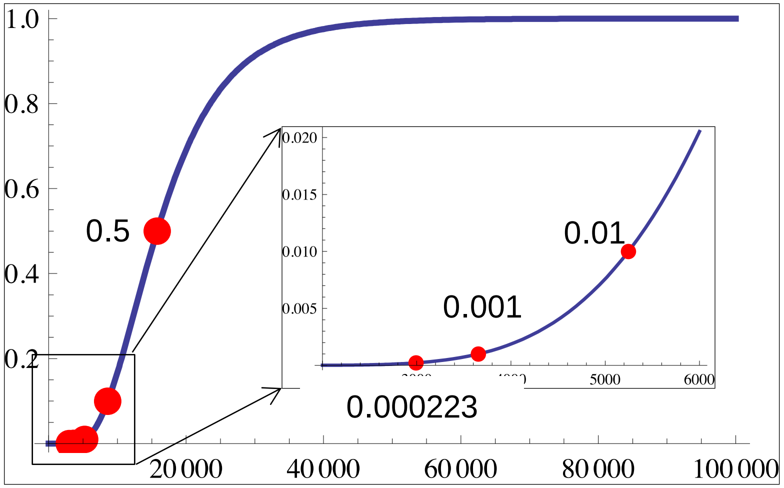
# Safe-life Results

## 20,000 Monte Carlo Samples

| | 95% CONFIDENCE BOUND | MEAN | 95% CONFIDENCE BOUND |
|---|---|---|---|
| **AC-23** | 41,109 | 41,277 | 41,445 |
| **ASTM** | 46,043 | 46,227 | 46,043 |

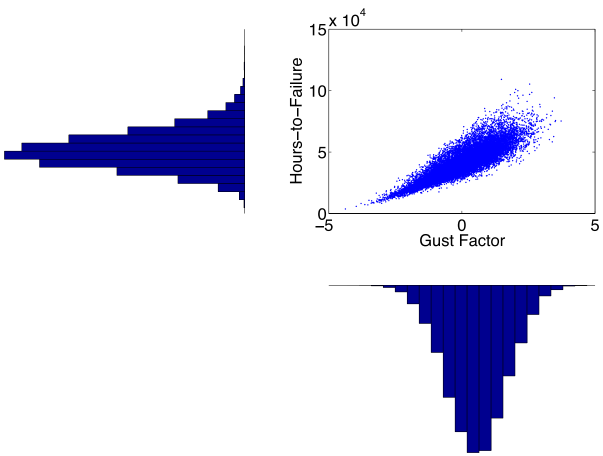
| | 95% CONFIDENCE BOUND | STANDARD DEVIATION | 95% CONFIDENCE BOUND |
|---|---|---|---|
| **AC-23** | 11,998 | 12,116 | 12,236 |
| **ASTM** | 13,180 | 13,309 | 13,441 |

| Probability | Hours-to-Failure AC23 | Hours-to-Failure ASTM |
|---|---|---|
| 0.5 | 40,445 | 44,343 |
| 0.1 | 26,462 | 30,332 |
| 0.01 | 16,314 | 21,533 |
| 0.001 | 10,280 | 16,391 |
| 0.000223 | 7,247 | 12,698 |

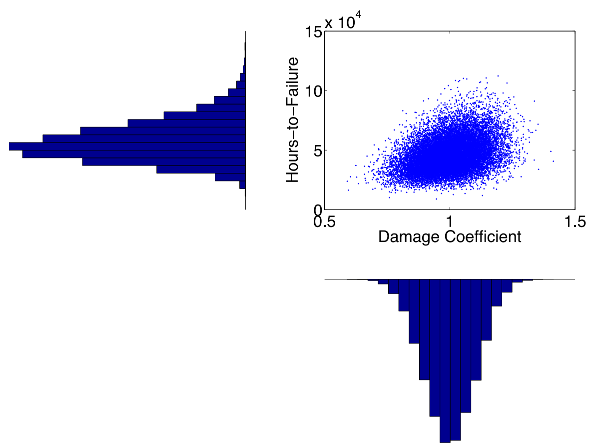


# Correlation Sensitivity Analysis wrt HTF

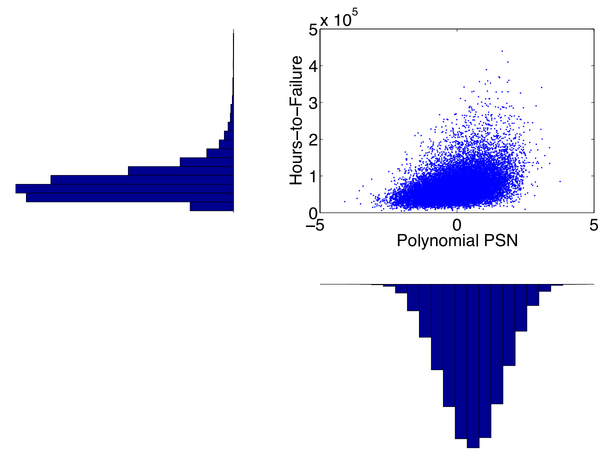
| | FLIGHTS DURATION | FLIGHT SPEED | SINK RATE | DAMAGE COEFFICIENT | GUST FACTOR | MANEUVER FACTOR | ONE-G STRESS | GROUND STRESS | PSN |
|---|---|---|---|---|---|---|---|---|---|
| **AC23** | 0.07 | -0.06 | -0.02 | 0.34 | 0.86 | 0.07 | -0.30 | 0.30 | 0.00 |
| **ASTM** | 0.00 | -0.10 | -0.01 | 0.35 | 0.66 | 0.07 | -0.28 | 0.28 | 0.41 |




AC23 SN Curve

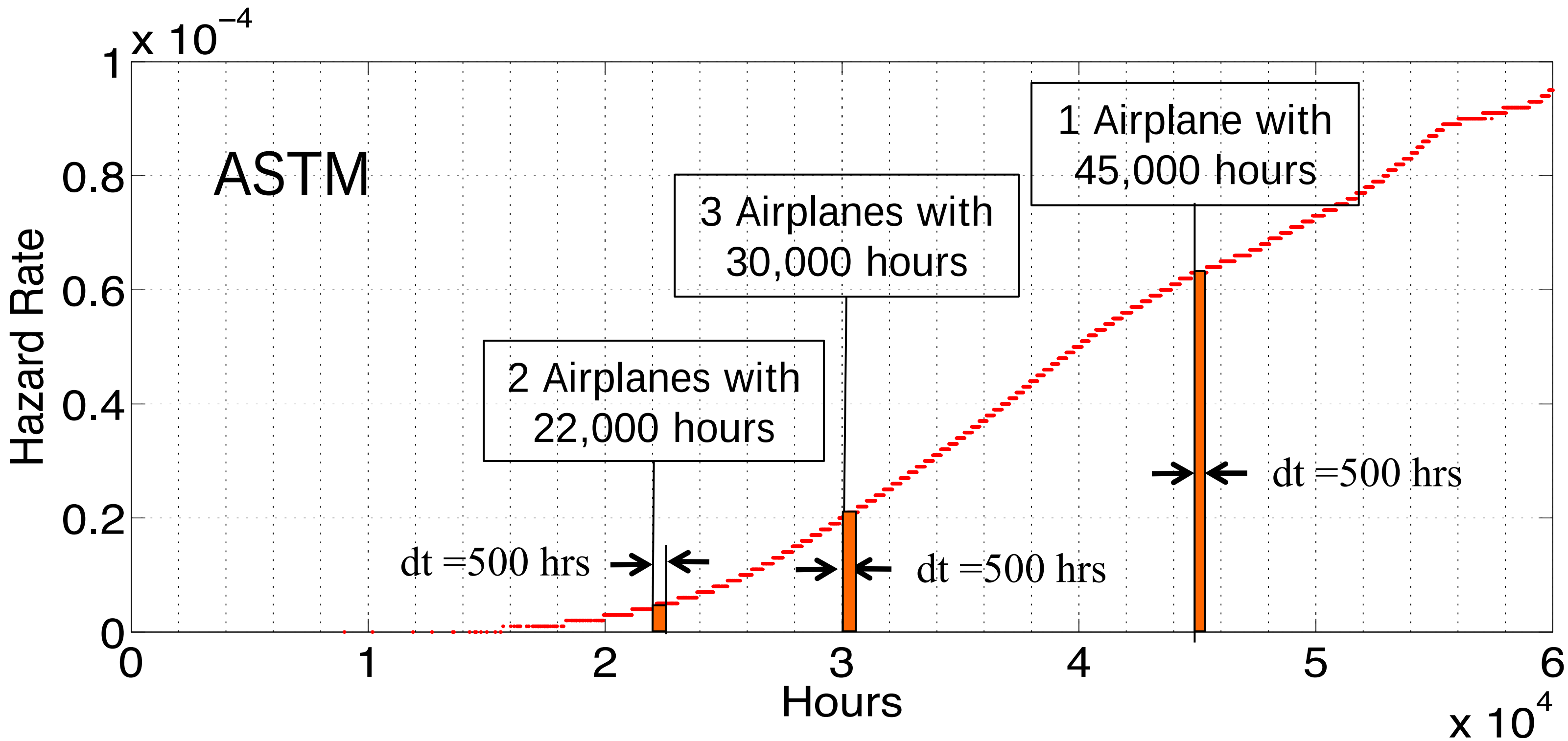



ASTM SN Curve




Poly SN Curve

# Hazard Function Example Application




| No A/C | Hours | Hz(t)*dt | H(t) |
|---|---|---|---|
| 2 | 22,000 | 0.002 | 0.004 |
| 3 | 30,000 | 0.01 | 0.03 |
| 1 | 45,000 | 0.0315 | 0.0315 |
| Fleet Total Hazard | | | 0.0655 |

- Fleet of 6 Airplanes.
- Calculate Hazard Next 500 hrs.

The hazard rate is defined as the probability per time unit that a case that has survived to the beginning of the respective interval will fail in that interval.

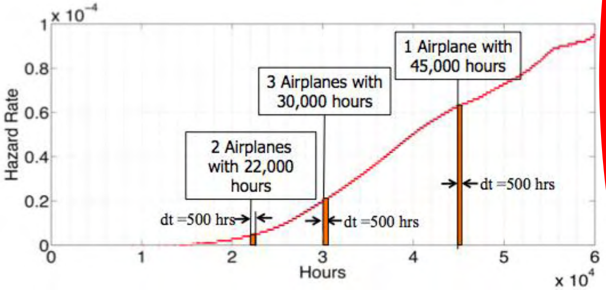
# Program Overview

| 2007-2011 | 2009-2013 | 2012-2016 |
|---|---|---|
| Phase I | Phase II | Phase III |

## Phase I (2007-2011)

### Probabilistic Fatigue Analysis for Small Airplanes ($SMART_{LD}$)

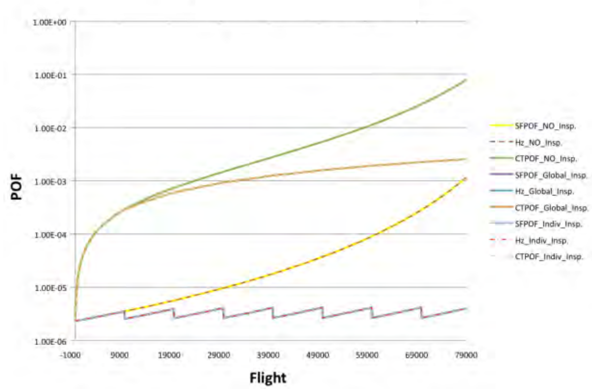
Safe-life Approach




- Prob. Life distribution
- Hazard Rate
- Sensitivity Analysis

## Phase II (2009-2013)

### Probabilistic Damage Tolerance Analysis for Small Airplane ($SMART_{DT}$)




- SFPOF, Hz, CTPOF
- Inspection/Repair Effect
- Sensitivity Analysis

## Phase III (2012-2016)

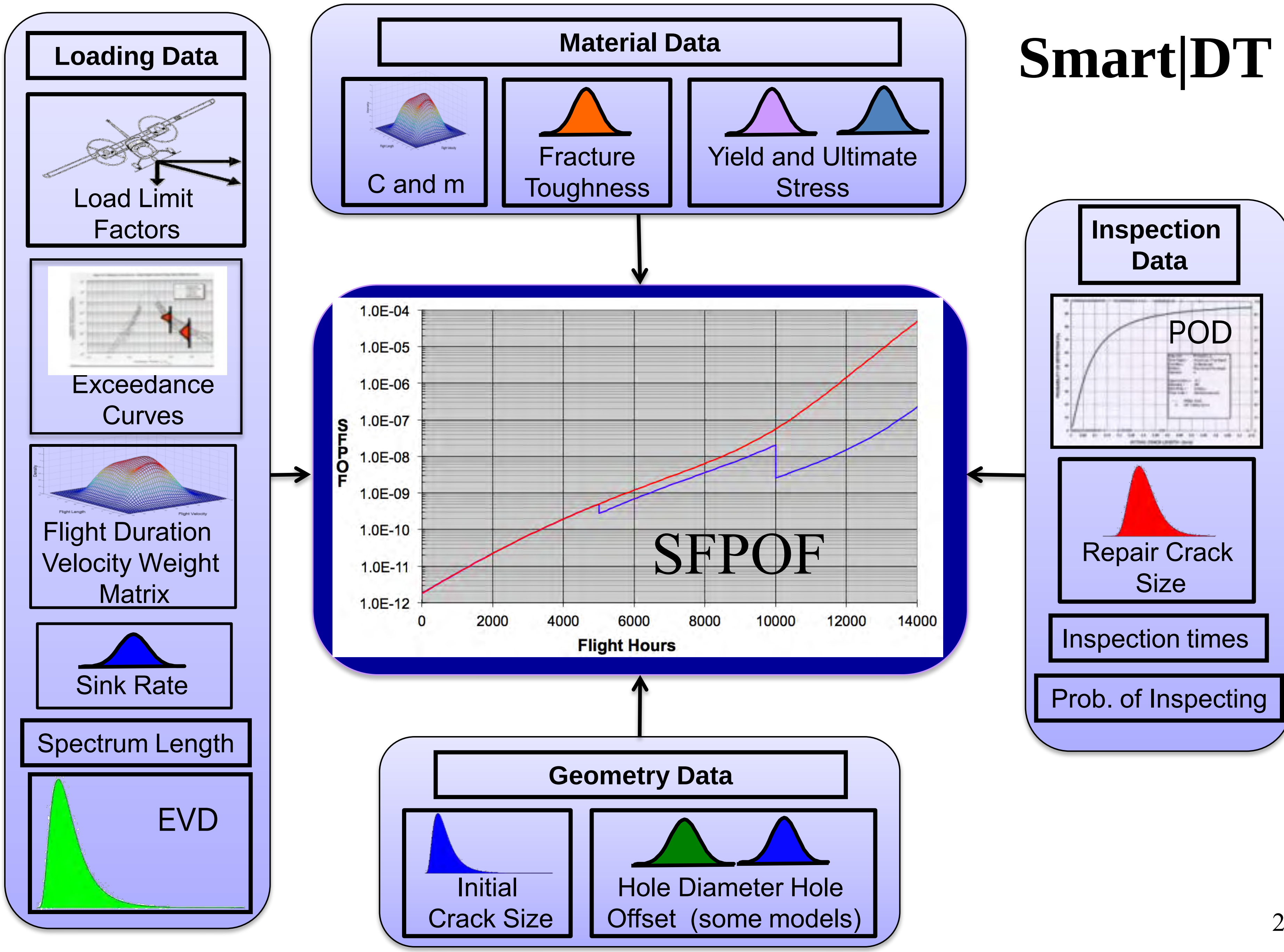
### Probabilistic Fatigue Management Program for General Aviation


- Develop experience and familiarity with probabilistic approaches within engineering personnel that design, manufacture and maintain general aviation aircraft.
- Verification with in-service findings.
- Develop a Probabilistically-based fatigue management plan (PFMP) for general aviation

# Smart|DT Capabilities

- Loading Generation
  - Computed from exceedance curves (Internal library and user exceedance option) – Weighted usage available.
  - Flight Duration and weight matrices, Design load limit factors, one-g stress, and ground stress as user input.
  - Stresses and/or flights randomizations
  - Spectrum editing option (Rainflow, rise/fall, Dead band)
  - User-defined spectra (Afgrow format)
- Extreme Value Distribution
  - User input, e.g., Gumbel, Frechet , and Weibull.
  - Ultimate/Limit load (deterministic)
  - Computed from exceedance curves, weight matrix, etc. (Gumbel, Frechet , and Weibull)
- Probability calculations
  - SFPOF (no survival term)
  - Hazard fn. (with survival term)
  - Cumulative (with survival term)
- Crack growth
  - Direct Nasgro link (for all computations – as an option)
  - Extension to Afgrow (Current Work)
  - Through, Corner, Surface crack growth geometry options
  - Master curve for 2D (ai and Kc) interpolation (user input or developed from Nasgro/Afgrow)
  - Kriging for efficient probabilistic fracture analysis
- Probabilistic methods
  - Standard Monte Carlo
  - Numerical integration
- Inspection capabilities
  - Any number of inspections (arbitrary limit set to 15)
  - Arbitrary repair crack size distribution (lognormal, tabular, deterministic)
  - Arbitrary POD (lognormal, tabular)
  - Deterministic POD
  - User defined probability of inspection
  - Extension to different repairs scenarios (Future Work)
- Random variables
  - ai, Kc, Evd – all cases
  - Crack growth parameters, hole diameter, crack aspect ratio
- Computational implementation
  - Standard Fortran 95/03 (ifort) - Unix, Windows
  - GUI (Windows)

# Smart|DT



Loading Data
Load Limit Factors
Exceedance Curves
Flight Duration Velocity Weight Matrix
Sink Rate
Spectrum Length
EVD
Material Data
C and m
Fracture Toughness
Yield and Ultimate Stress
Inspection Data
POD
Repair Crack Size
Inspection times
Prob. of Inspecting
Geometry Data
Initial Crack Size
Hole Diameter Hole Offset (some models)
SFPOF
1.0E-04
1.0E-05
1.0E-06
1.0E-07
1.0E-08
1.0E-09
1.0E-10
1.0E-11
1.0E-12
0
2000
4000
6000
8000
10000
12000
14000
Flight Hours
SFPOF

# POF

**SMART$_{DT}$**

**SMall Aircraft Risk Technology - Damage Tolerance Analysis**

# POF Calculations


The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength $\sigma_{RS}$ of the aircraft component

$$P_f = P\left[\sigma_{Max} > \left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right] = P\left[\sigma_{Max} > \sigma_{RS}\right]$$

The CDF of the maximum stress in a flight ($F_{EVD}$) can be determined using extreme value theory

$$P_f(t|a_i K_C) = 1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)$$

$$POF(t) = \int_0^{\infty}\int_{-\infty}^{\infty}\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right] f_{a_0}(a_0) f_{K_c}(K_c)\, da_0\, dK_c$$

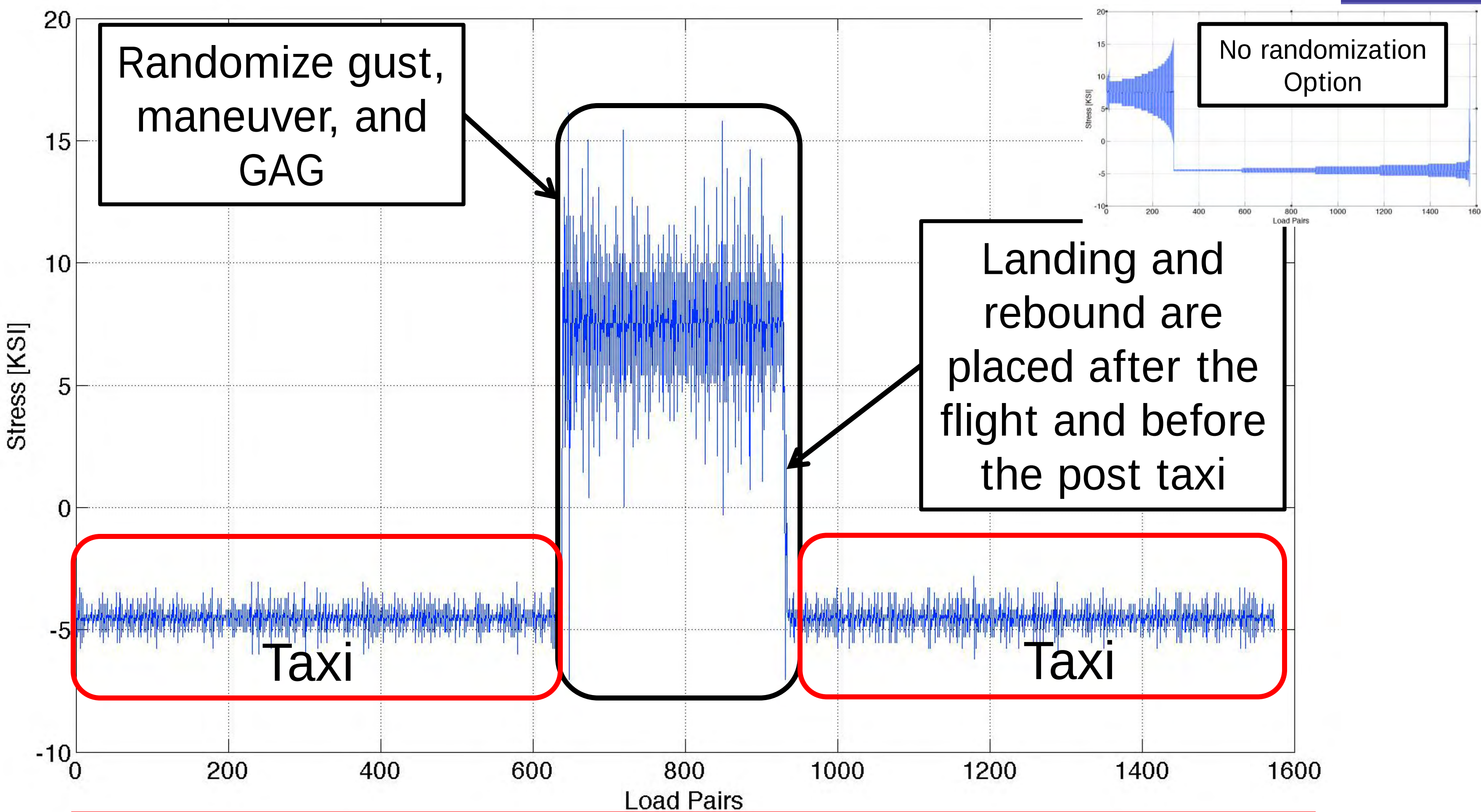
Given these POF calculations, other auxiliary results can be obtained such as the SFPOF (Lincoln and Freudenthal) Cumulative POF and the hazard function.

# Loading Generation

**SMART**$_{DT}$

**SMall Aircraft Risk Technology - Damage Tolerance Analysis**

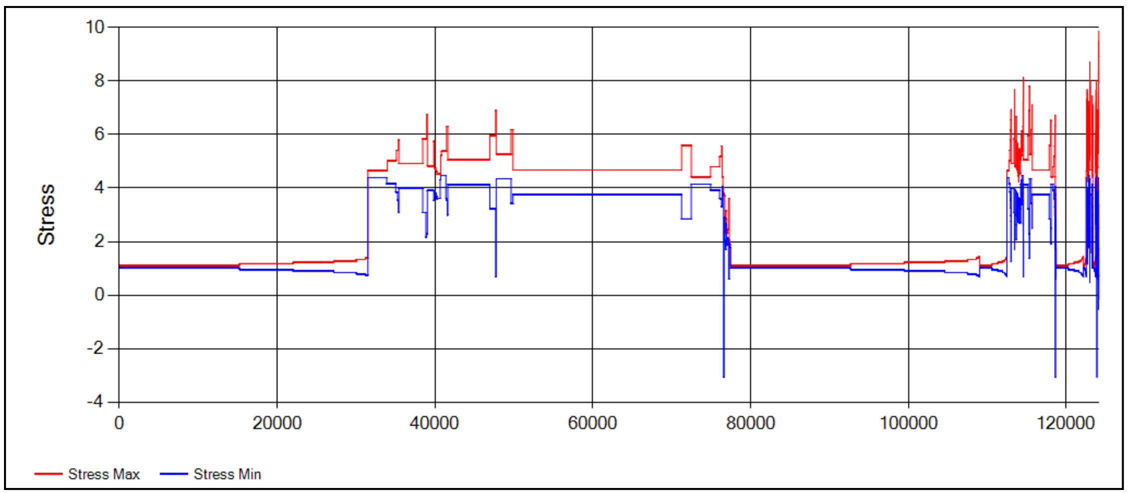
# Loading Example

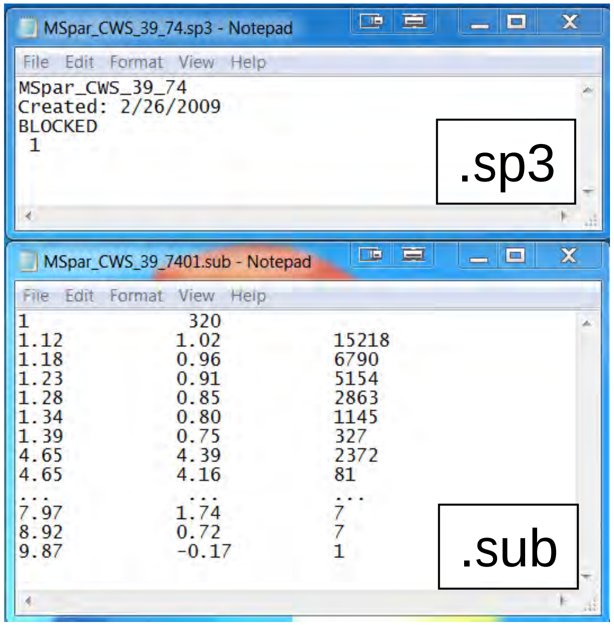



Randomize taxi loads and split half before the flight and half after the flight, Taxi load can be excluded from the analysis.

# Loading Generation (User Defined)

Smart allows the user to load Afgrow spectra files (.sp3 and .sub). The GUI will read the ".sp3"



# EVD Generation

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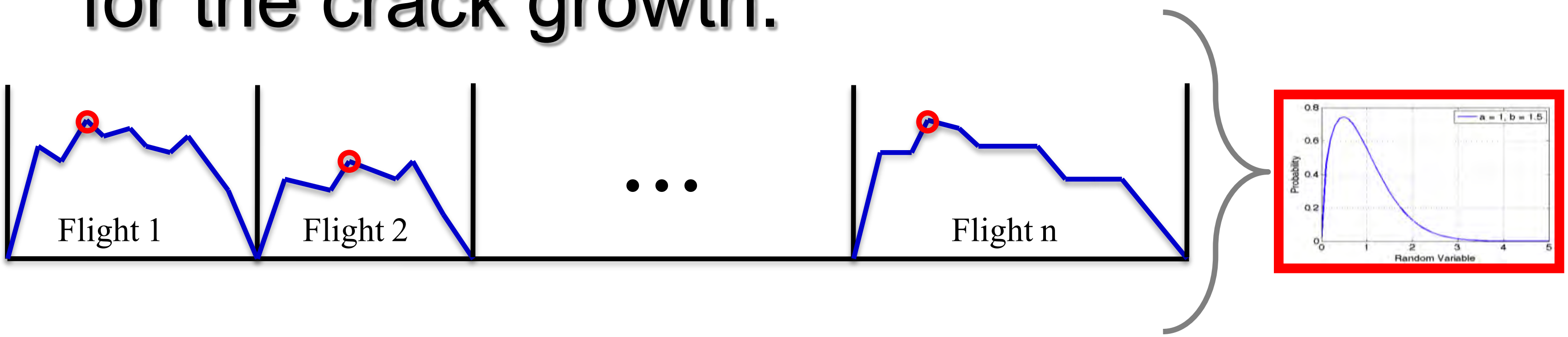
# EVD Generation (I)

- A critical component is the extreme load per flight. This extreme load is (should be) determined from the same spectrum used for the crack growth.



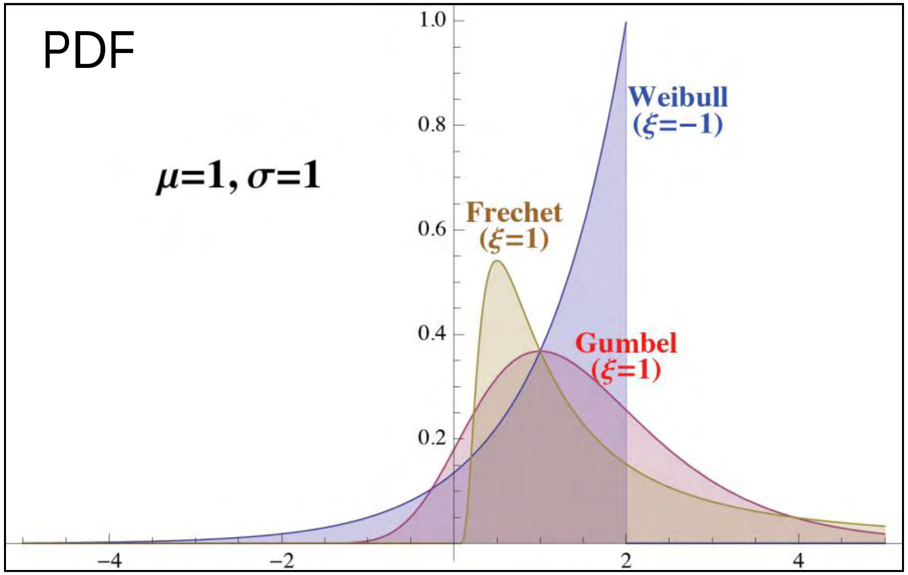

$$POF(T) = \int_{-\infty}^{\infty}\int_{0}^{\infty}\left[1 - F_{EVD}\left(\frac{K_C}{\beta(T)\sqrt{\pi a(T)}}\right)\right] f_{a_0}(a_0)\, f_{K_c}(K_c)\, da_0\, dK_c$$

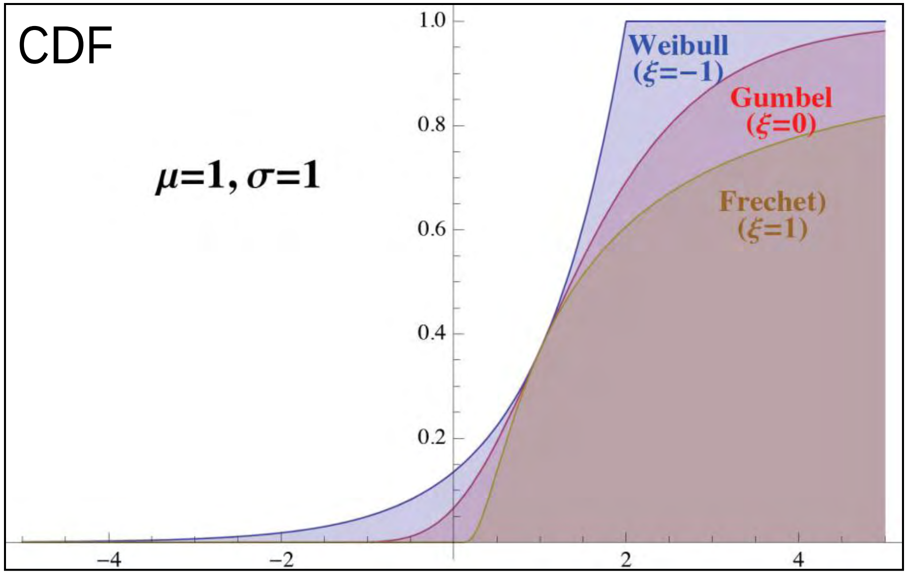
# Generalized EVD Formulation

- Weibull, Frechet, or Gumbel can be written in terms of the Generalized Extreme Value Distribution as

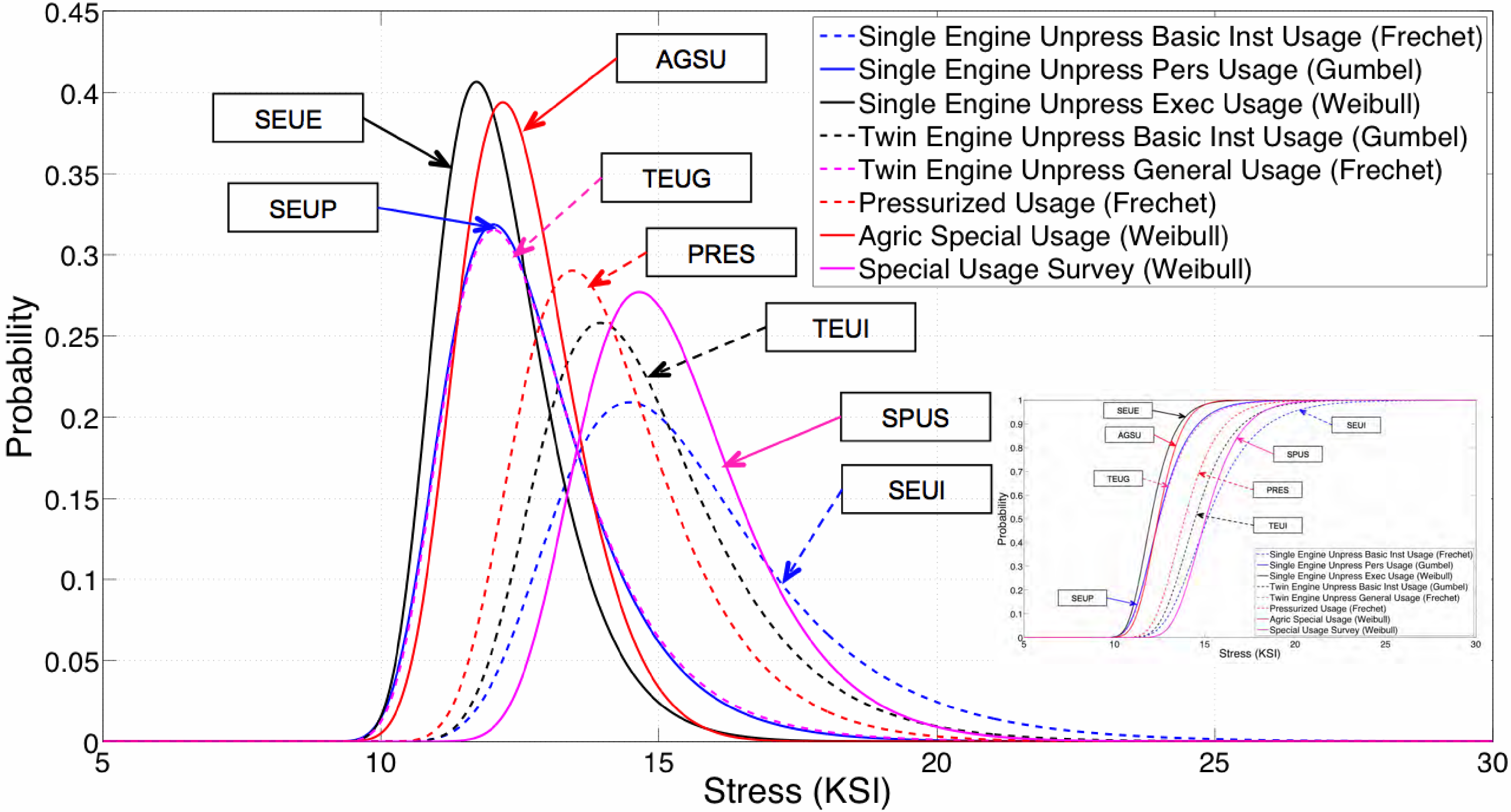
$$F(x) = \exp\left\{ -\left[ 1 + \xi \left( \frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\} \qquad \begin{array}{ll} \xi = 0 & \text{Gumbel} \\ \xi > 0 & \text{Frechet} \\ \xi < 0 & \text{Weibull} \end{array}$$

- Parameters $(\mu, \sigma, \xi)$ location, scale, and shape define the distribution.



# EVD Results

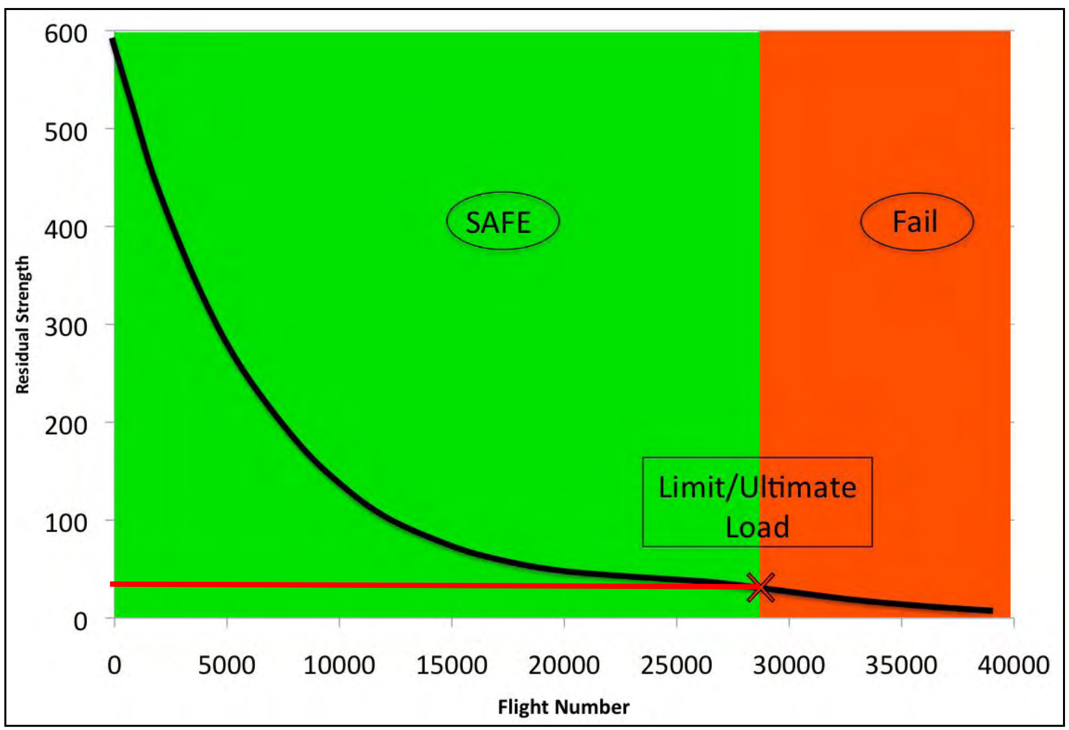


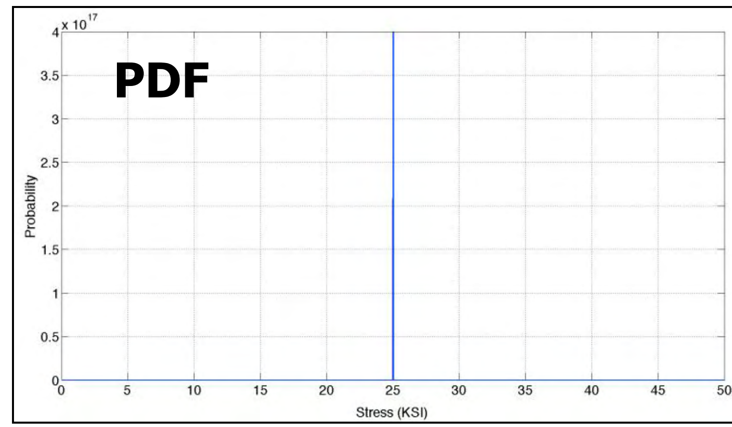
Probability
Stress (KSI)
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
5
10
15
20
25
30
AGSU
SEUE
TEUG
SEUP
PRES
TEUI
SPUS
SEUI
Single Engine Unpress Basic Inst Usage (Frechet)
Single Engine Unpress Pers Usage (Gumbel)
Single Engine Unpress Exec Usage (Weibull)
Twin Engine Unpress Basic Inst Usage (Gumbel)
Twin Engine Unpress General Usage (Frechet)
Pressurized Usage (Frechet)
Agric Special Usage (Weibull)
Special Usage Survey (Weibull)

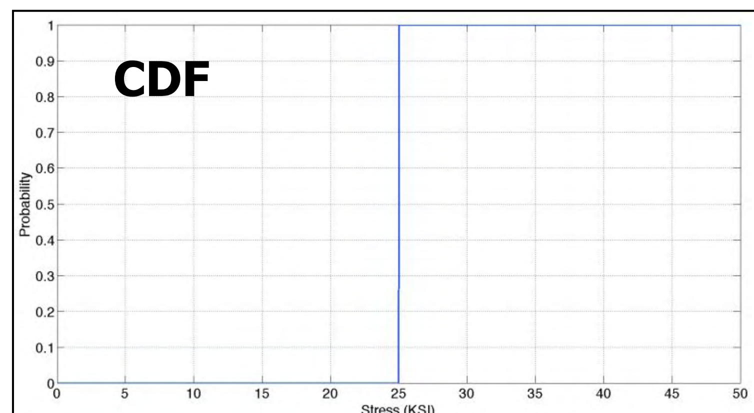
# Limit Load EVD


Smart|DT allows the user to input the limit load as EVD input.
The limit load behaves as an step function, residual Strength smaller or equal than the limit load has a POF = 1 and , residual Strength bigger than the limit load has a POF = 0

EVD is set to a deterministic values
equal to the airplane limit/ultimate load



# Crack Growth Methods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**SMall Aircraft Risk Technology - Damage Tolerance Analysis**

# Analysis Methods

- ✓Only $a_i$ and $K_c$ Random
- ✓Representative Spectrum
- ✓Inspection/Repair

→ **Master Curve**
Monte Carlo/
Numerical Integration
→ ✓Hz, SFPOF, CTPOF

**1X - Efficiency**

- ✓Multiple rnd variables
- ✓Multiple Spectrum
- ✓Inspection/Repair

→ **Full Nasgro/Afgrow**
Monte Carlo/
Numerical Integration
→ ✓Hz, SFPOF, CTPOF

**1000X - Efficiency**

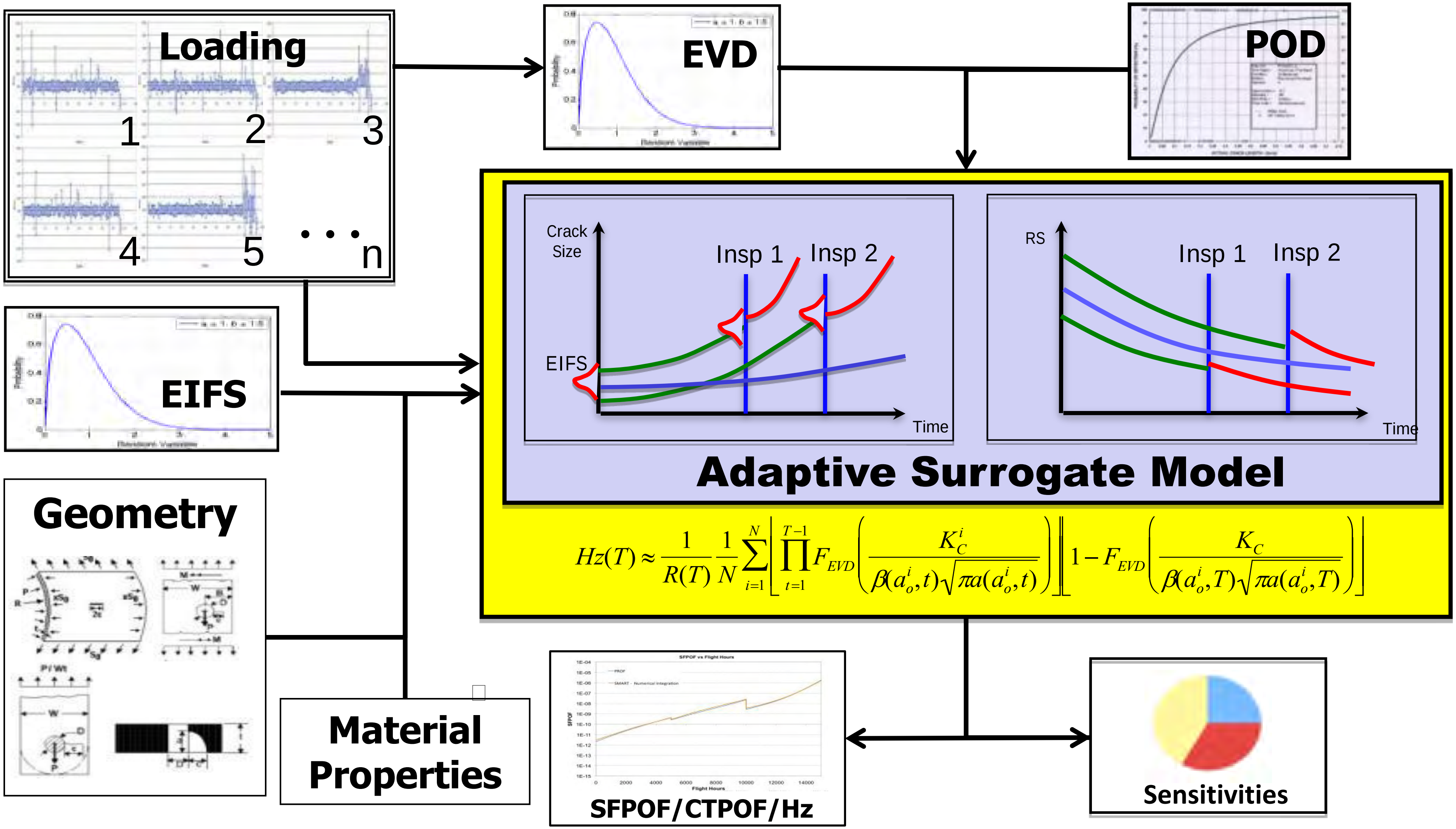
- ✓Multiple rnd variables
- ✓Representative Spectrum
- ✓Inspection/Repair

→ **Adaptive Kriging/Nasgro**
Monte Carlo/
Numerical Integration
→ ✓Hz, SFPOF, CTPOF

**10X - Efficiency**

# Methodology

## Probabilistic Damage Tolerance for Small Airplanes



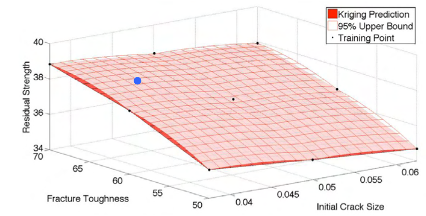

$$Hz(T) \approx \frac{1}{R(T)} \frac{1}{N} \sum_{i=1}^{N} \left[ \prod_{t=1}^{T-1} F_{EVD}\left( \frac{K_C^i}{\beta(a_o^i,t)\sqrt{\pi a(a_o^i,t)}} \right) \right] \left[ 1 - F_{EVD}\left( \frac{K_C}{\beta(a_o^i,T)\sqrt{\pi a(a_o^i,T)}} \right) \right]$$

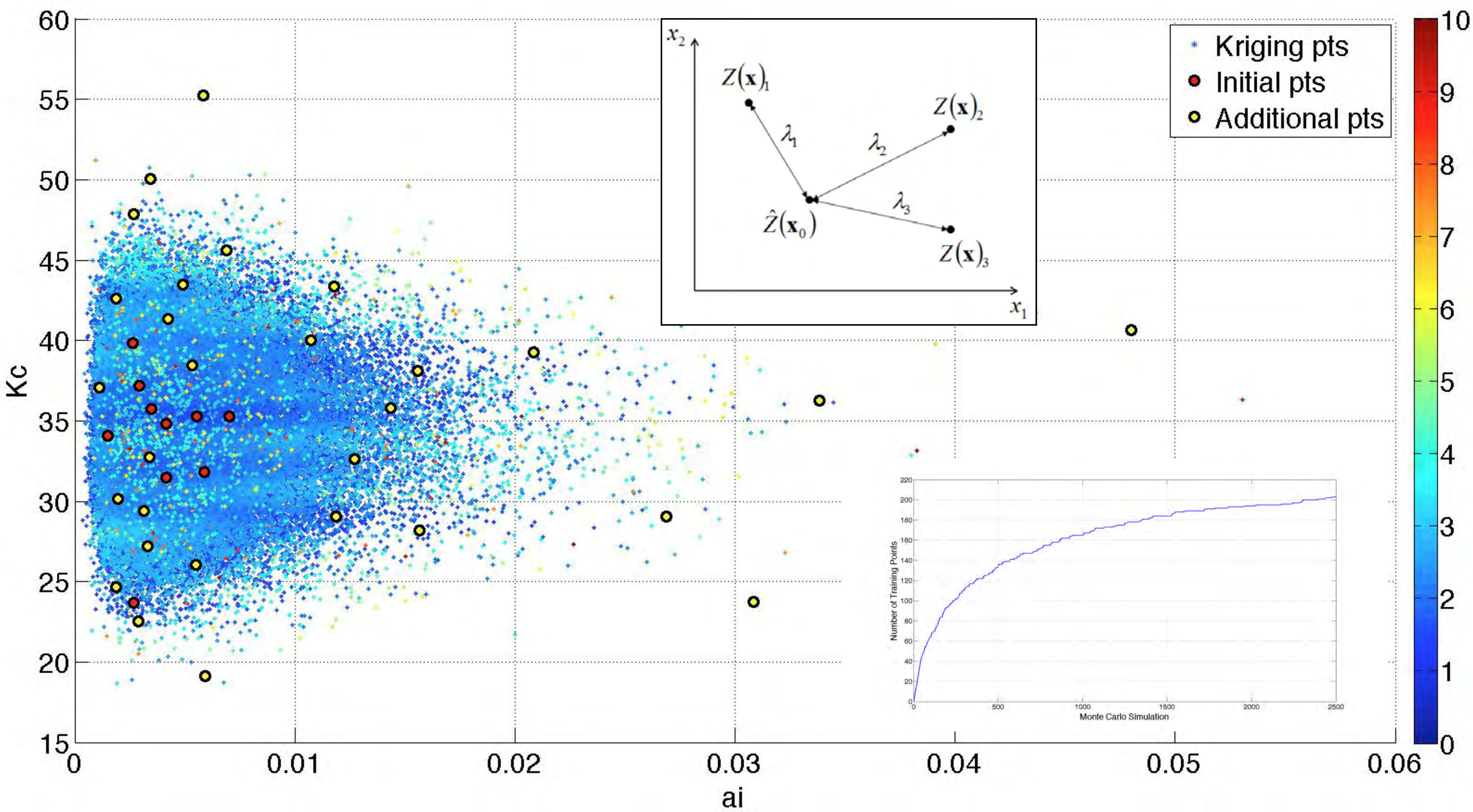
# Adaptive Residual Strength and Crack Growth Surrogate Model

$$POF(t) = \int_{-\infty}^{\infty} \left[ 1 - F_{EVD}\left( \sigma_{RS}(K_C, \beta, a_o, C, m, t) \right) \right] f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$

- An adaptive Kriging surrogate model is used to reduce physics-based crack growth function calls, e.g., AFGROW, FASTRAN, UniGrow
  - Applicable to both:
    - POF calculations (residual strength predictions) and inspections (crack growth predictions)
  - Adaptive (self correcting):
    - additional crack growth function calls added as needed per user-defined error threshold.

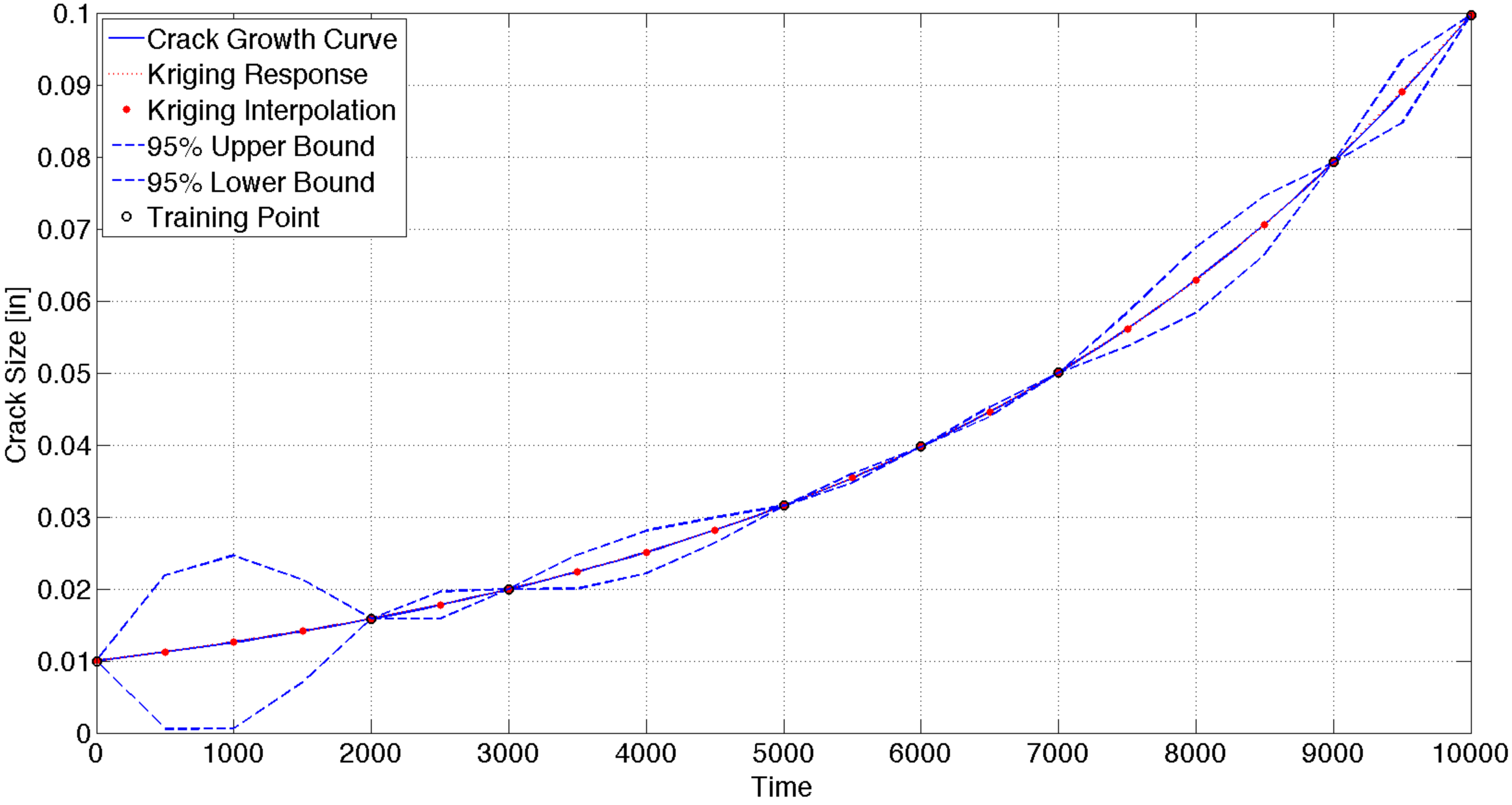
# Kriging Schemetic



# Kriging Error Prediction

Compute prediction variance and confidence bounds



# Kriging Adaptive Model

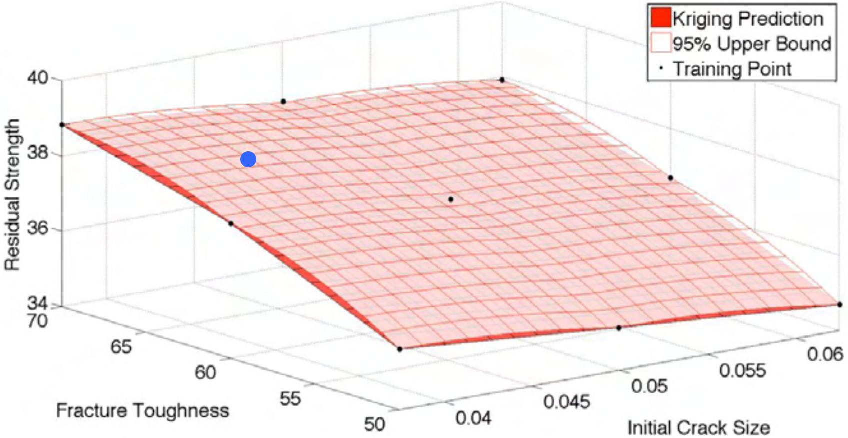
The error is calculated based on the Kriging variance and the assumption that $Z(\cdot)$ is Gaussian

The 95% confidence bound from the prediction value can be computed as

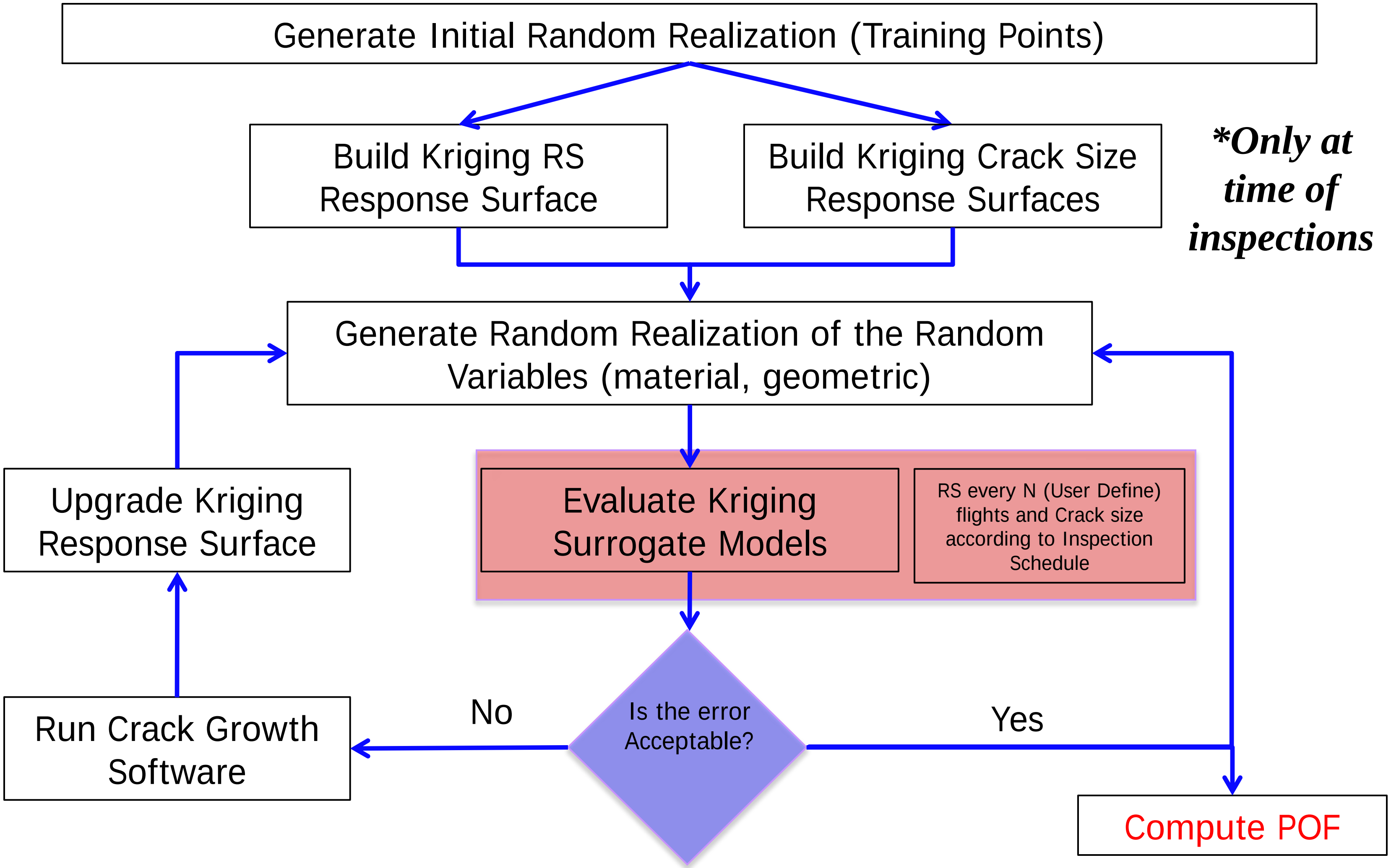
$$A \equiv (A_{LB}, A_{UB}) \equiv \left(\hat{Z}(x_0) - 1.96\sigma_{\varepsilon}(x_0), \hat{Z}(x_0) + 1.96\sigma_{\varepsilon}(x_0)\right)$$

The error based on the 95% (99%) confidence bound can be computed as

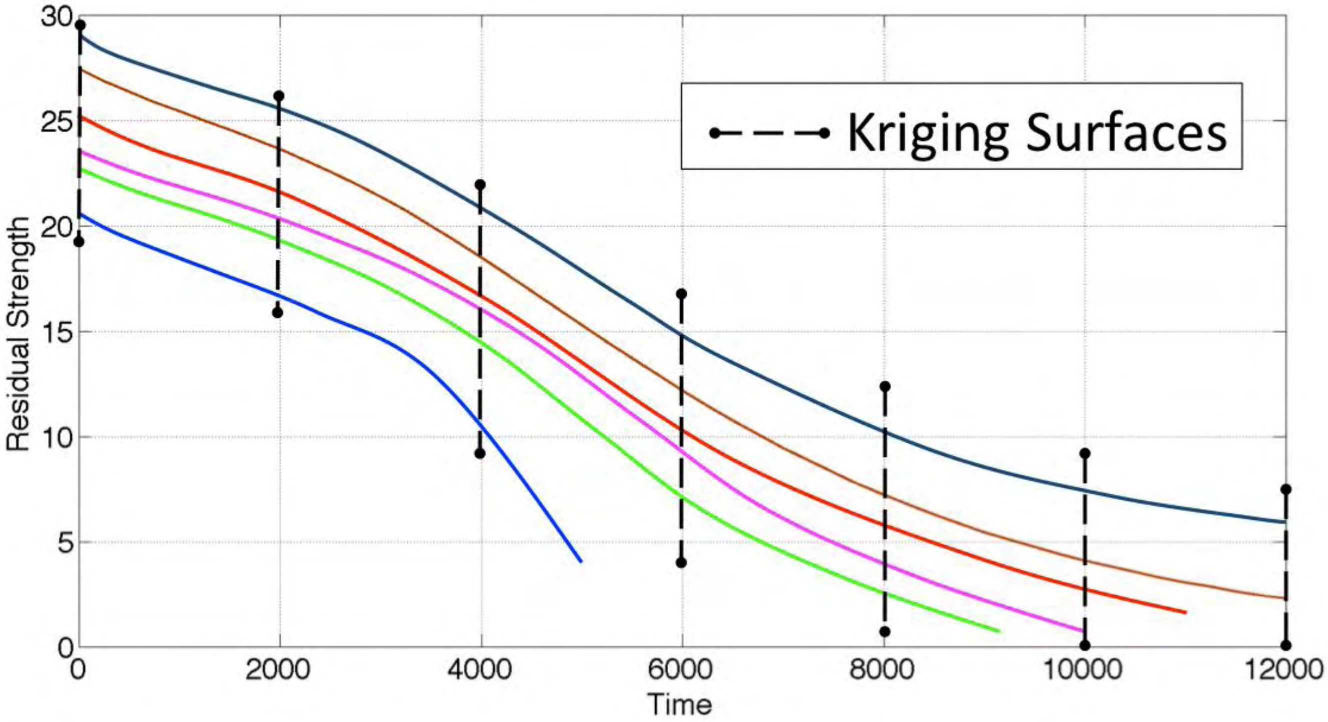
$$error = \frac{abs\left(A_{LB} - \hat{Z}(x_0)\right)}{A_{LB}}$$



# Kriging Adaptive Model

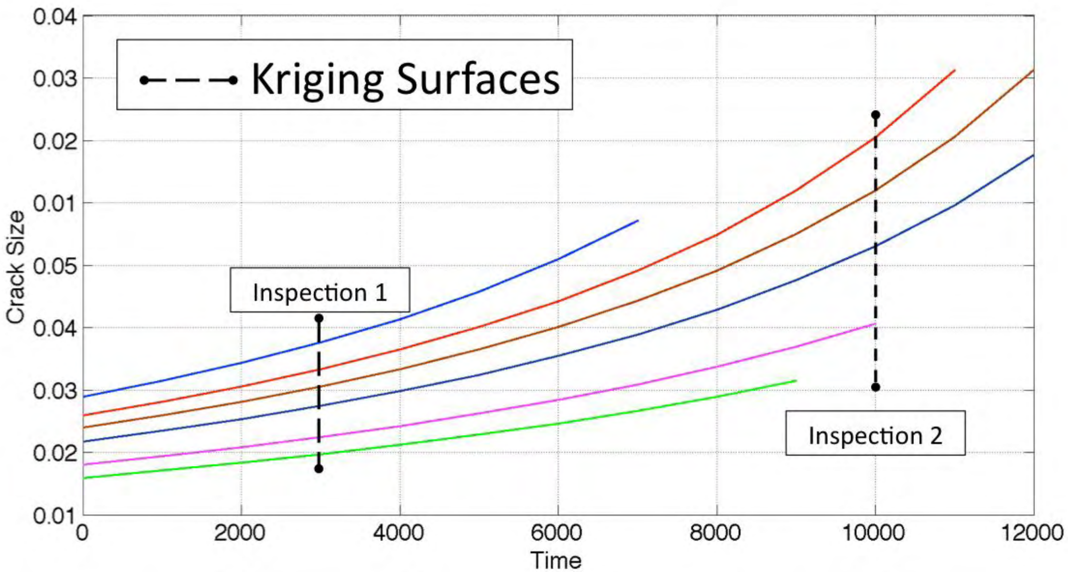


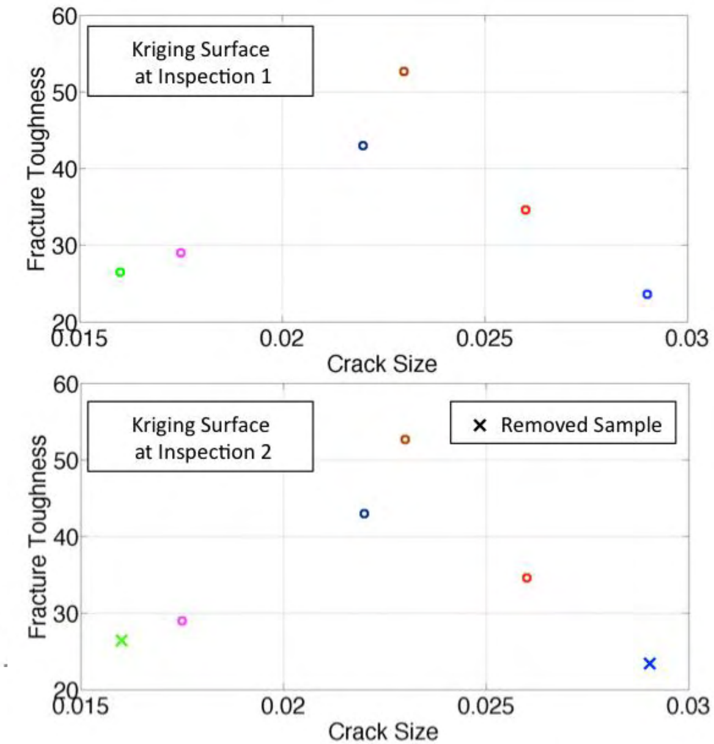
# Time Dependent Surrogates




Residual strength Kriging surfaces are created anew at each time step requested by the user using non-failed realizations. Similarly for crack size estimates.

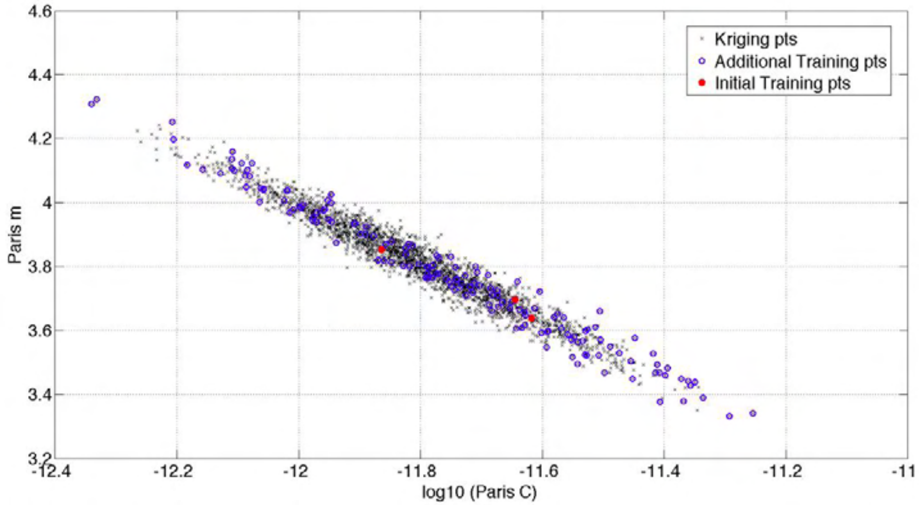
# Crack Growth Surrogates

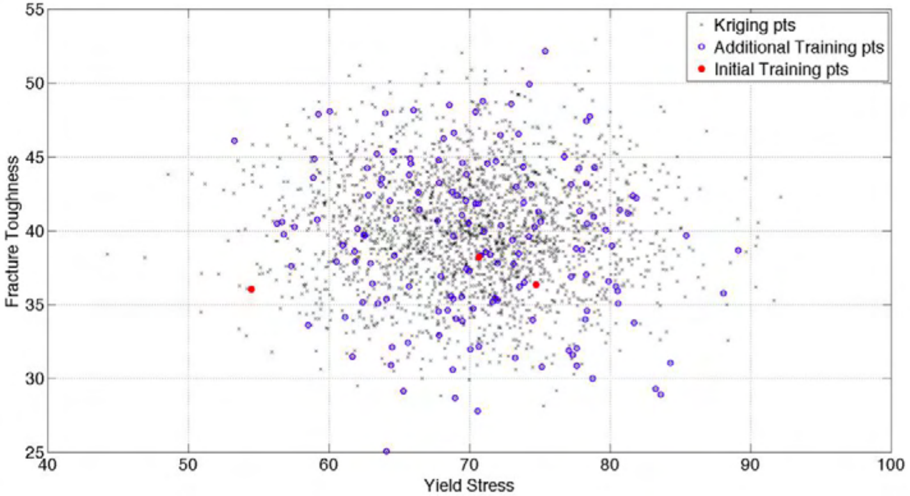


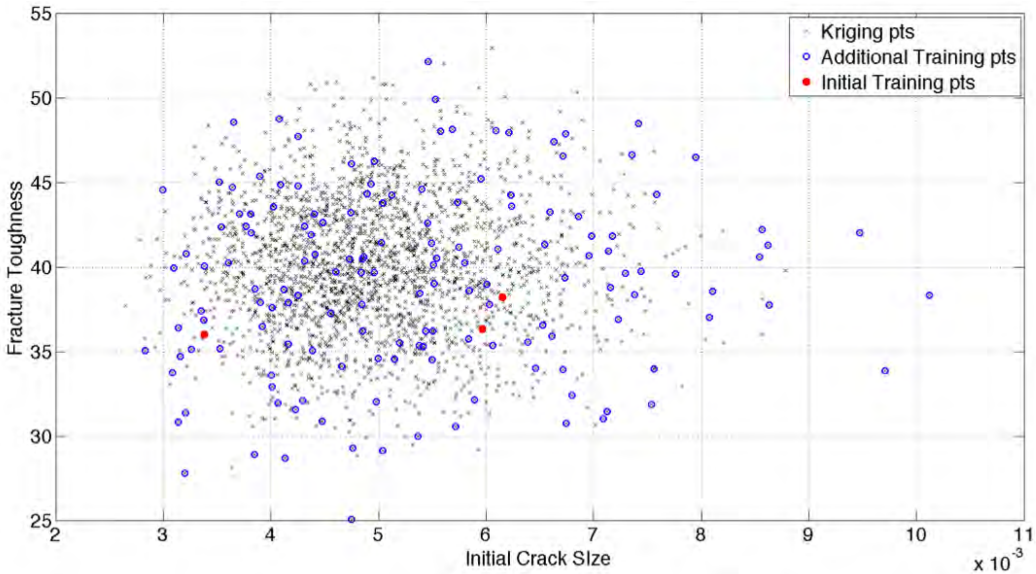




If an inspection occurs at time t, crack size Kriging surfaces are created at each inspection time

# Adaptive Kriging
## Multiple Random Variables



# Inspections and Repair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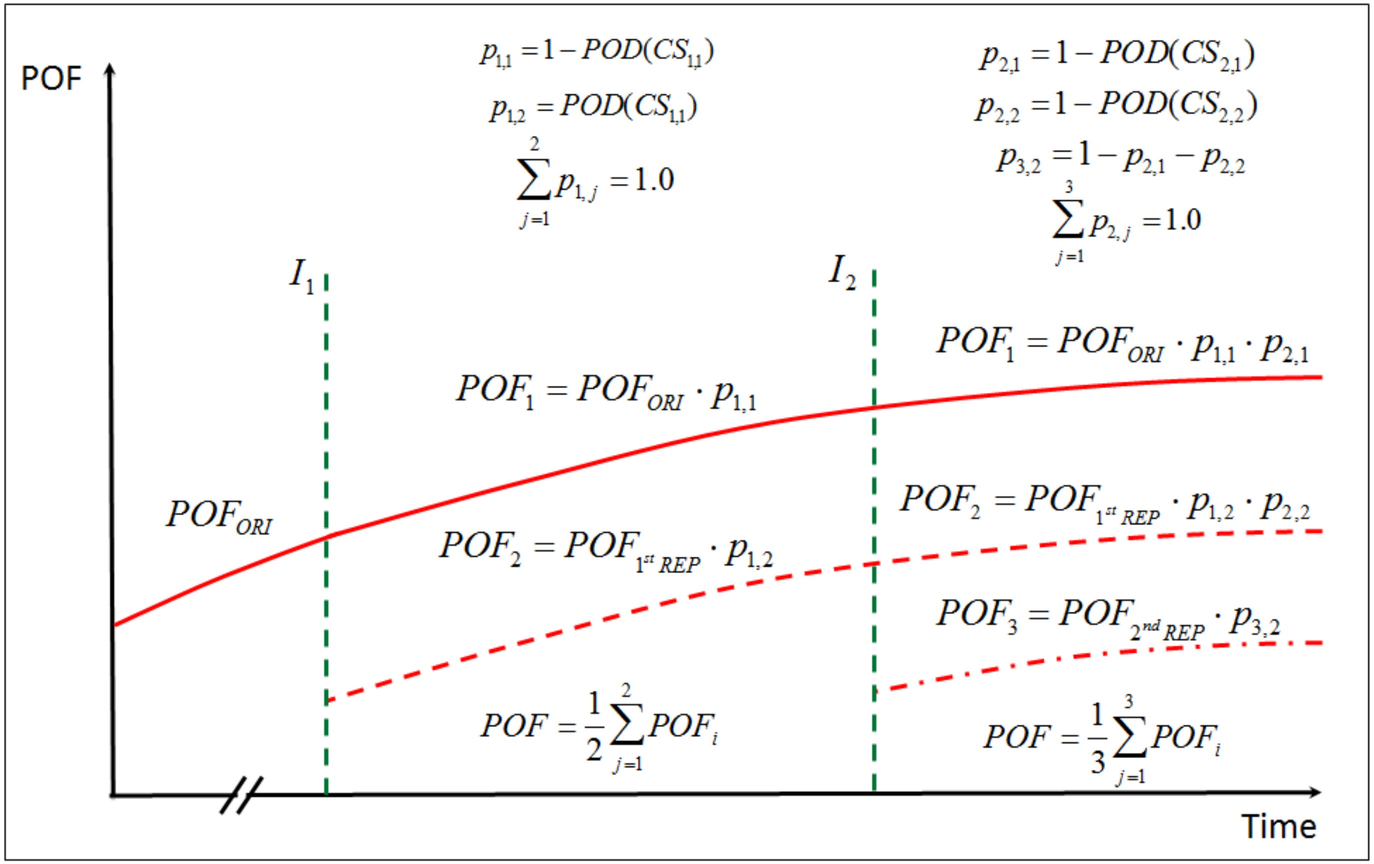
# Implementation
## Monte Carlo

- Weighted sum of possible crack growth paths
- 1 additional path for each inspection

For Each Realization



# Post-Inspection Analysis

- After inspection, some cracks are detected and repaired. The post-inspection crack size distribution becomes a combination of a “before” and a “repair” distribution

$$f_{after}(a) = P_{\det} f_R(a) + [1 - POD(a)] f_{before}(a)$$

$$P_{\det} = \int_0^{\infty} POD(a) f_{before}(a)\,da$$ - % of cracks detected

$f_{before}$ - crack size at the time of inspection

$f_{after}$ - crack size after inspection

# Example Problem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**SMall Aircraft Risk Technology Damage Tolerance Analysis**

# High Performance Aircraft no Inspection


| Quantity | Definition |
|---|---|
| Nasgro Crack Growth Model. | TC03 – Through crack in a hole |
| Geometric Variables | Width = 2.5 in.<br>Thickness = 0.09 in.<br>Hole Diameter = 0.10 in.<br>Hole Offset = 0.5 in. |
| Fracture Toughness Distribution | Normal:<br>Mean = 34.8ksi√in.<br>Standard Deviation = 3.9 ksi√in. |
| Initial Crack Size Distribution | Lognormal<br>Median = 0.00163 in.<br>Mean = ln(median) = -6.420<br>Standard Deviation = 1.113 |
| Extreme Value Distribution (Weibull) | Location = 5.0, Scale = 10.0, and Shape = 5.0 |
| Material | Al-2024 |

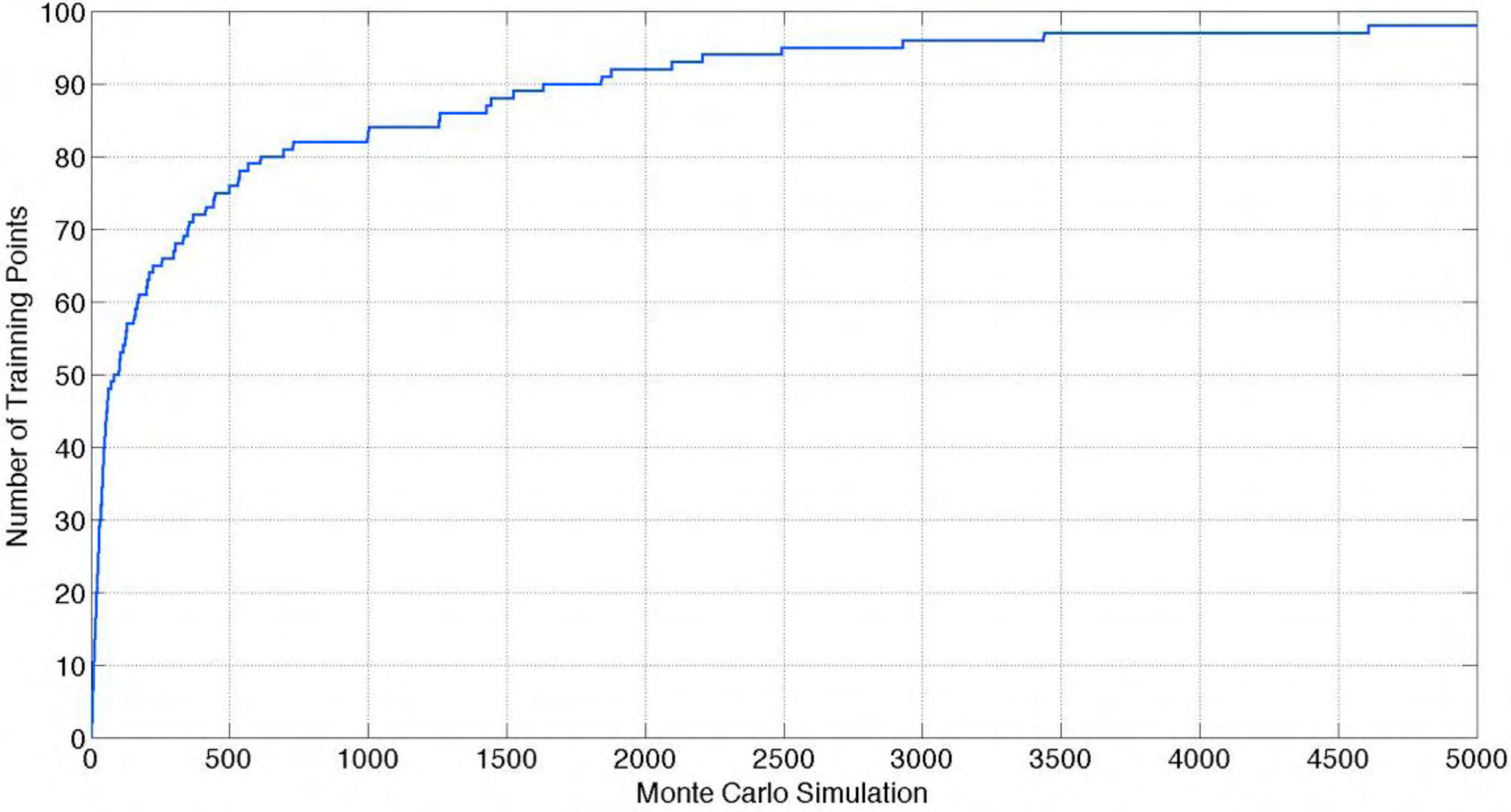
# High Performance Aircraft no Inspection


| Variable | Value |
|---|---|
| Usage | Single Engine Unpressurized Basic Executive Usage |
| Design LLF Maneuver | 3.8, -1.52 |
| Design LLF Gust | 3.155, -1.155 |
| Ground Stress (psi) | -4,550 |
| One-g stress (psi) | 7,100 |
| Flight Length and Velocity Matrix | |
| Flight Length and Weight Matrix | |

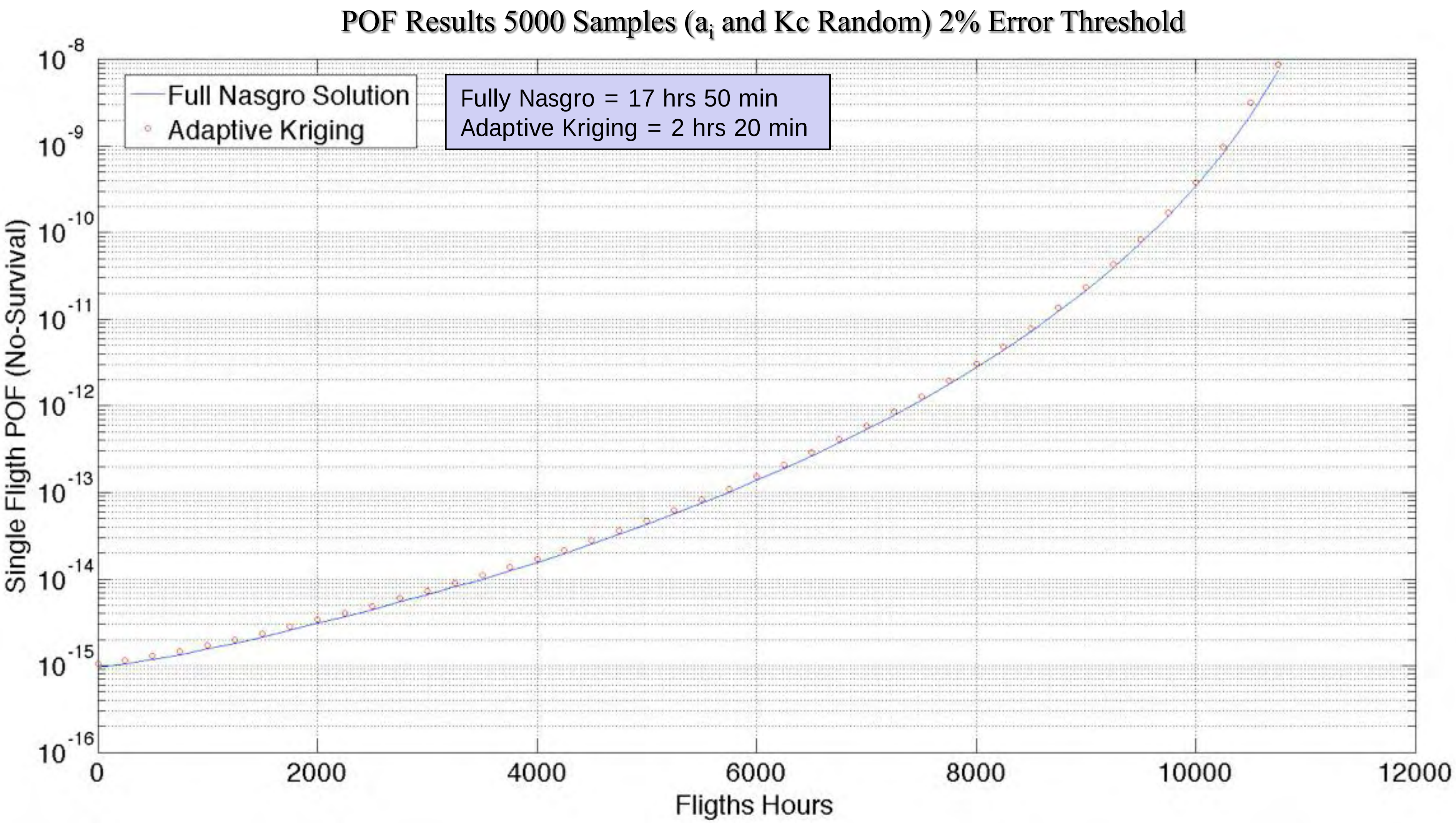
| Dur/Wei | | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 |
|---|---|---|---|---|---|---|
| 0.50: | 0.05 | 0.00 | 0.00 | 0.00 | 0.20 | 0.80 |
| 0.60: | 0.05 | 0.00 | 0.00 | 0.00 | 0.20 | 0.80 |
| 0.70: | 0.10 | 0.00 | 0.00 | 0.00 | 0.15 | 0.85 |
| 0.80: | 0.15 | 0.00 | 0.00 | 0.00 | 0.15 | 0.85 |
| 0.90: | 0.20 | 0.00 | 0.00 | 0.00 | 0.10 | 0.90 |
| 1.00: | 0.25 | 0.00 | 0.00 | 0.00 | 0.10 | 0.90 |
| 1.10: | 0.15 | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |
| 1.20: | 0.05 | 0.00 | 0.00 | 0.00 | 0.05 | 0.95 |

| Variable | Value |
|---|---|
| Average Velocity (Vno/Vmo (Knots)) | 165 |

# High Performance Aircraft no Inspection



# High Performance Aircraft no Inspection



# Commuter Aircraft with Inspections

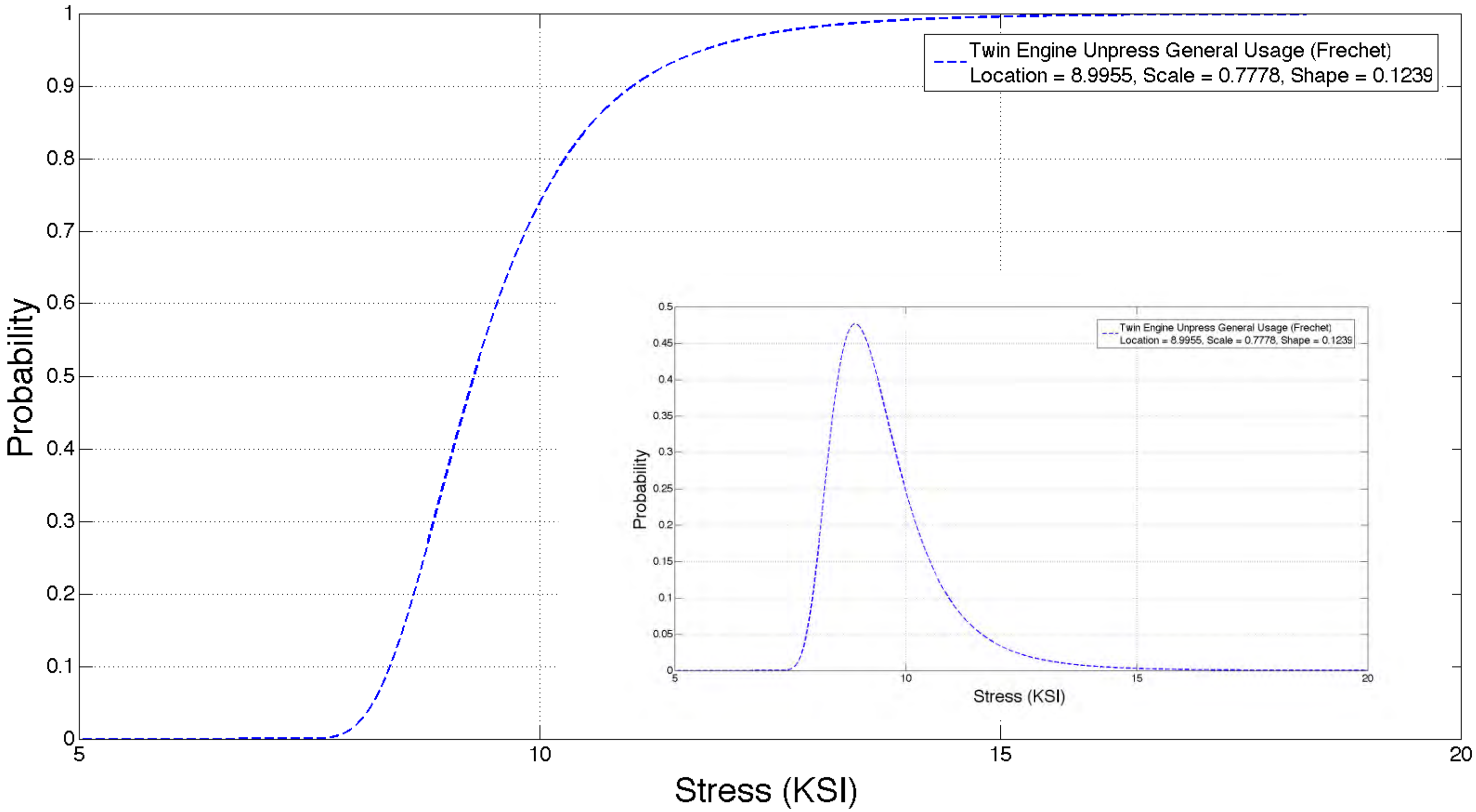

| Quantity | Definition |
|---|---|
| Nasgro Crack Growth Model. | TC03 – Through crack in a hole |
| Geometric Variables | Width = 2.5 in.<br>Thickness = 0.15 in.<br>Hole Diameter = 0.10 in.<br>Hole Offset = 0.5 in. |
| Fracture Toughness Distribution | Normal:<br>Mean = 40.0 ksi√in.<br>Standard Deviation = 4.0 ksi√in. |
| Initial Crack Size Distribution | Lognormal<br>Median = 0.050 in.<br>Mean = ln(median) = -2.995<br>Standard Deviation = 0.001 |
| Material | Al-2024 |

# Commuter Aircraft with Inspections

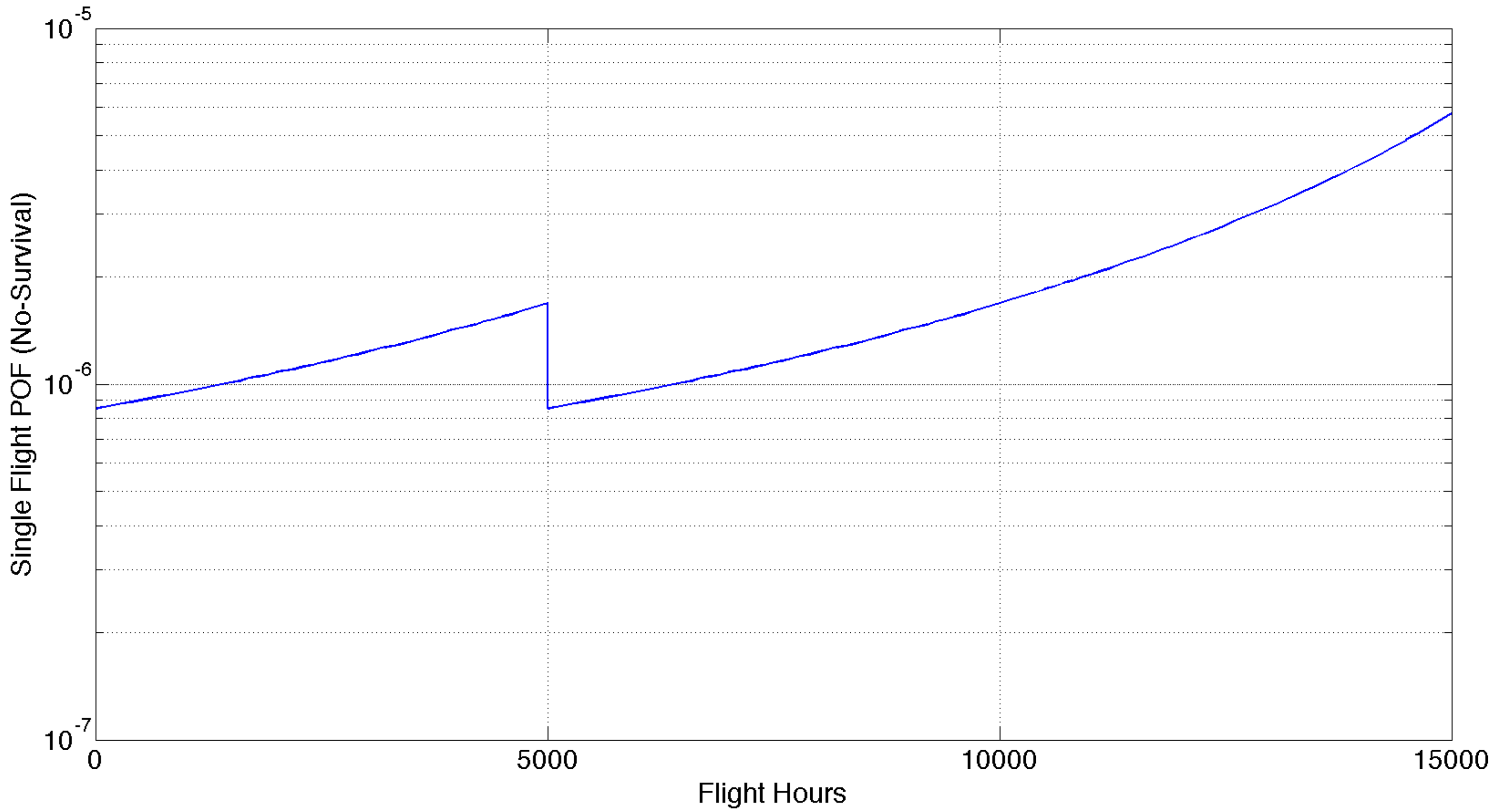
| Variable | Value |
|---|---|
| Usage | Twin Engine Unpressurized Basic Executive Usage |
| Design LLF Maneuver | 3.2, -1.5 |
| Design LLF Gust | 3.2, -1.2 |
| Ground Stress (psi) | -4,000 |
| One-g stress (psi) | 5,100 |
| Flight Length and Velocity Matrix | Deterministic (1 hr. Duration) |
| Flight Length and Weight Matrix | deterministic |
| Average Velocity (Vno/Vmo (Knots)) | 165 |

| Quantity | Definition |
|---|---|
| Inspection Time | 5,000 |
| Probability of Inspection | 1.0 |
| Probability of Detection | Lognormal<br>Median = 0.00390 in.<br>Mean = ln(median) = -5.545 in.<br>Standard Deviation = 1.113 in. |
| Repair Crack Size Distribution | Lognormal<br>Median = 0.050 in.<br>Mean = ln(median) = -2.995<br>Standard Deviation = 0.001 |

# Commuter Aircraft with Inspections



# Commuter Aircraft with Inspections



# Future & Current Work

**SMART$_{DT}$**

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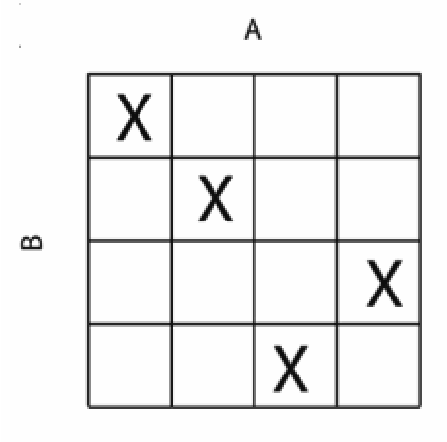
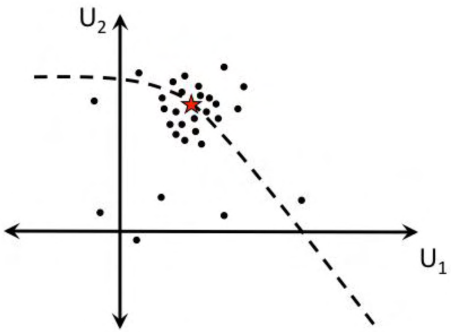
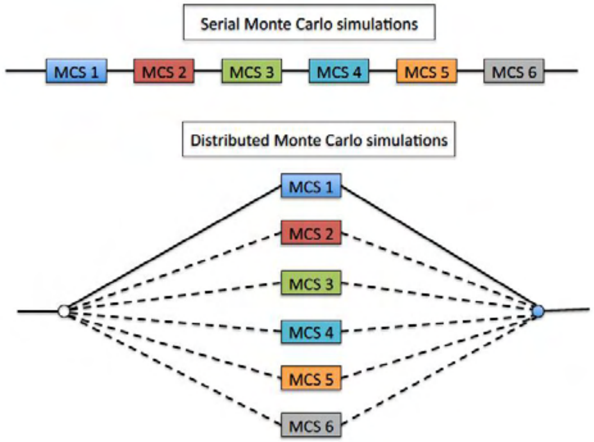
# Current/Future Work

✓ Improved sampling methods:




✓ High Performance Computing:

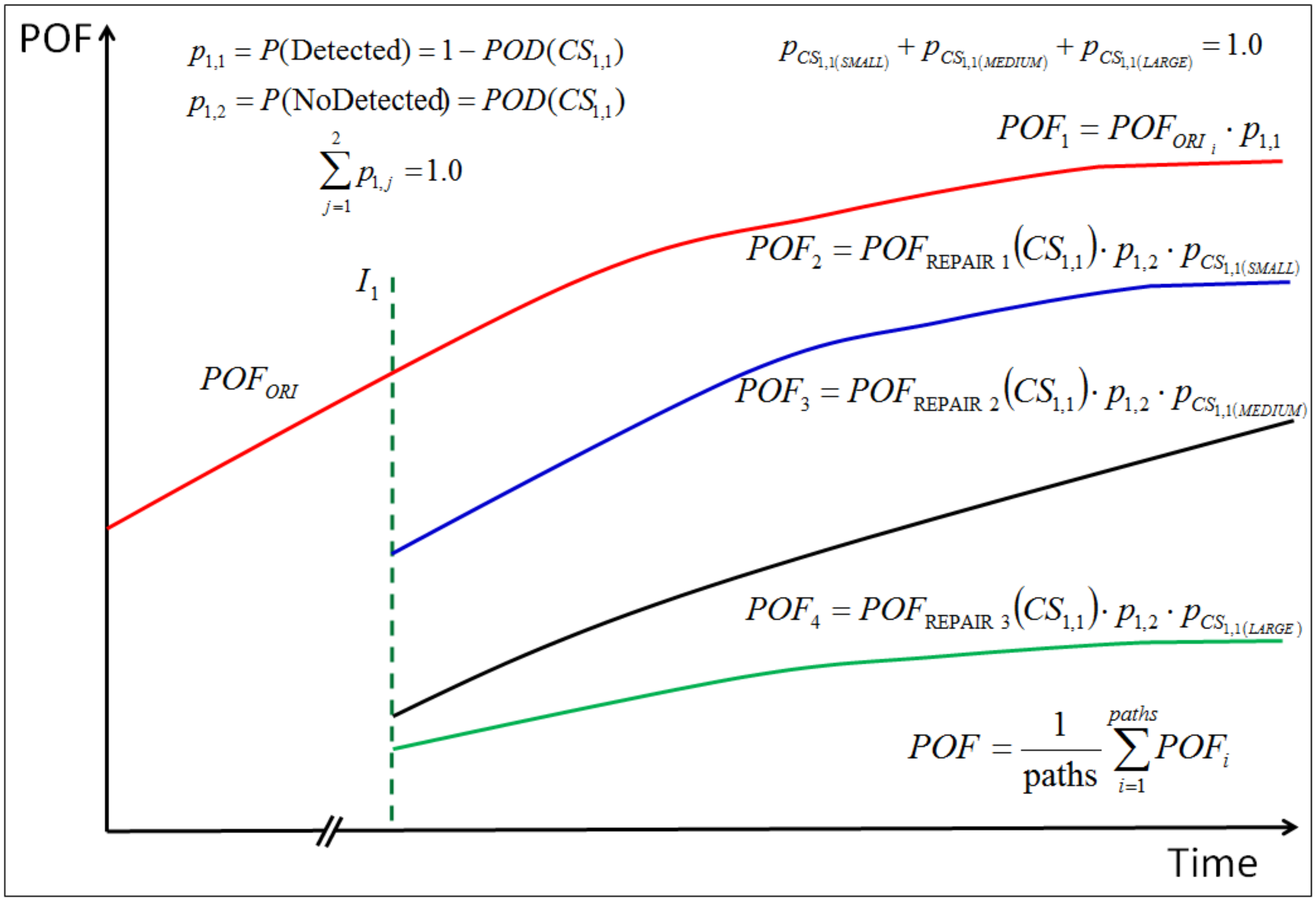



NVIDIA CUDA

CUDA Fortran

# Current/Future Work

✓ Extension to Different repair scenarios

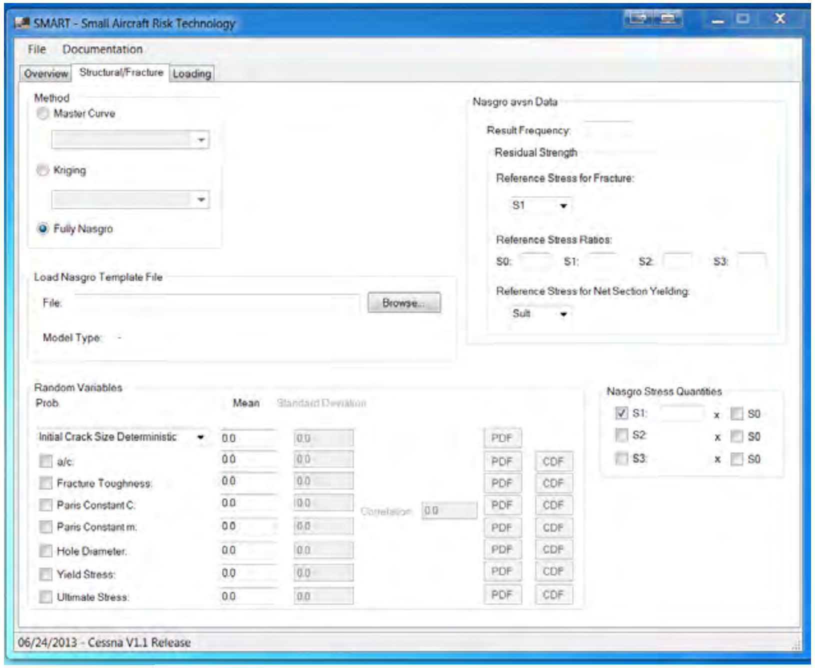



Simple Oversize

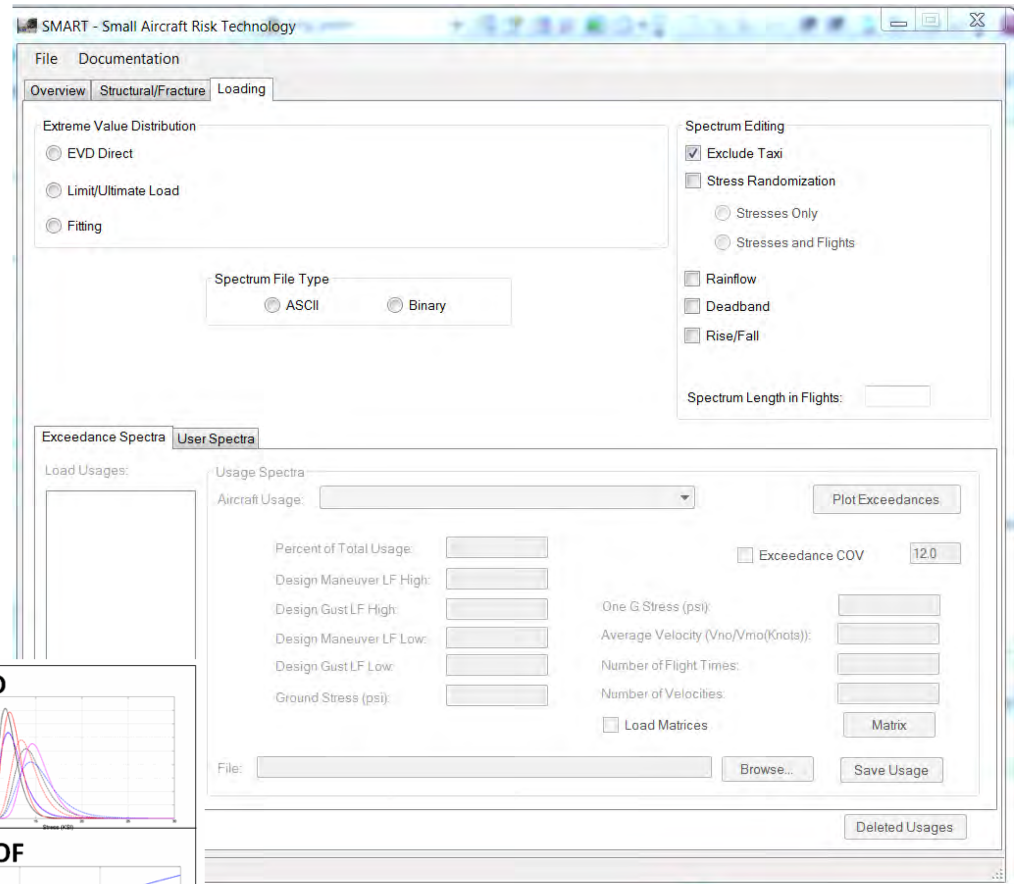
Minor Repair (patch)

Major Repair (Replacement)

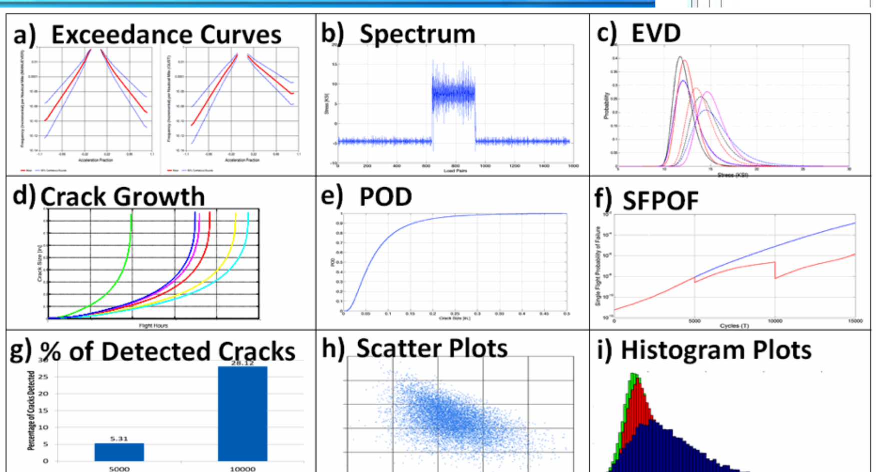
# GUI



SMART - Small Aircraft Risk Technology
File Documentation
Overview Structural/Fracture Loading
Method
Master Curve
Kriging
Fully Nasgro
Nasgro avsn Data
Result Frequency:
Residual Strength
Reference Stress for Fracture:
S1
Reference Stress Ratios:
S0: S1: S2: S3:
Reference Stress for Net Section Yielding:
Sult
Load Nasgro Template File
File:
Browse...
Model Type:
Random Variables
Prob.
Mean
Standard Deviation
Initial Crack Size Deterministic
a/c:
Fracture Toughness:
Paris Constant C:
Paris Constant m:
Hole Diameter:
Yield Stress:
Ultimate Stress:
Correlation
PDF
CDF
Nasgro Stress Quantities
S1: x S0
S2: x S0
S3: x S0
06/24/2013 - Cessna V1.1 Release

SMART - Small Aircraft Risk Technology
File Documentation
Overview Structural/Fracture Loading
Extreme Value Distribution
EVD Direct
Limit/Ultimate Load
Fitting
Spectrum Editing
Exclude Taxi
Stress Randomization
Stresses Only
Stresses and Flights
Spectrum File Type
ASCII
Binary
Rainflow
Deadband
Rise/Fall
Spectrum Length in Flights:
Exceedance Spectra
User Spectra
Load Usages:
Usage Spectra
Aircraft Usage:
Plot Exceedances
Percent of Total Usage:
Design Maneuver LF High:
Design Gust LF High:
Design Maneuver LF Low:
Design Gust LF Low:
Ground Stress (psi):
Exceedance COV
12.0
One G Stress (psi):
Average Velocity (Vno/Vmo(Knots)):
Number of Flight Times:
Number of Velocities:
Load Matrices
Matrix
File:
Browse...
Save Usage
Deleted Usages

a) Exceedance Curves
b) Spectrum
c) EVD
d) Crack Growth
e) POD
f) SFPOF
g) % of Detected Cracks
5.31
28.12
5000
10000
Inspection Time
Percentage of Cracks Detected
h) Scatter Plots
i) Histogram Plots

# Acknowledgements



- Probabilistic Structural Risk Assessment and Risk Management for Small Airplanes, Sep 2007- Dec 2010, Federal Aviation Administration, Grant 07-G-011
- Probabilistic Damage Tolerance-Based Maintenance Planning for Small Airplanes, Sep. 2009-Aug. 2012, Federal Aviation Administration, Grant 09-G-016
- Probabilistic Fatigue Management Program for General Aviation, Sep. 2012-Aug. 2016, Federal Aviation Administration, Grant 12-G-012

# Questions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