PROBABILISTIC DAMAGE TOLERANCE WITH BAYESIAN UPDATING

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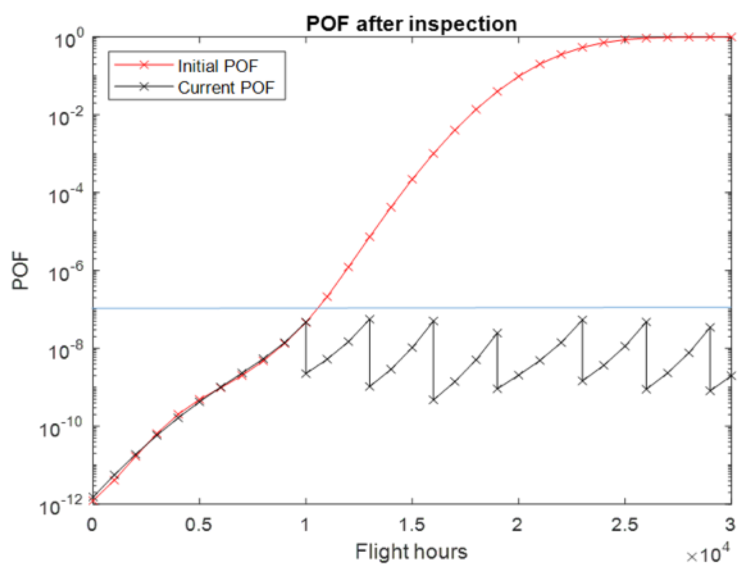
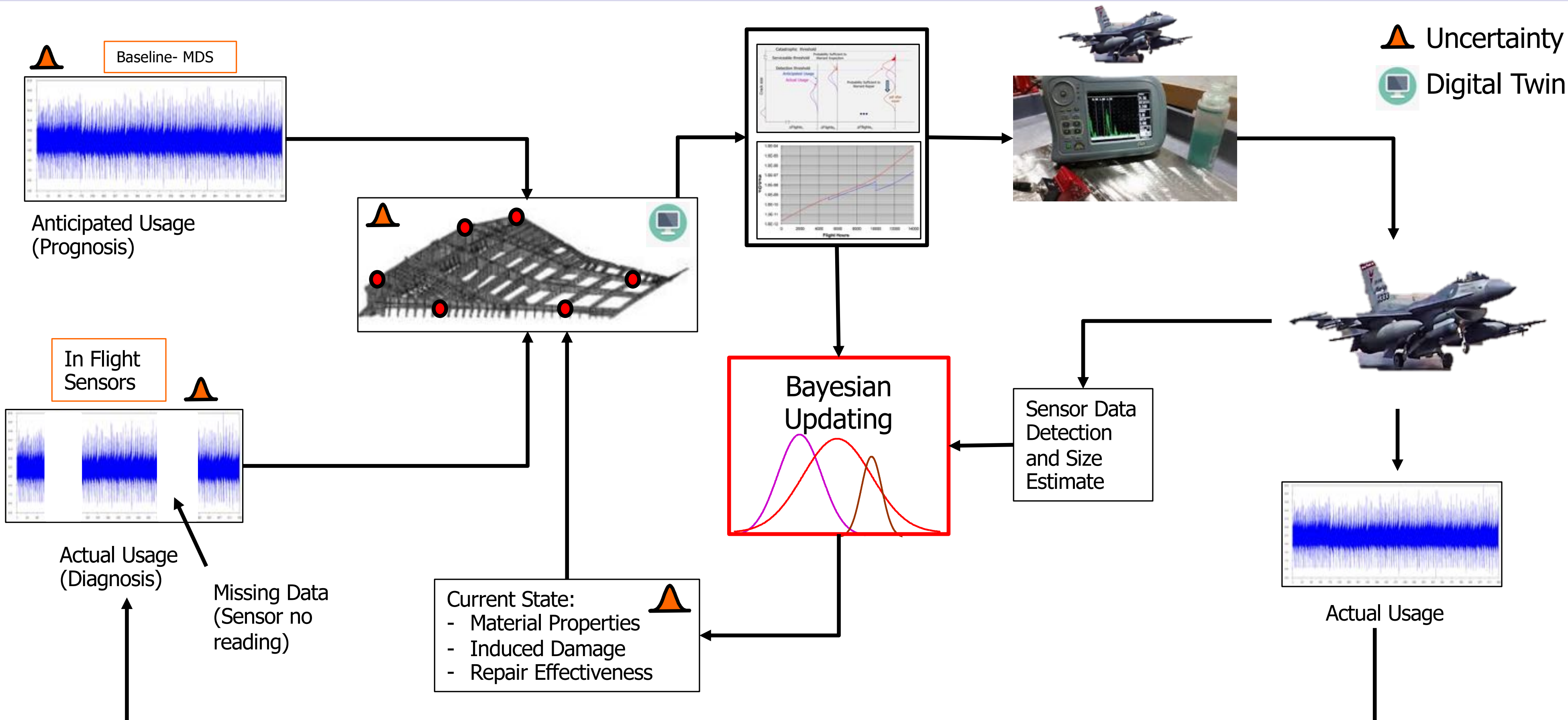
Overview

- Motivation
- SMART Probabilistic Damage Tolerance Analysis Quick Review
- Bayes Theorem
- Script
- Examples
- Conclusion

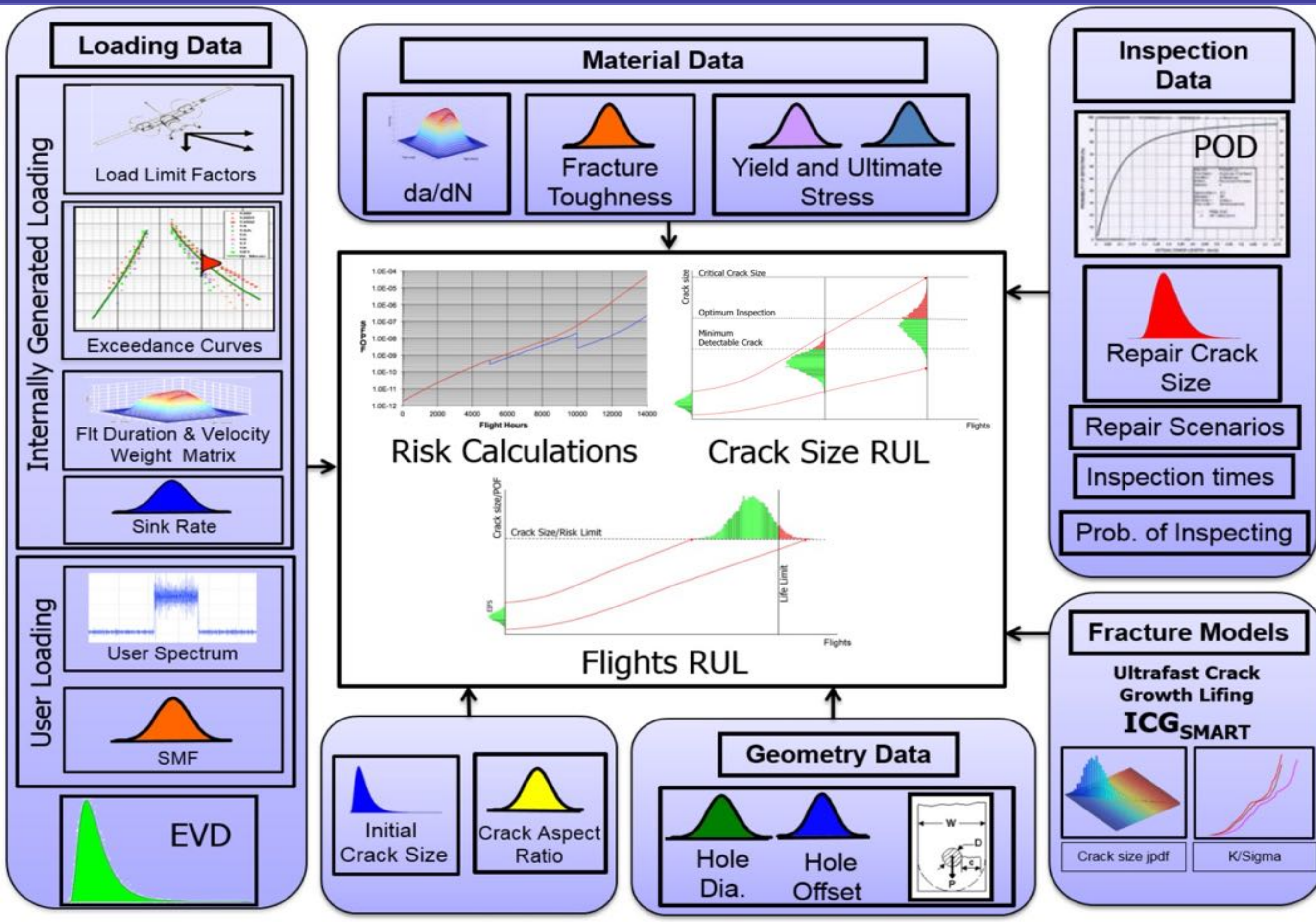


Motivation

Motivation - Digital Twin



SMART|DT



Loading Data
Internally Generated Loading
Load Limit Factors
Exceedance Curves
Flt Duration & Velocity Weight Matrix
Sink Rate
User Loading
User Spectrum
SMF
EVD
Material Data
da/dN
Fracture Toughness
Yield and Ultimate Stress
Risk Calculations
Crack Size RUL
Critical Crack Size
Optimum Inspection
Minimum Detectable Crack
Flights RUL
Crack Size/Risk Limit
Life Limit
Flights
Inspection Data
POD
Repair Crack Size
Repair Scenarios
Inspection times
Prob. of Inspecting
Fracture Models
Ultrafast Crack Growth Lifing
ICG SMART
Crack size jpdf
K/Sigma
Initial Crack Size
Crack Aspect Ratio
Geometry Data
Hole Dia.
Hole Offset

Bayes theorem

Likelihood, Prior distribution

$$P^{+}(\theta|\mathbf{D}) = \frac{L(\mathbf{D}|\theta)\cdot P^{-}(\theta)}{NF}$$

Posterior distribution

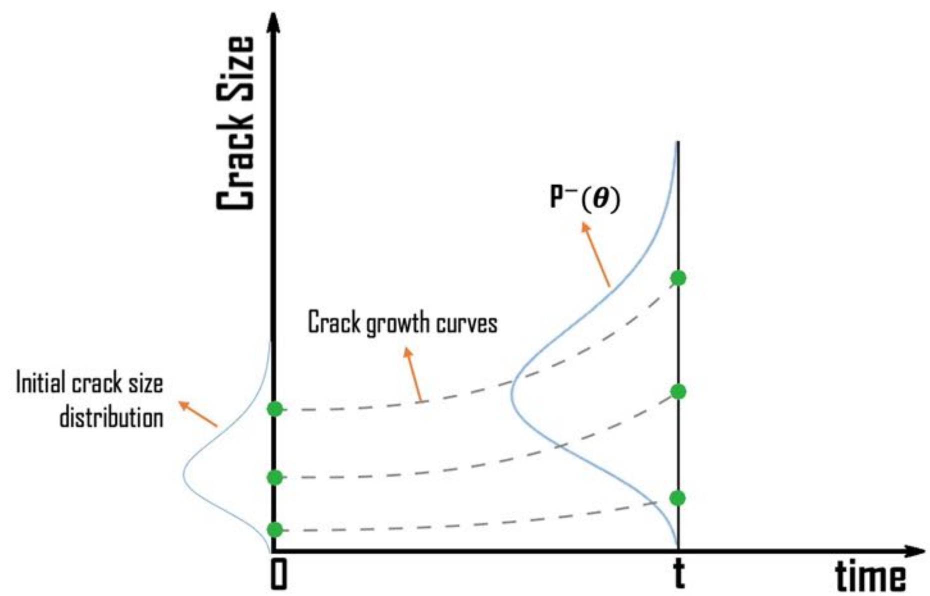
Normalization factor

- $\boldsymbol{\theta}$ represents the parameters mean(μ) →independent variable and standard deviation(σ)→assumed, it will be fixed,
- $\mathbf{D}$ represents the vector of the measurements (or inspections),
- $\mathbf{P}^{-}$ represents the prior distribution → Distribution of crack size at the time,
- $\mathbf{L}(\mathbf{D}|\boldsymbol{\theta})$ represents the likelihood function of the parameters.
- **NF** Normalization Factor, used to get a probability density function.
- $\mathbf{P}^{+}$ represents the posterior distribution given the detected crack sizes.

Bayesian Updating

$$\underbrace{P^{+}(\theta|\mathbf{D})}_{\text{Posterior distribution}} = \frac{\overbrace{L(\mathbf{D}|\theta)}^{\text{Likelihood}} \cdot \overbrace{P^{-}(\theta)}^{\text{Prior distribution}}}{NF}$$

NF: Normalization factor



Bayesian Updating

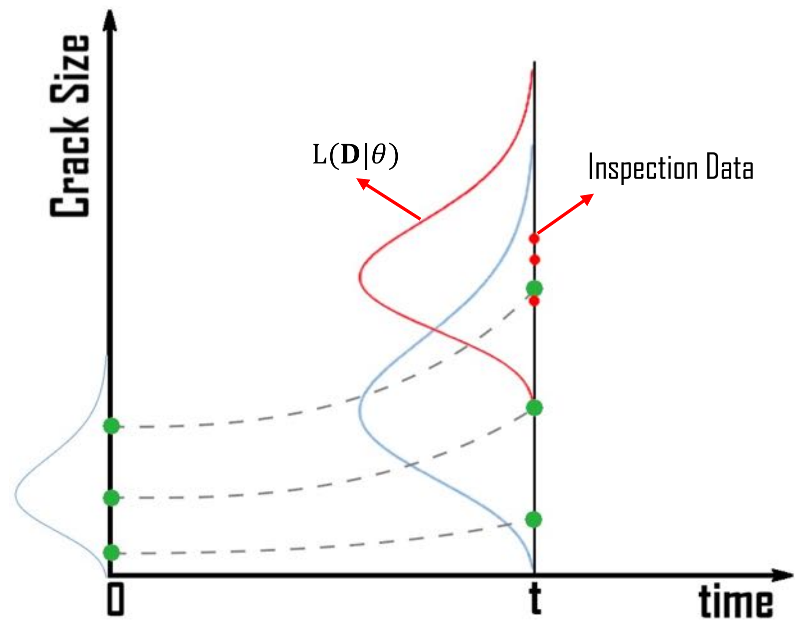
$$P^{+}(\theta|\mathbf{D}) = \frac{L(\mathbf{D}|\theta)\cdot P^{-}(\theta)}{NF}$$

Likelihood: $L(\mathbf{D}|\theta)$

Prior distribution: $P^{-}(\theta)$

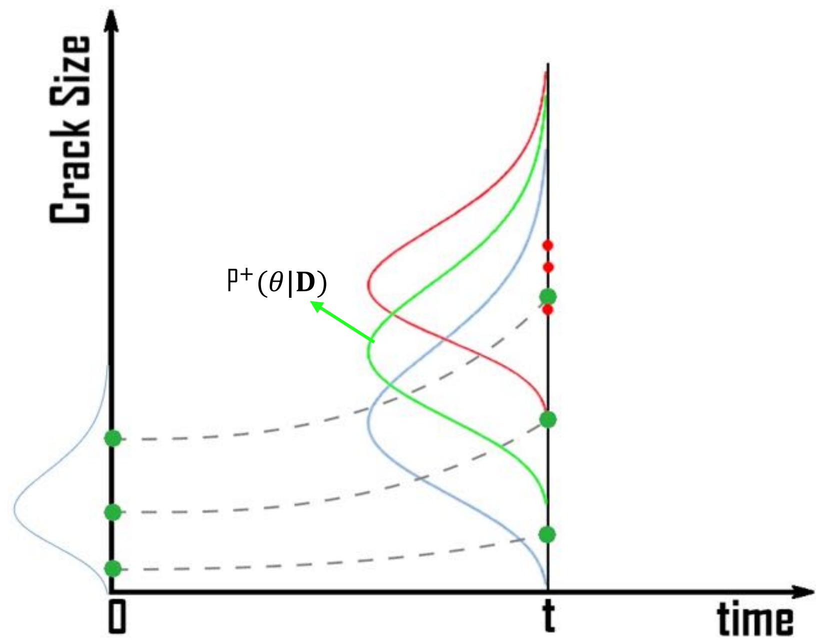
Posterior distribution: $P^{+}(\theta|\mathbf{D})$

Normalization factor: NF

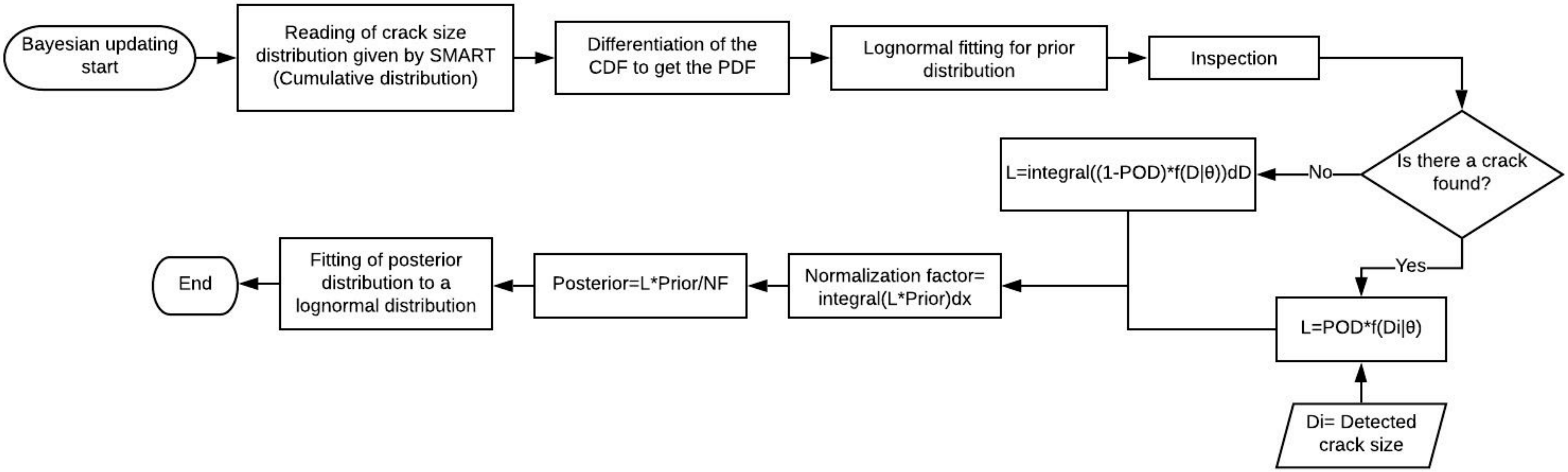


Bayesian Updating

$$\underbrace{P^{+}(\theta|\mathbf{D})}_{\text{Posterior distribution}} = \frac{\overbrace{L(\mathbf{D}|\theta)}^{\text{Likelihood}} \cdot \overbrace{P^{-}(\theta)}^{\text{Prior distribution}}}{\underset{\text{Normalization factor}}{NF}}$$



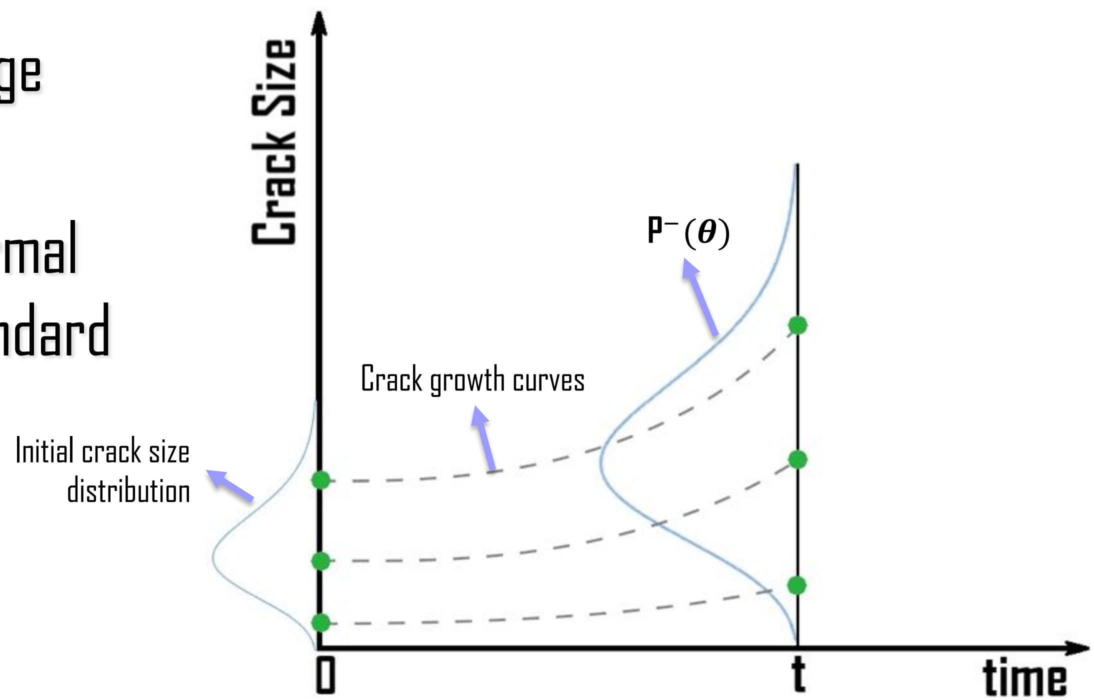
Flowchart



$f(D|\boldsymbol{\theta})$ Represents the distribution of means for the crack found D_i.

1. Prior distribution $\mathbf{P}^{-}(\boldsymbol{\theta})$

- It is known, based on the damage tolerance model at time t.
- Assumed that follows a log-normal distribution with mean and standard deviation known.



2. Likelihood $\mathrm{L}(\mathbf{D}|\theta)$

$$\mathrm{L}(\mathbf{D}|\theta)=\underbrace{\mathrm{L}_{D}(\theta)}_{\text{Likelihood of Detection}} \cdot \underbrace{\mathrm{L}_{ND}(\theta)}_{\text{Likelihood of \textbf{NO} Detection}}$$

- The likelihood function reflects the degree of agreement between the obtained measurements, D, and the output obtained from the mathematical model (Log-normal distribution) used to physically describe the system.
- It will be dependent on each inspection, and whether a crack is found or not.

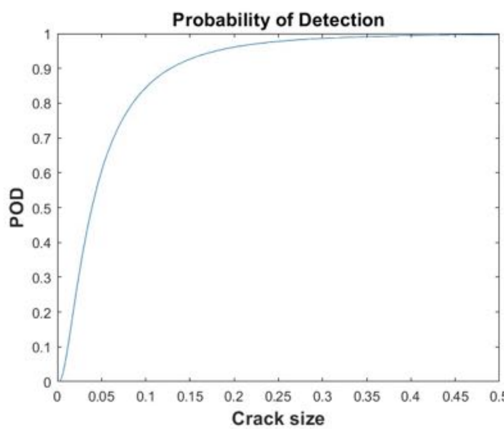
[1] A. Lye, A. Cicirello, and E. Patelli, "Sampling methods for solving Bayesian model updating problems: A tutorial," *Mech. Syst. Signal Process.*, vol. 159, p. 107760, 2021, doi: 10.1016/j.ymssp.2021.107760.

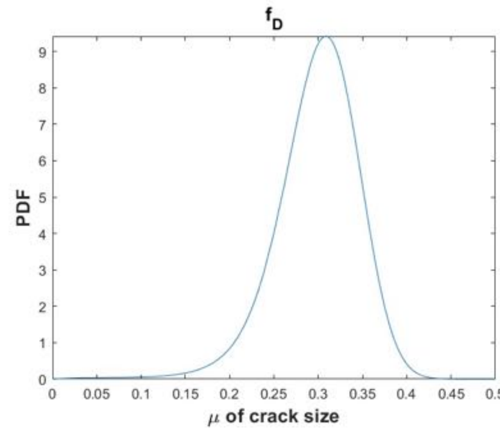
Likelihood of Detection $\mathrm{L}_D(\theta)$

$$\mathrm{L}_D(\theta) = \prod_{i=1}^{N_D} \underbrace{POD_i(D_i(t_i))}_{} \cdot \underbrace{f(D_i(t_i)|\theta)}_{}$$

Eddy current testing

Taken from: https://www.olympus-ims.com/en/insight/ndt-level-3-eddy-current-testing/





Represents the distribution of means for the crack found D_i.

Function $POD_i(D_i(t_i))$

- First, the probability of detection curve from the inspection method is used. It will follow a log-normal distribution with mean μ and standard deviation of σ

$$\text{LogNormal}(x|\varphi, h) = \frac{1}{x \cdot h \cdot \sqrt{2\pi}} \cdot exp\left\{\frac{-(\log(x) - \varphi)^2}{2 \cdot h^2}\right\} \quad \text{for } x > 0$$

$$\varphi = Log\left(\frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}}\right)$$

$$h = \sqrt{Log\left(\frac{\sigma^2}{\mu^2} + 1\right)}$$

Function $f(D_i(t_i))$

- It is defined as:

$$\text{LogNormal}(D|\varphi, h) = \frac{1}{D\cdot h\cdot\sqrt{2\pi}}\cdot exp\left\{\frac{-(\log(D)-\varphi)^2}{2\cdot h^2}\right\}$$, for D>0

$\varphi = Log\left(\frac{\mu^2}{\sqrt{\sigma^2+\mu^2}}\right)$, where μ →Independent variable and $\sigma = \sigma_{\text{prior}}$

$$h = \sqrt{Log\left(\frac{\sigma^2}{\mu^2}+1\right)}$$

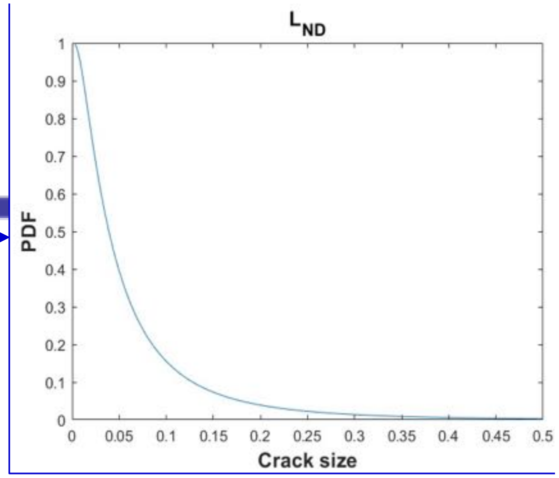
Likelihood for detected cracks

$$f(D_i(t_i) \mid \theta)$$

- Is a probability density function that represents the distribution of parameters for the crack detected.
- For this pdf, we know the crack size D, as random variable with mean D.
- The standard deviation can be:
  - Estimated as the mean squared error of (D_k - M($\boldsymbol{\theta}$)).
  - Set it as a fixed parameter based on prior calculations or knowledge.

Likelihood of **NO** Detection $L_{ND}(\theta)$

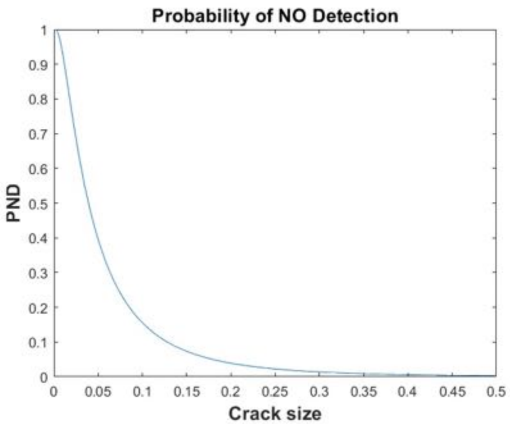
$$L_{ND}(\theta) = \prod_{i=1}^{N_{ND}} \int_0^{\infty} PND_i(D(t_i)) \cdot f(D(t_i)|\theta) dD$$

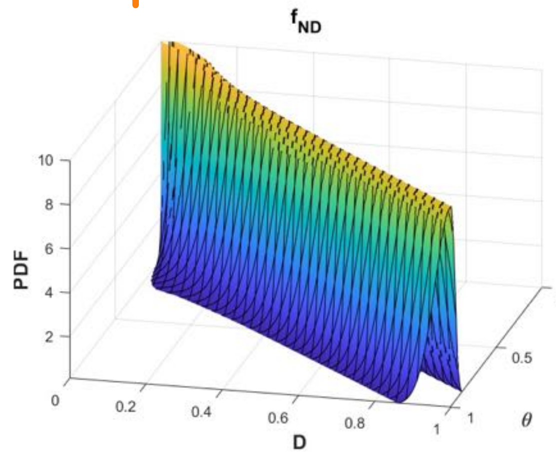


Eddy current testing

Taken from: https://www.olympus-ims.com/en/insight/ndt-level-3-eddy-current-testing/

PND=1-POD



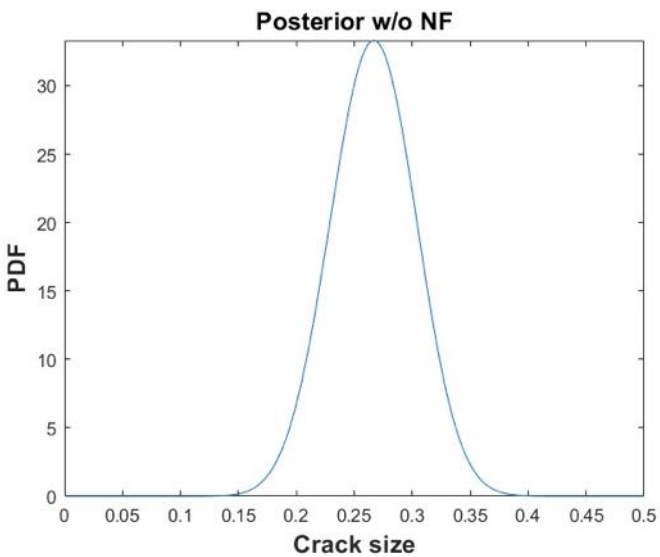


For the PDF we followed the same methodology than for detected data, but now we have D as random variable.
The integral considers all the values D can take.

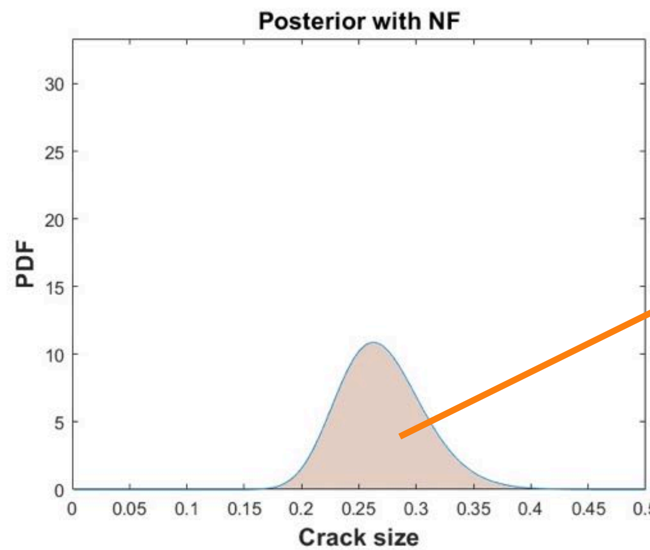
3. Normalization Factor

$$NF=\int_0^\infty \underbrace{L(\mathbf{D}|\theta)}_{\text{Likelihood}}\cdot \underbrace{P^-(\theta)}_{\text{Prior distribution}}\cdot d\theta$$

It's a normalization factor, so when we integrate the posterior distribution the cumulative density function is equal to 1.



NF →

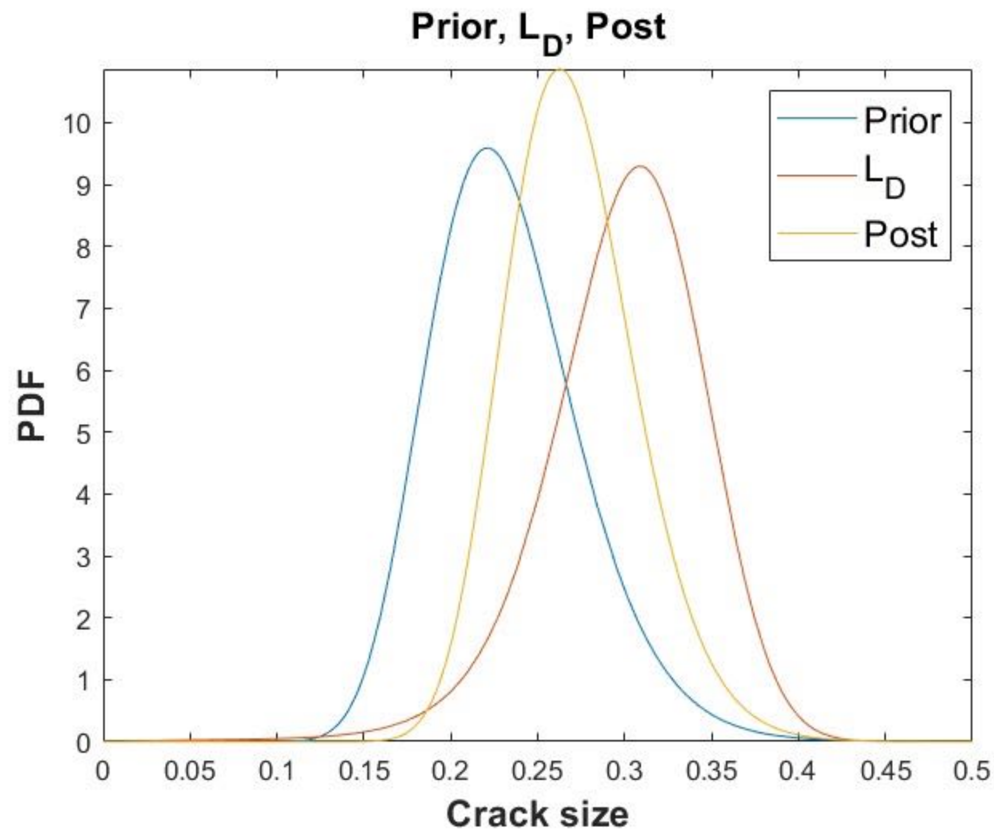


$$\int_0^\infty P^+(\theta|\mathbf{D}) = 1$$

3. Posterior $P^{+}(\theta|\mathbf{D})$

- After computing the posterior distribution, we fit that expression to a log-normal distribution and get the new parameters.

$$P^{+}(\theta|\mathbf{D}) = \frac{L(\mathbf{D}|\theta) \cdot P^{-}(\theta)}{NF}$$

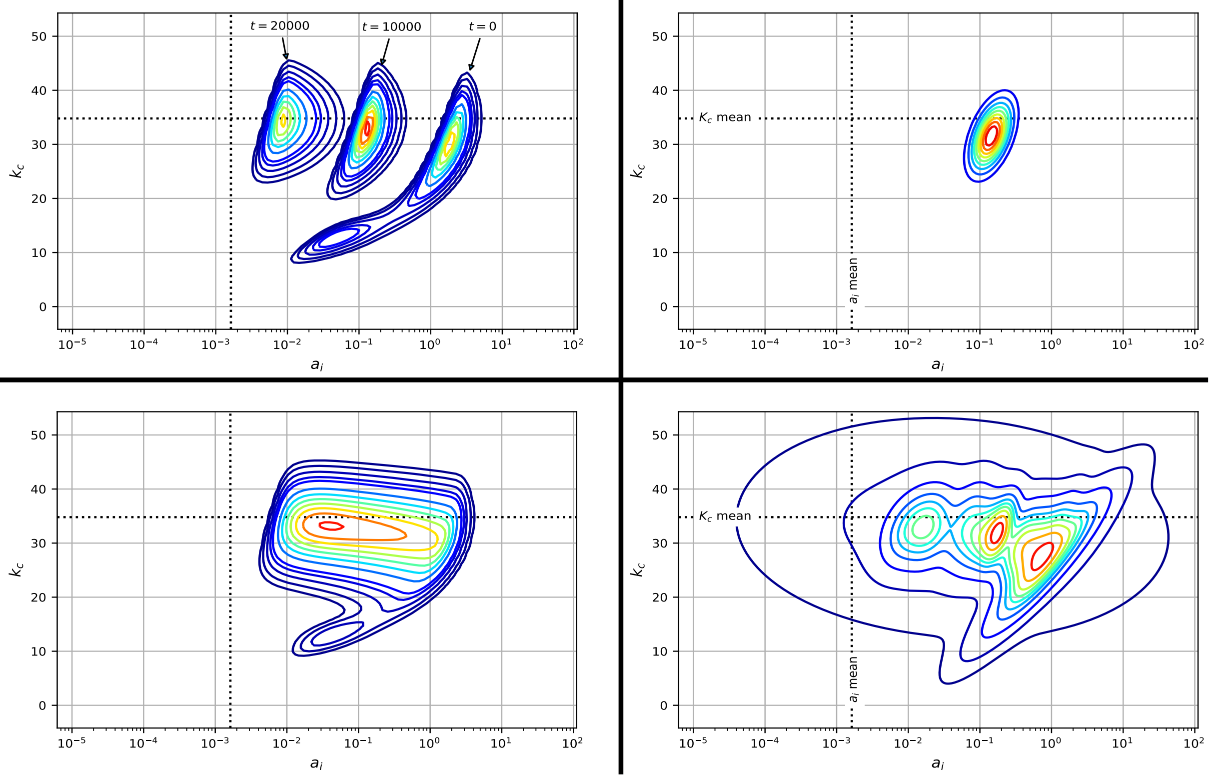


Multiple Importance Sampling Approach for PDTA

Important Region(s) | Adapted Sampling Region

Individual Times

Multiple Times

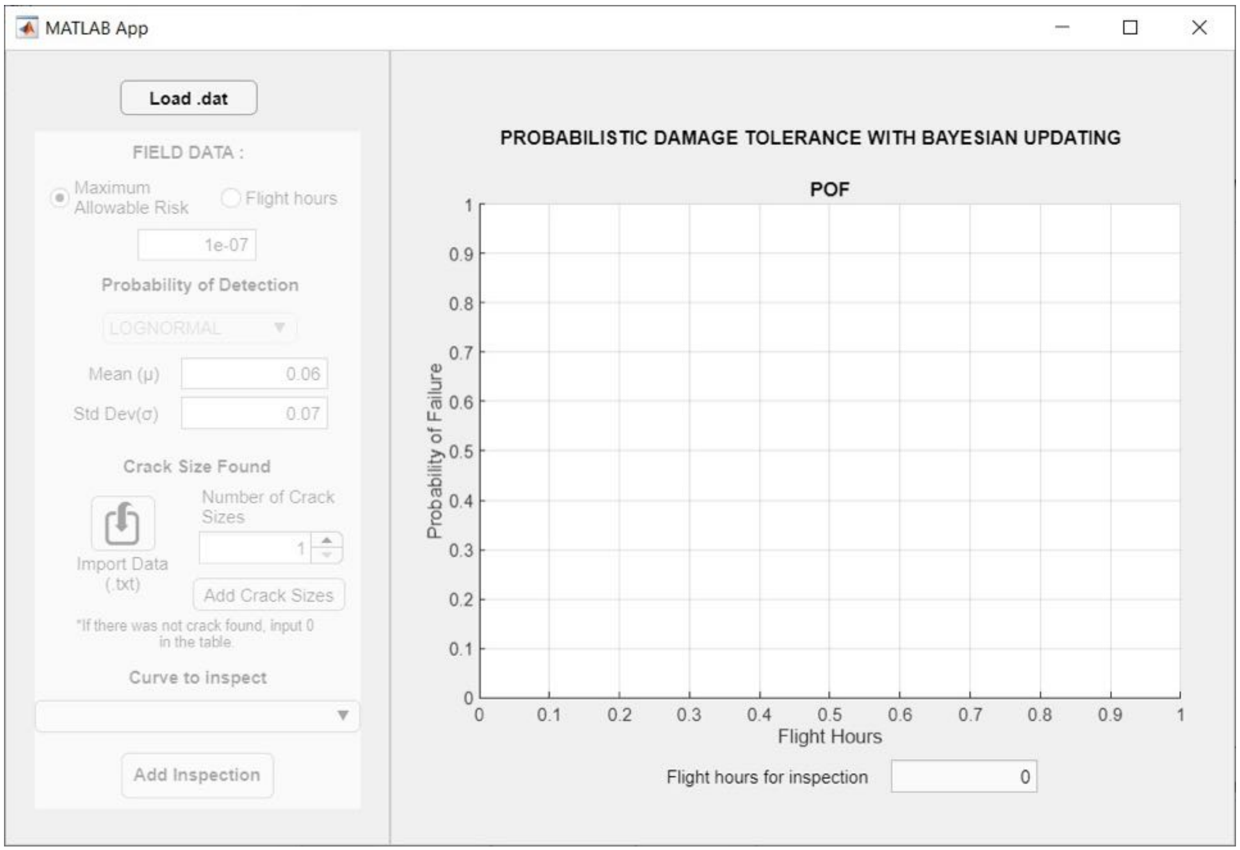


- Basic Importance sampling
  - Adapt single sampling densities for individual evaluation times
- Multiple Importance Sampling
  - Adapt a mixture density for a range of evaluation times

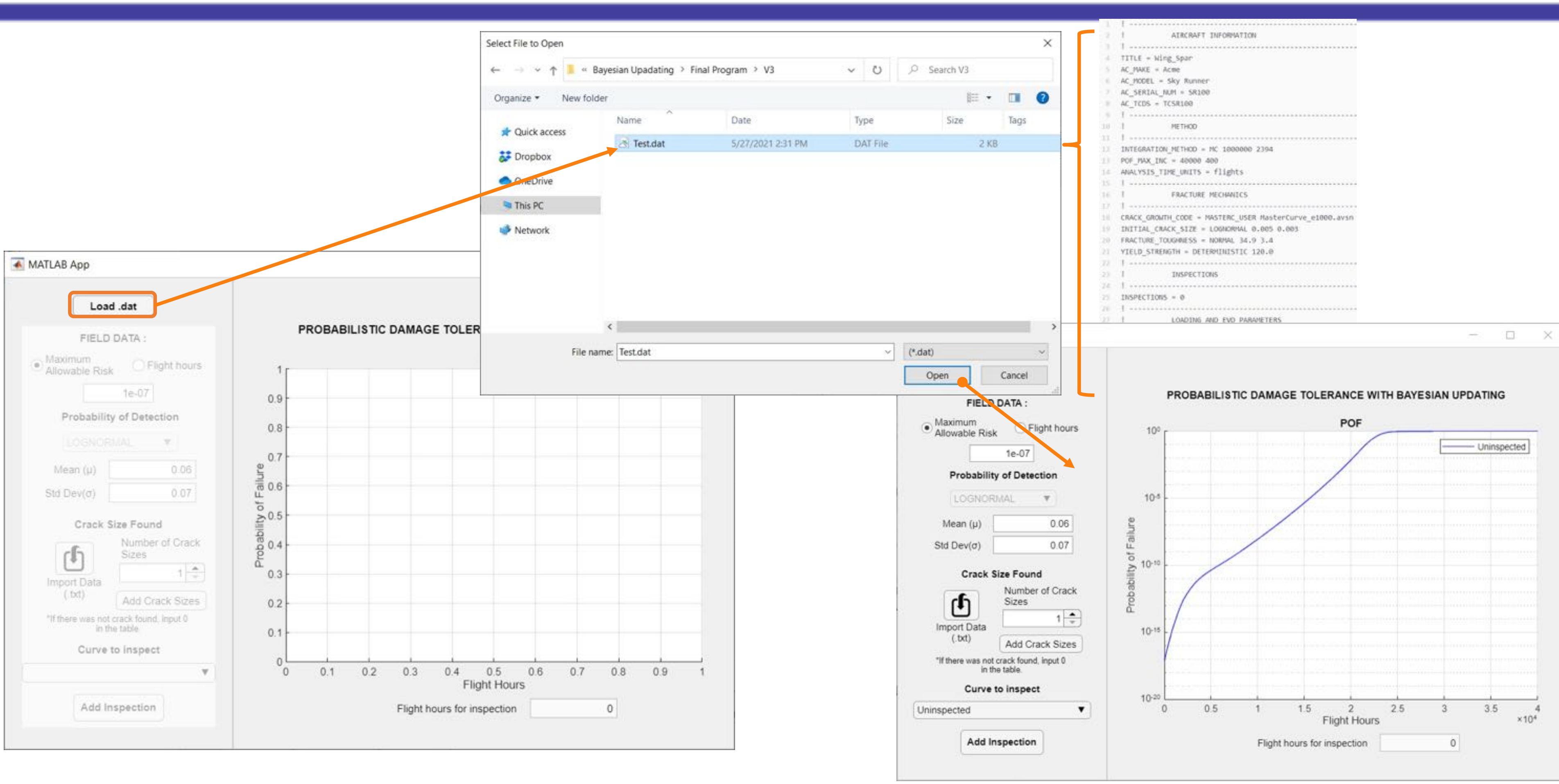
The PDTA AMIS algorithm estimates POF for PDTA using 6 orders of magnitude fewer samples compared to SMC for probabilities of 10^{-7}

MATLAB Script

Program interface



1. Reading .dat

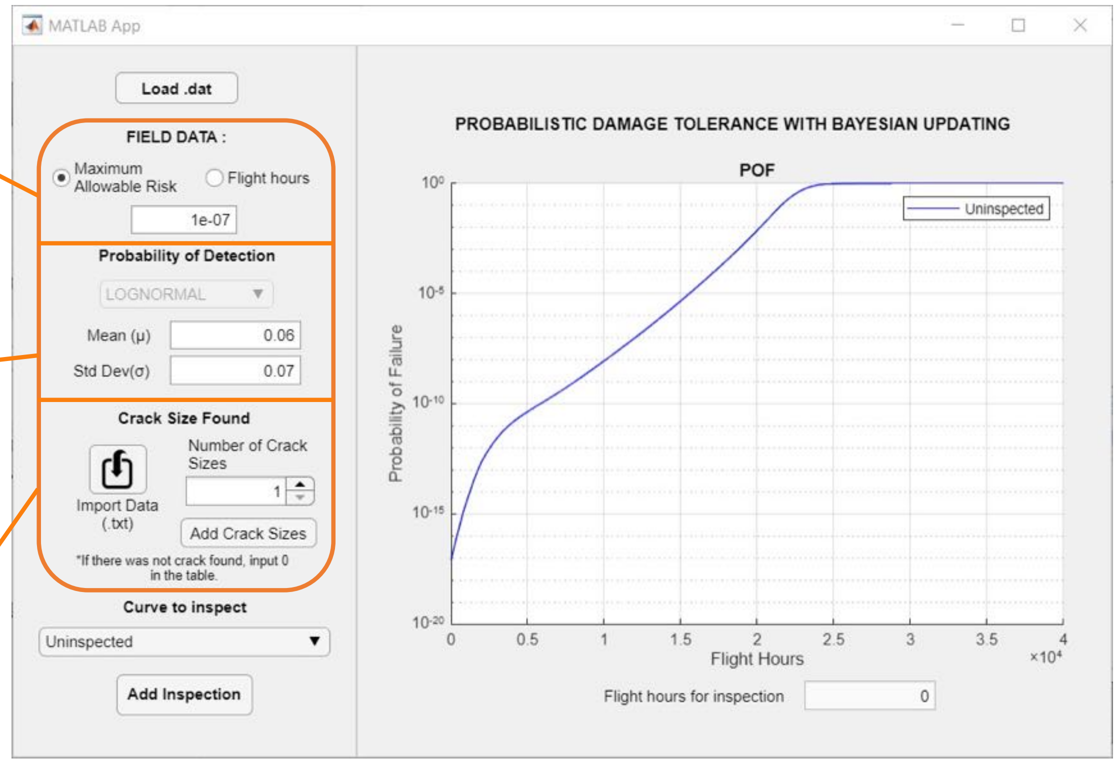


2. Input Field Data

Choose the risk or time to perform an inspection

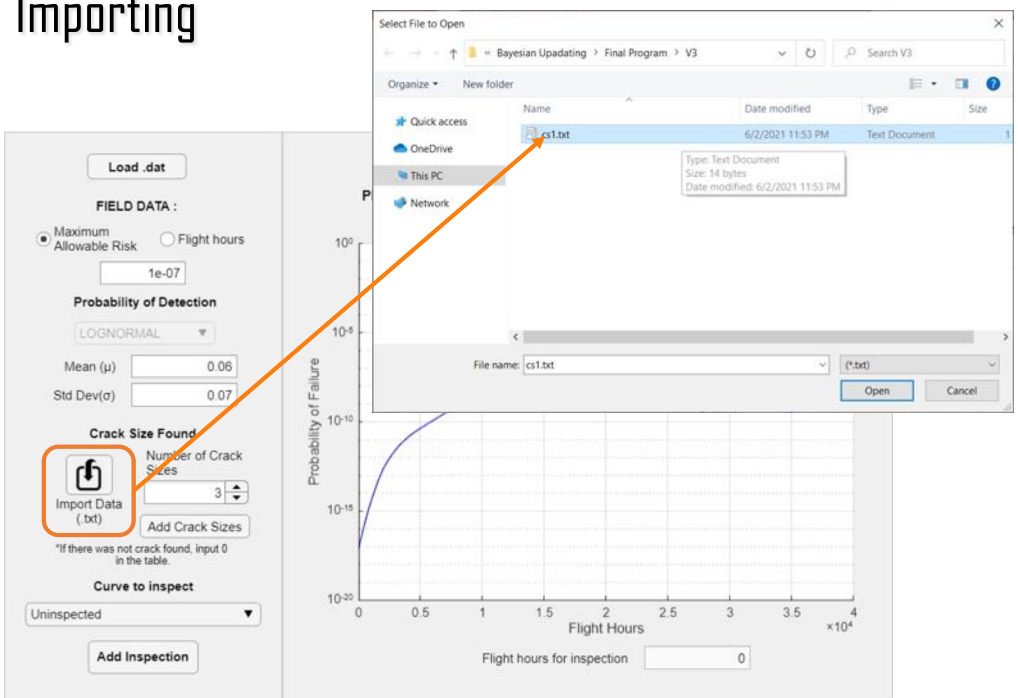
Input the Probability of Detection (POD) parameters from the inspection method

Two options to add crack sizes: Importing a txt file or manually inputting them. (No limit on the number of crack sizes)

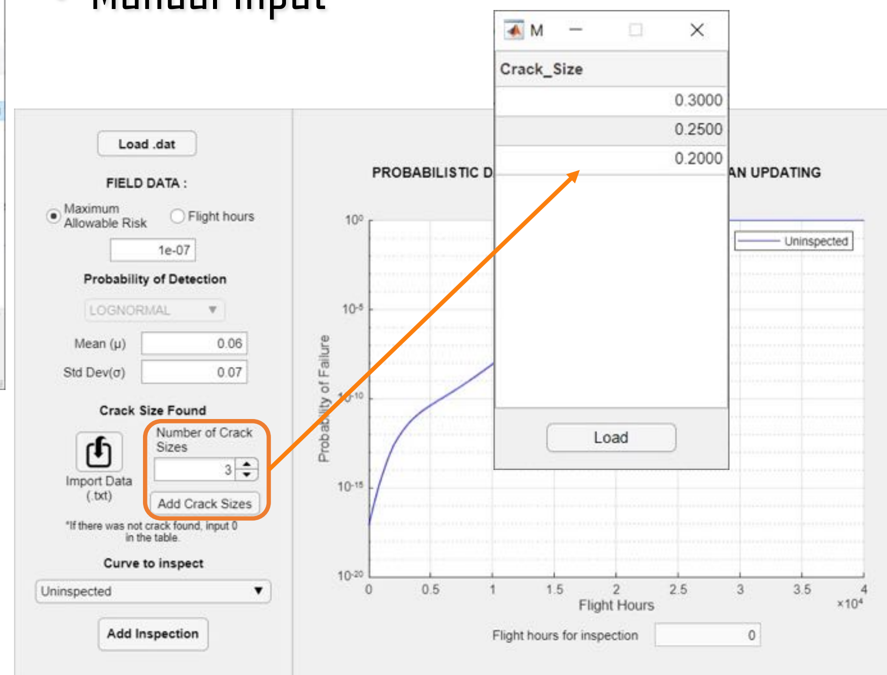


2.1 Crack Size Found During Inspection

- Importing



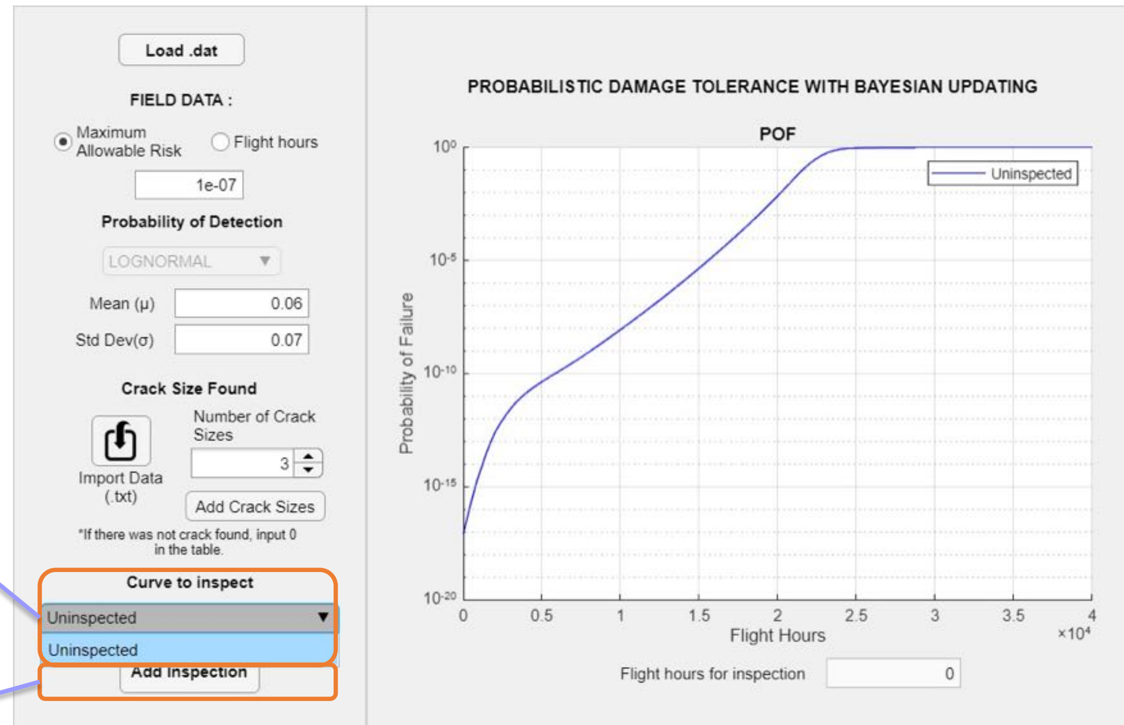
- Manual Input



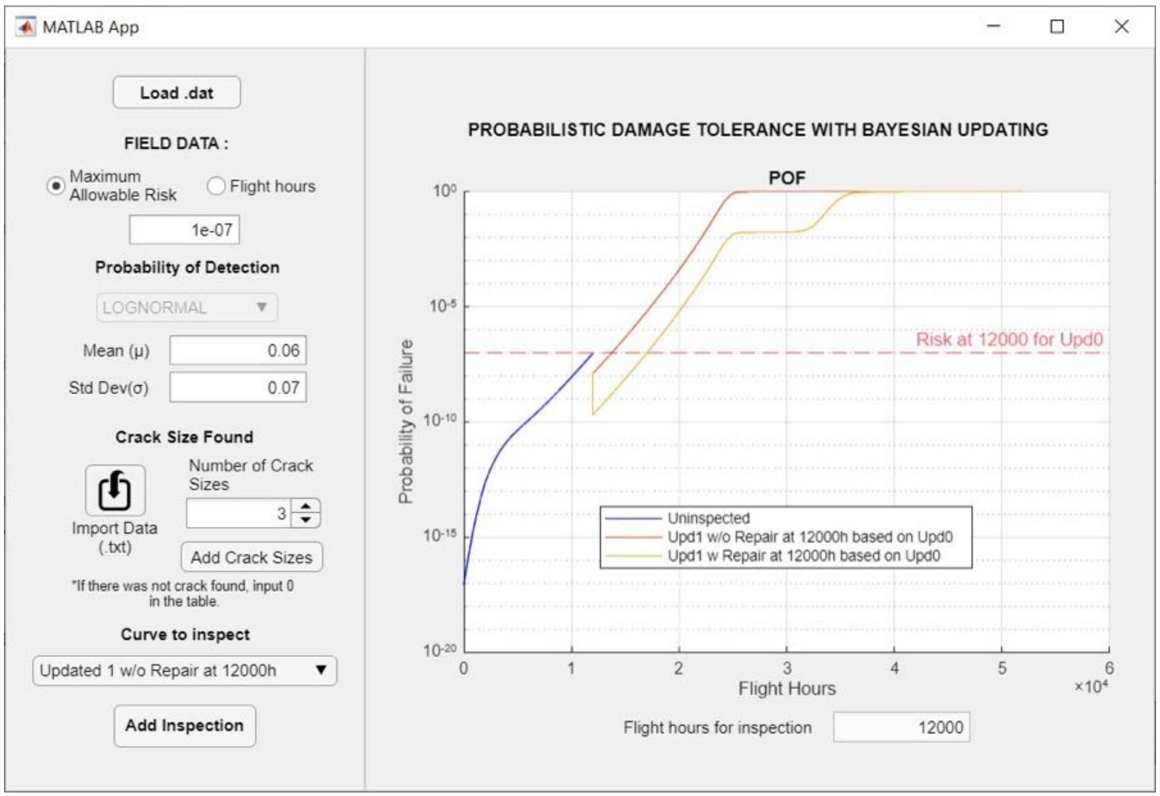
2.2. Select a POF curve in which the Bayesian is performed

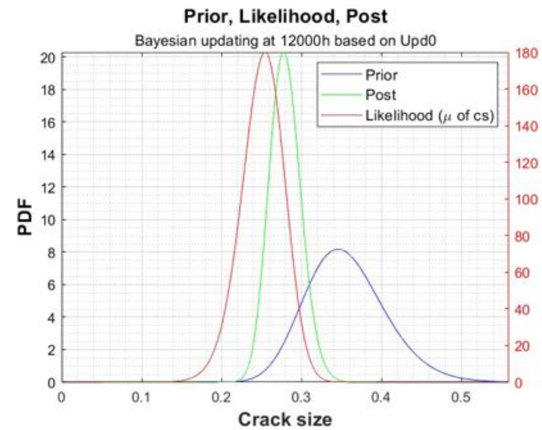
Select a POF curve in which the Bayesian updating inspection will be performed.

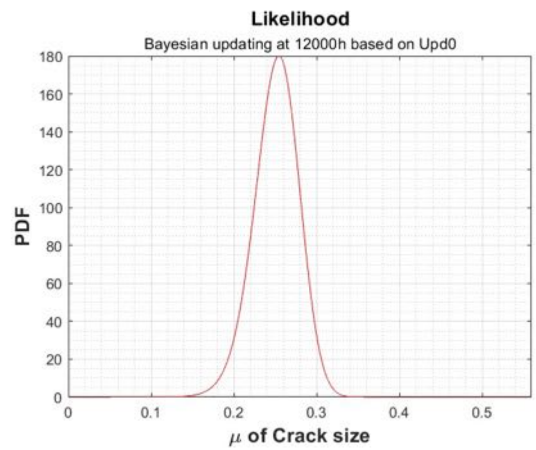
Click "Add Inspection" to perform Bayesian Updating



3. Result of inspection added

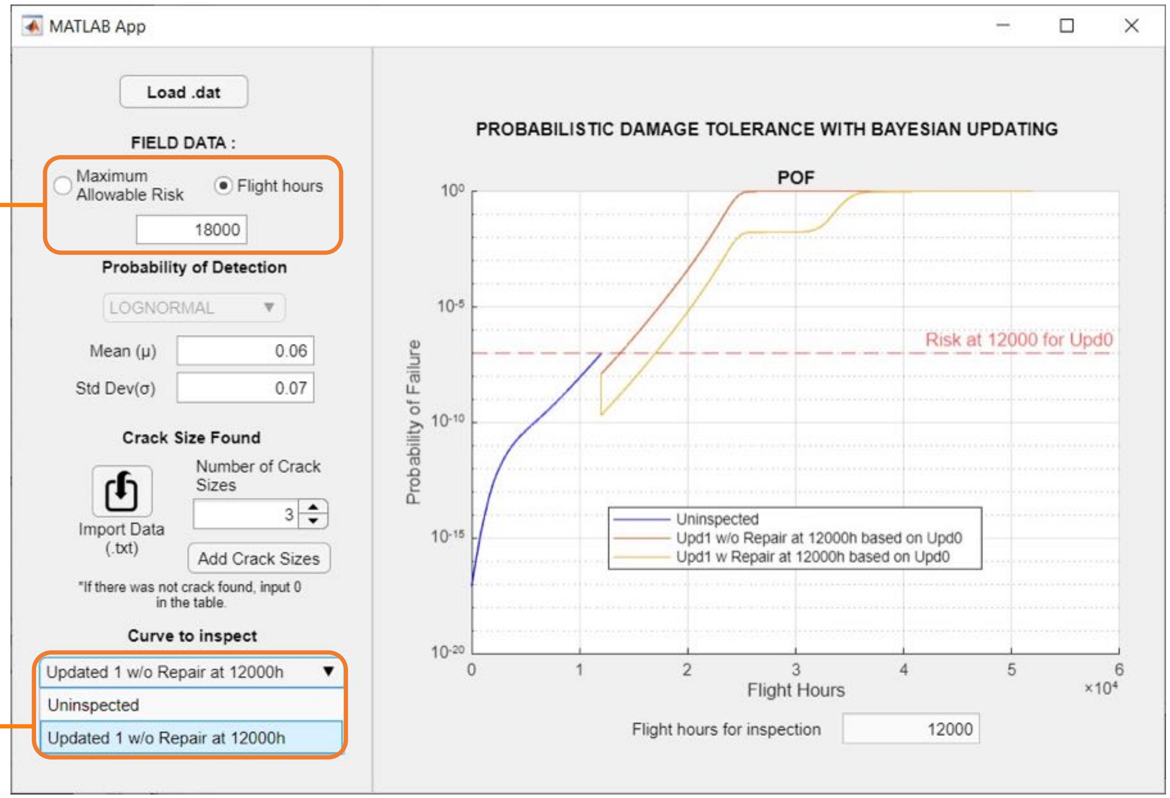






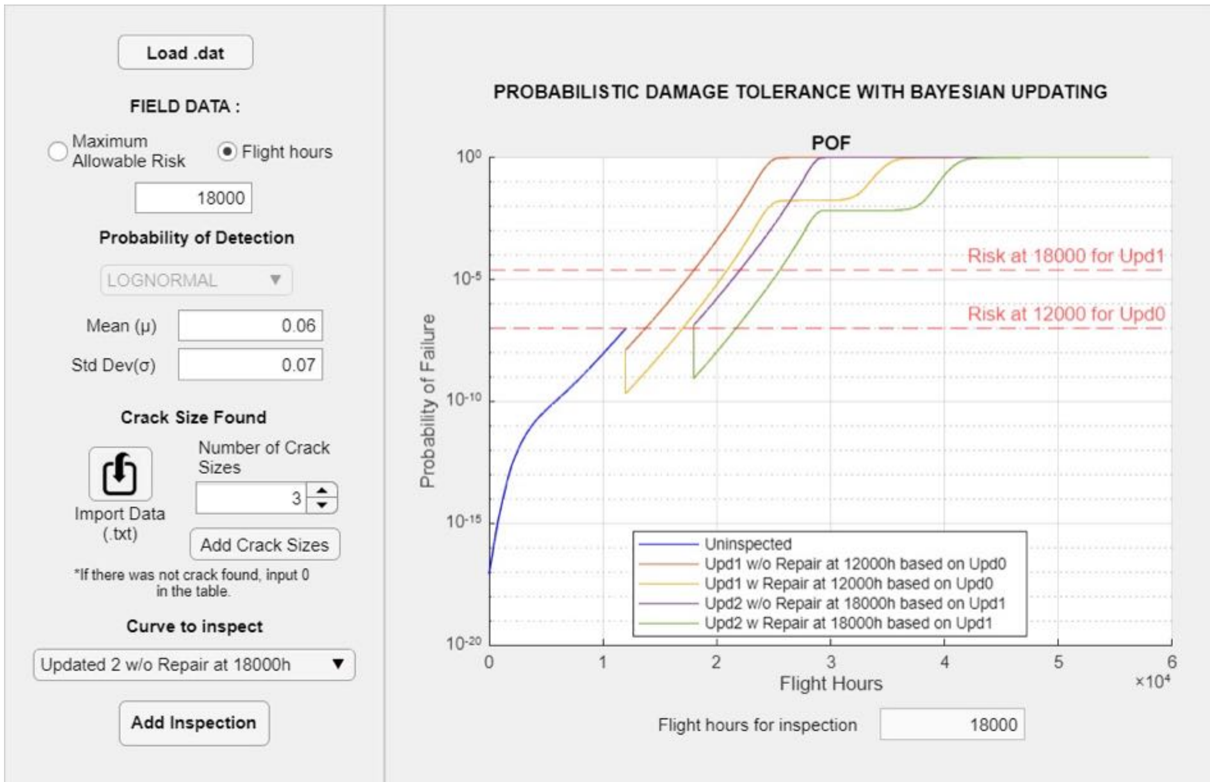
4. New Inspection

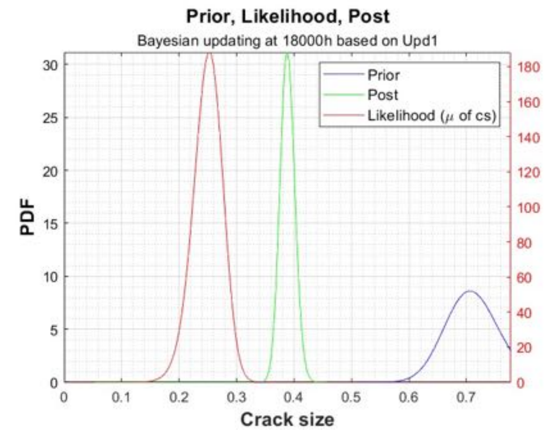
With Flight hours: 18000

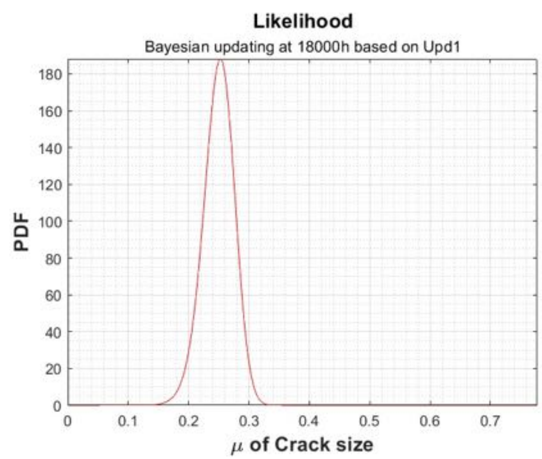


Curve to Inspect: Updated 1

5. Results of new inspection







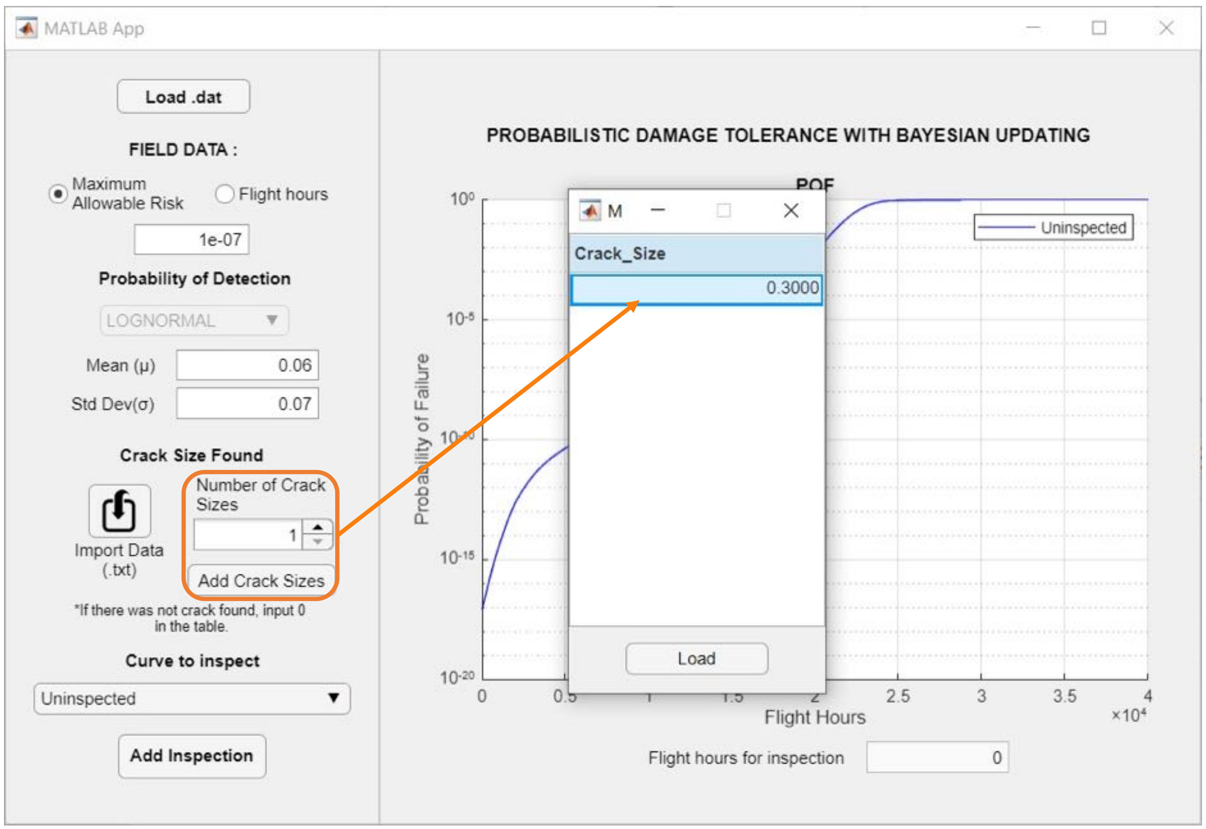
EXAMPLES

	INSPECTION		
	1 AIRPLANE	2 AIRPLANES	
1	D	D-D	3
2	ND	D-ND	4
		ND-ND	5

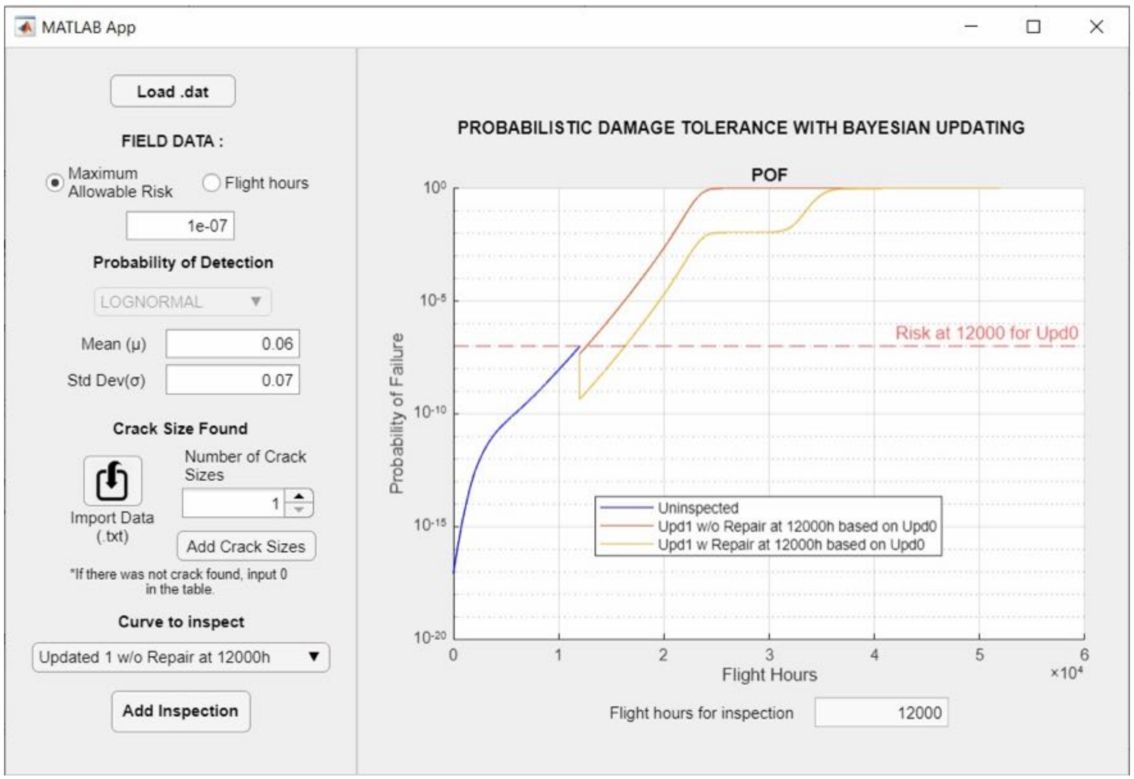
- 1 Detection = 1 crack in 1 airplane = Di
- D=Detection
- ND= No Detection

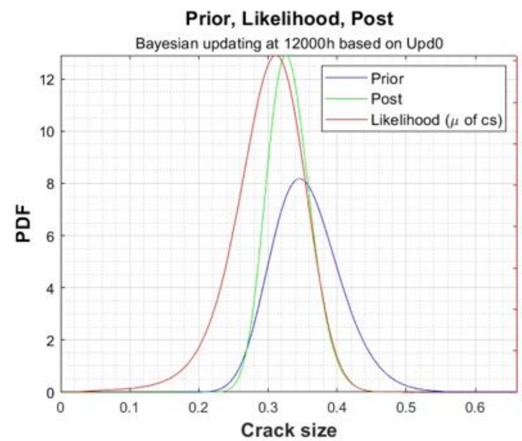
Crack size detection = 0.3 in.

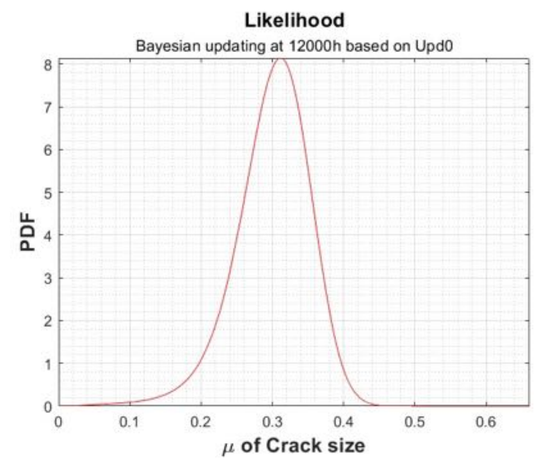
1



Results Crack size detection = 0.3 in.

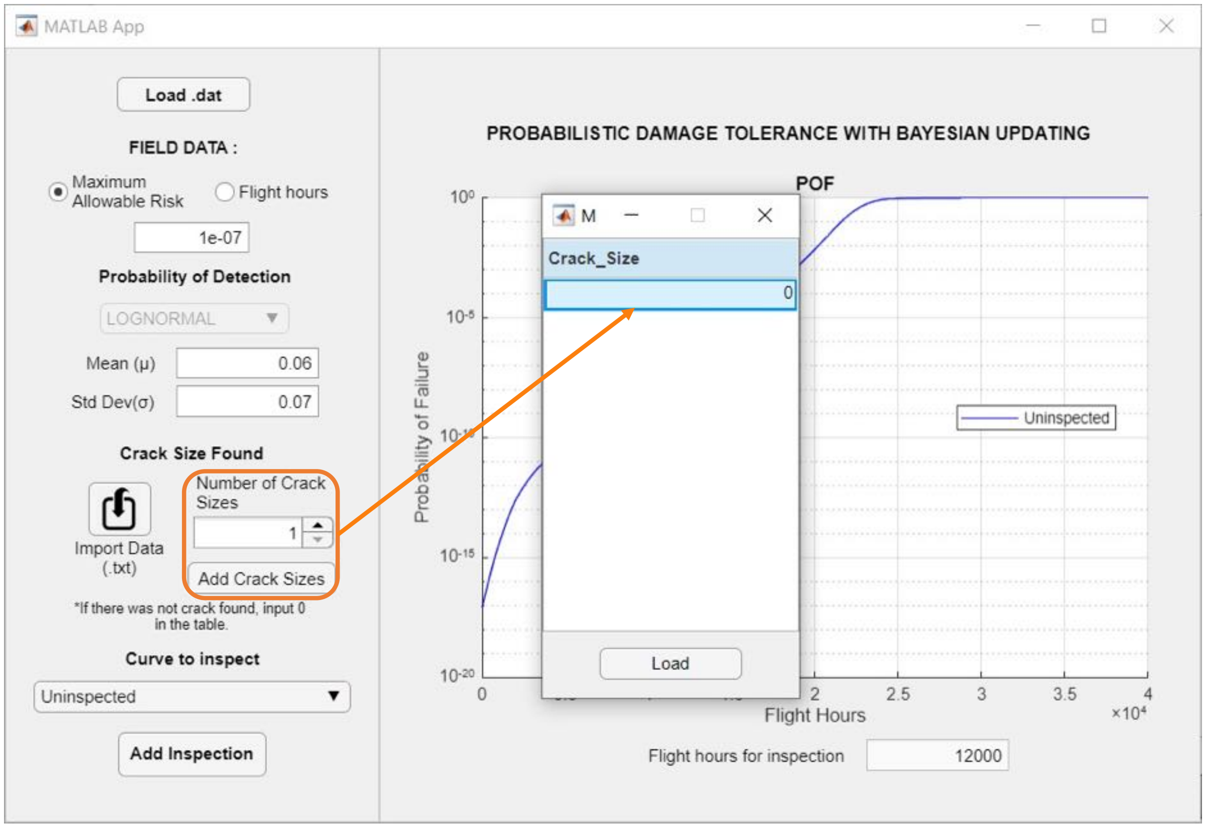




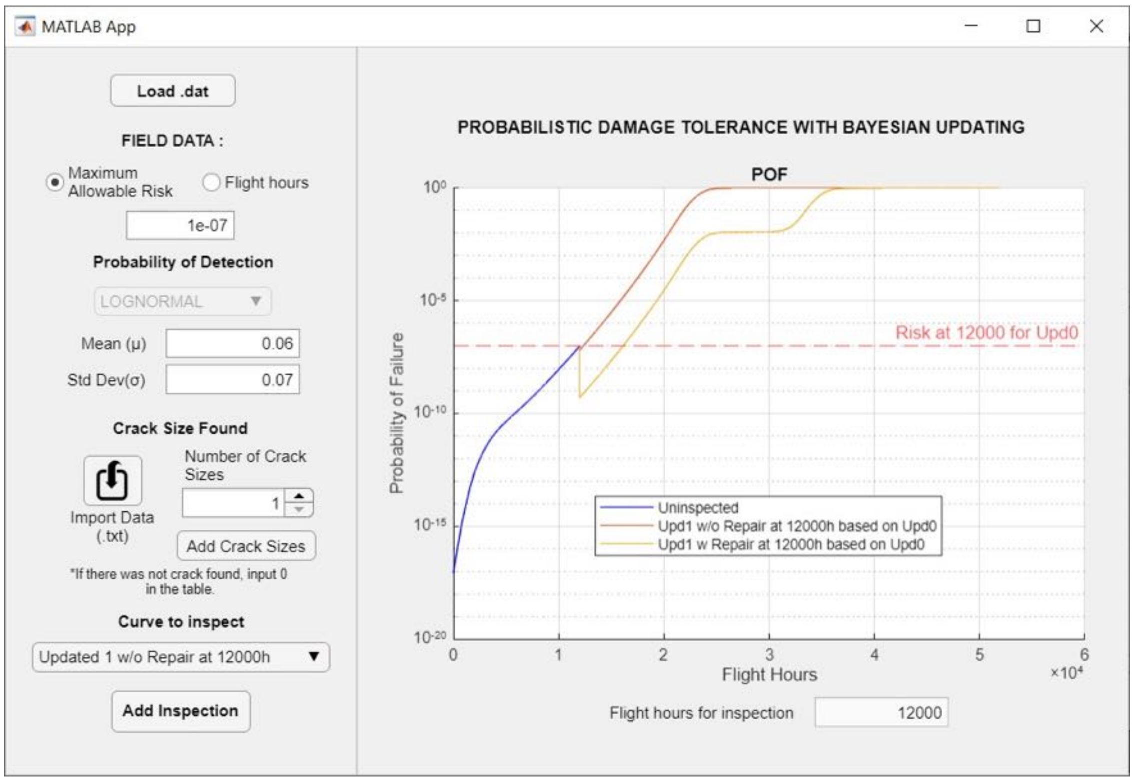


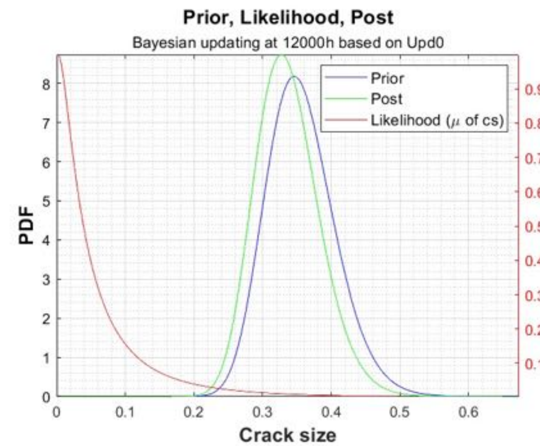
No detection

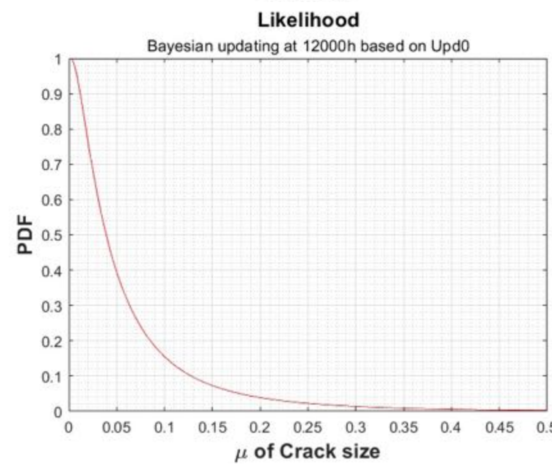
2



Results Crack size No detection

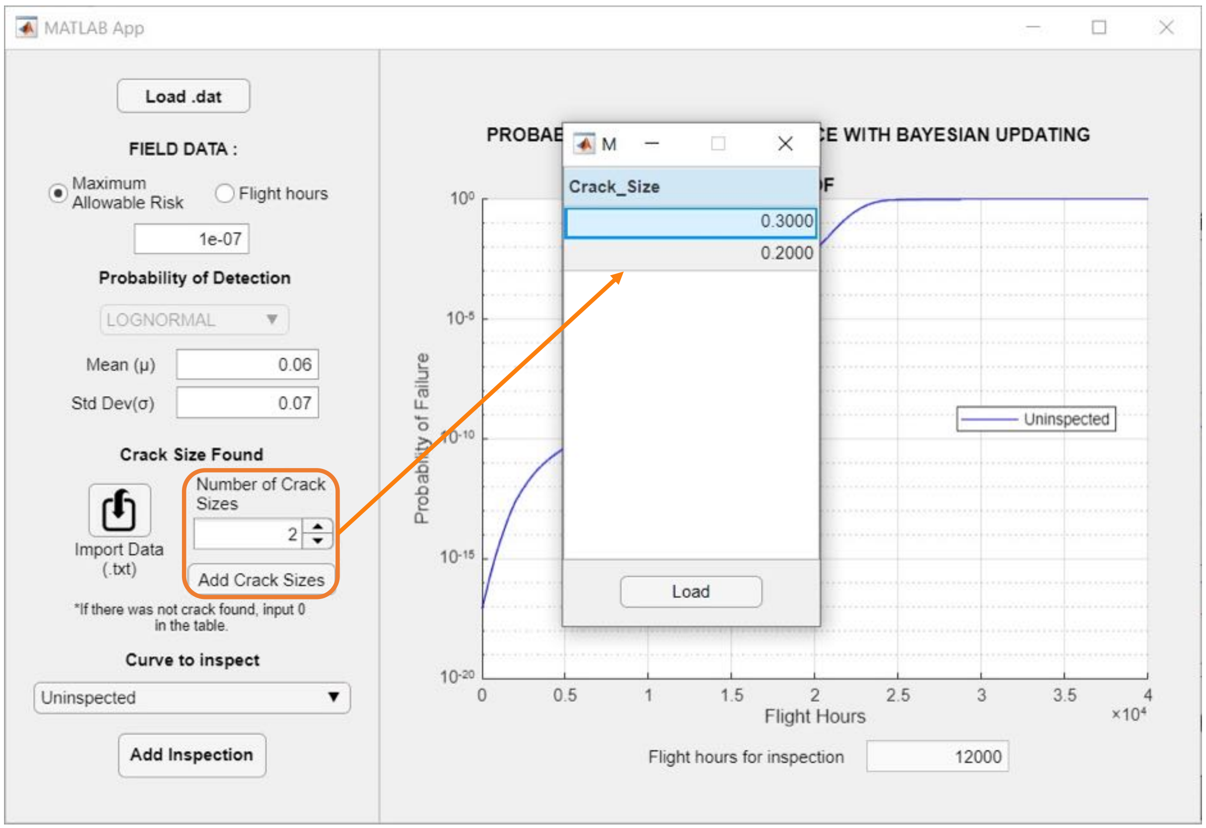




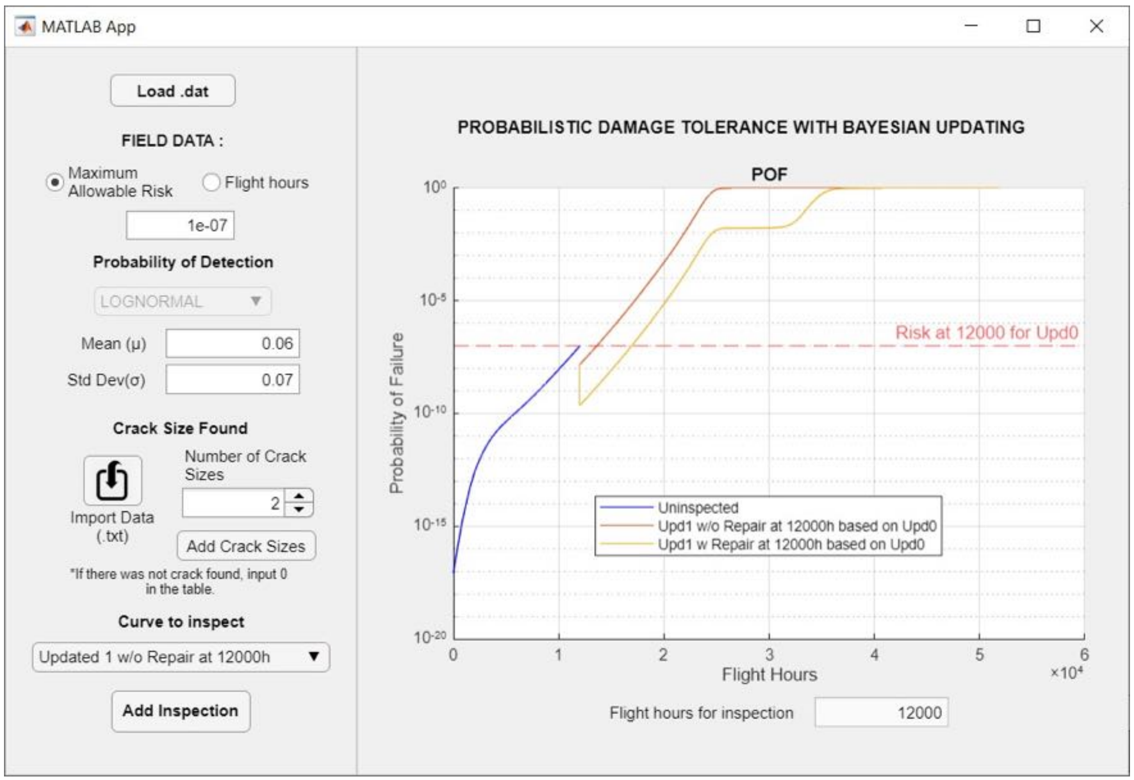


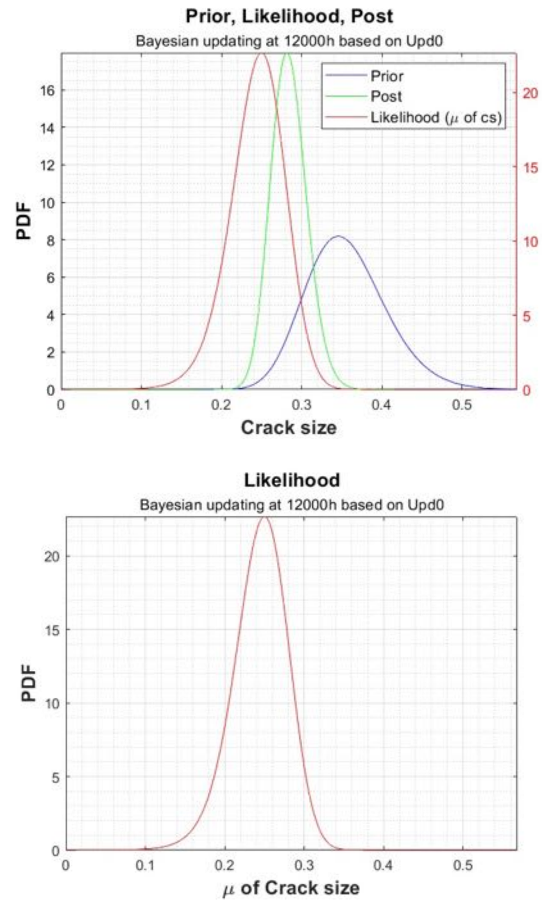
Crack size detections = 0.3 and 0.2 in.

3



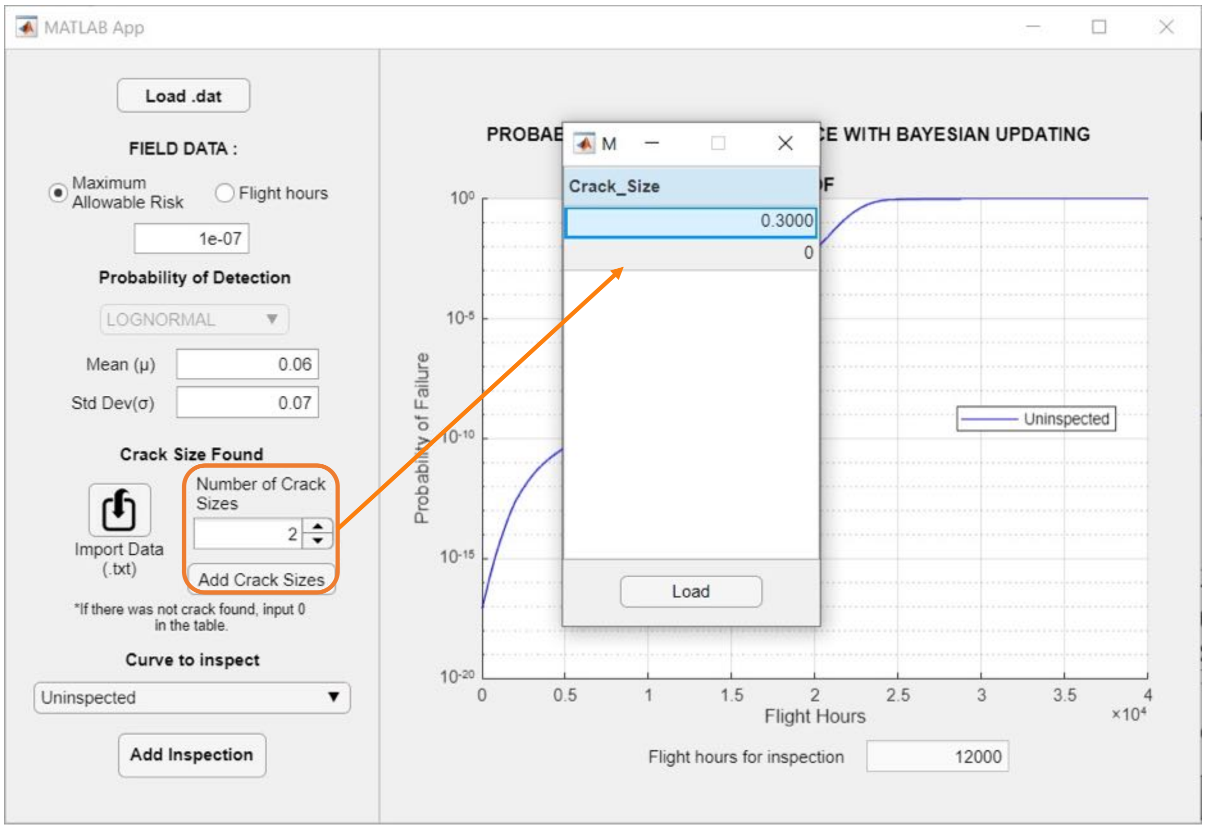
Results Crack size det.= 0.3 and 0.2 in



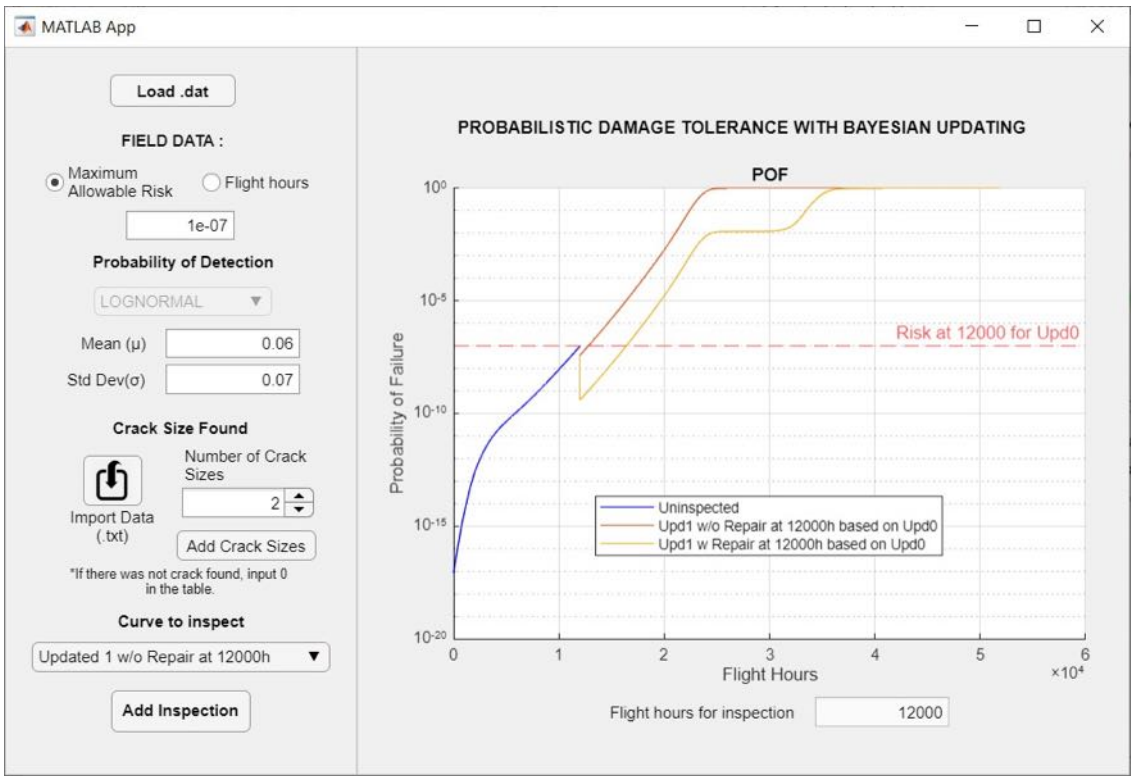


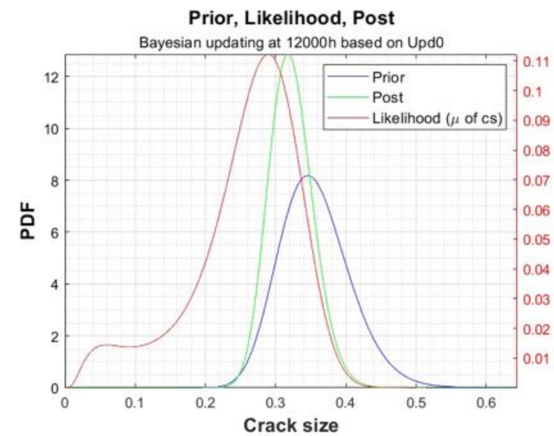
Crack size detections = 0.3 in. and one no detection

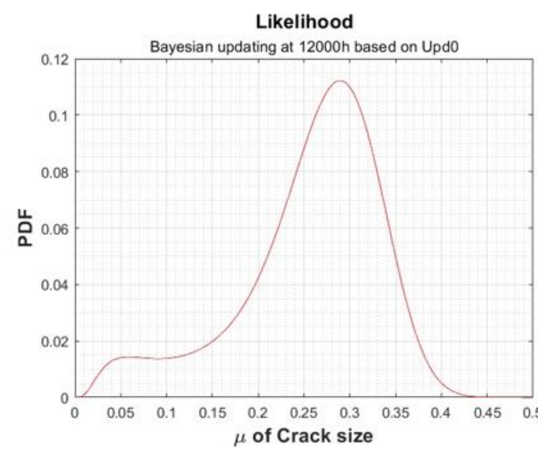
4



Results Crack size det.= 0.3 in. and ND

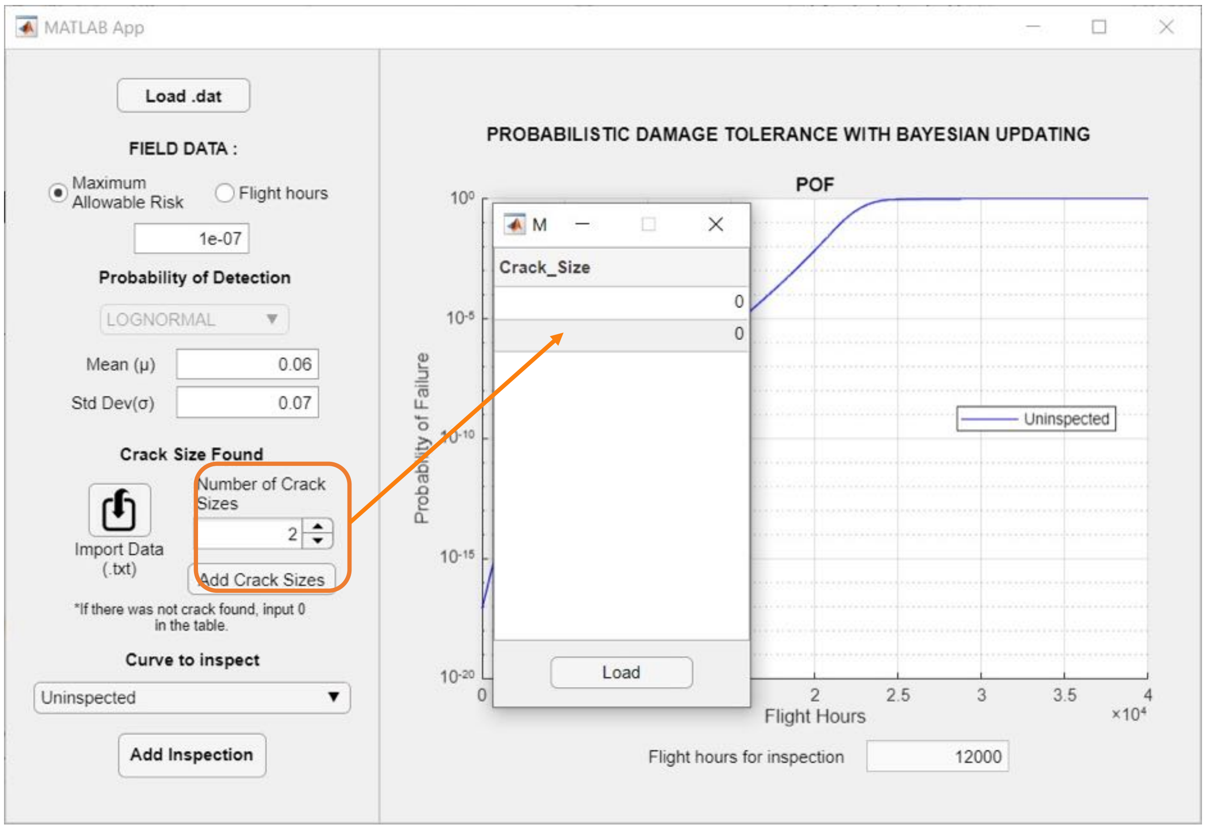




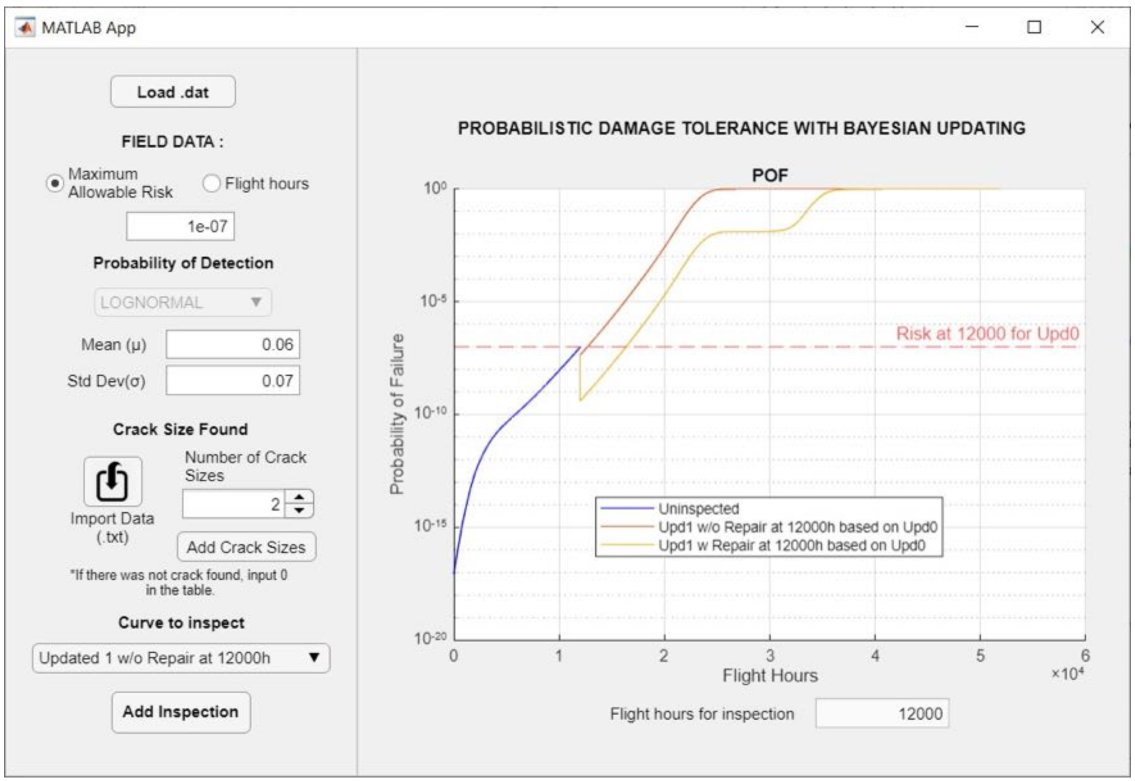


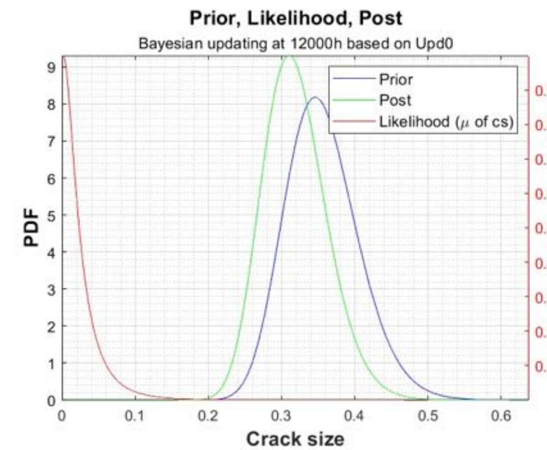
Posterior for two inspections and no detected cracks

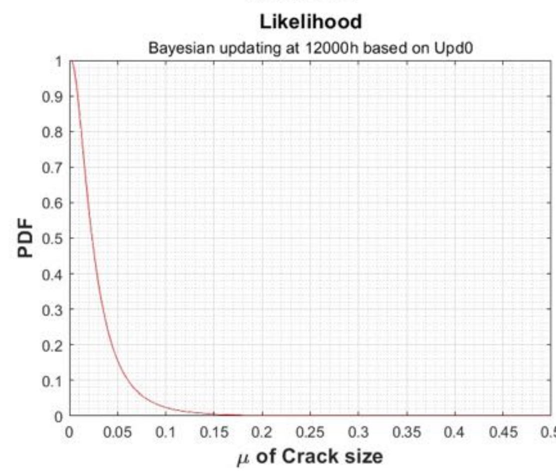
5



Results Crack size ND-ND







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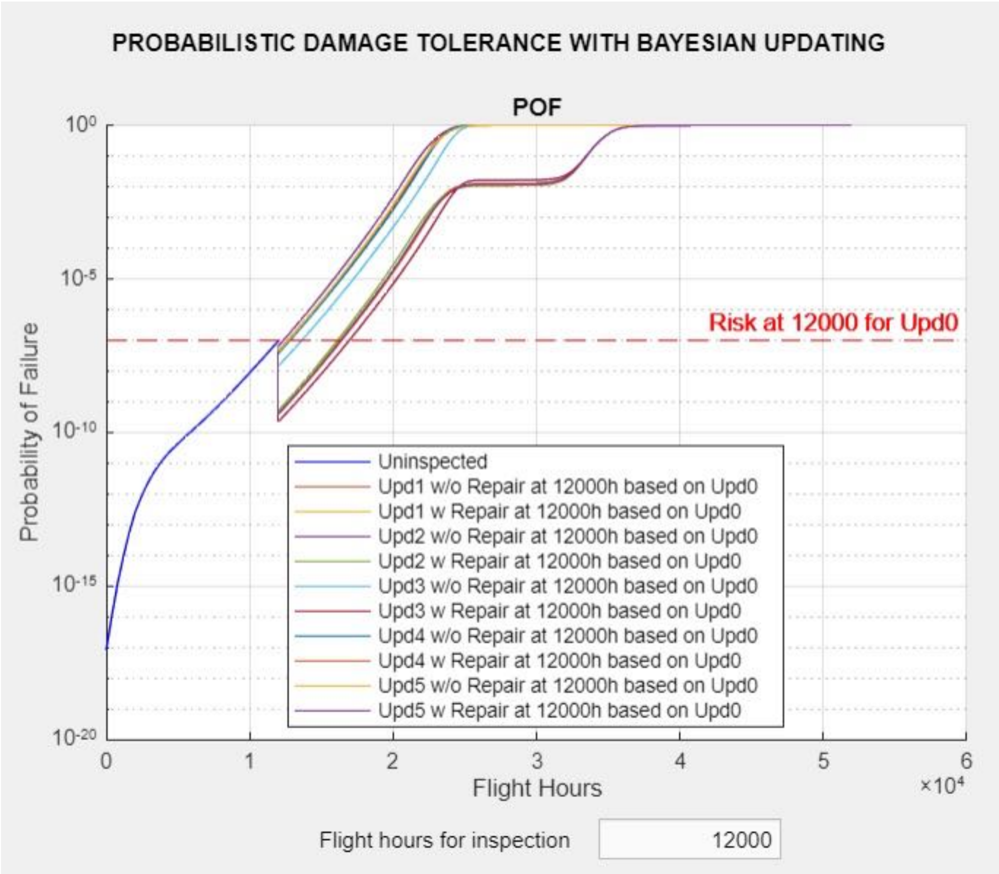
SUMMARY



PROBABILISTIC DAMAGE TOLERANCE WITH BAYESIAN UPDATING
POF
Probability of Failure
Flight Hours
×10^4
Risk at 12000 for Upd0
Uninspected
Upd1 w/o Repair at 12000h based on Upd0
Upd1 w Repair at 12000h based on Upd0
Upd2 w/o Repair at 12000h based on Upd0
Upd2 w Repair at 12000h based on Upd0
Upd3 w/o Repair at 12000h based on Upd0
Upd3 w Repair at 12000h based on Upd0
Upd4 w/o Repair at 12000h based on Upd0
Upd4 w Repair at 12000h based on Upd0
Upd5 w/o Repair at 12000h based on Upd0
Upd5 w Repair at 12000h based on Upd0
Flight hours for inspection
12000

Conclusions

- A Bayesian updating methodology was integrated within the FAA-sponsored SMART|DT.
  - Bayesian code can also be used as an stand alone code.
- A “finding” or “no finding” can be used to update the PDTA distribution modeling assumptions – Not limits on the number of “finding” or “no finding”.
- Bayesian updating provides a powerful tool to incorporate inspection data into the PDTA/DT analysis.

Acknowledgments

Probabilistic Fatigue Management Program for General Aviation, Federal Aviation Administration, Grant 16-G-005.

- Sohrob Mattaghi (FAA Tech Center) – Program Manager
- Michael Reyer (Kansas City) – Sponsor

Thank you

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