

# The Single Flight Probability of Failure: Past, Present, and Future: An Open Discussion



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# Outline



- ✓ The SFPOF
  - ✓ Original Definition
  - ✓ PROF V3.#
  - ✓ SMART|DT
  - ✓ Example Problems
- ✓ Risk Assessment Survey
- ✓ SMART|DT Overview
  - ✓ Current Developments
    - ✓ Adaptive Multiple Important Sampling
    - ✓ Bayesian Updating
    - ✓ Optimized Inspections
    - ✓ Fleet Management
- ✓ Discussion and Where to Go Next

# Traditional Hz. Fn.



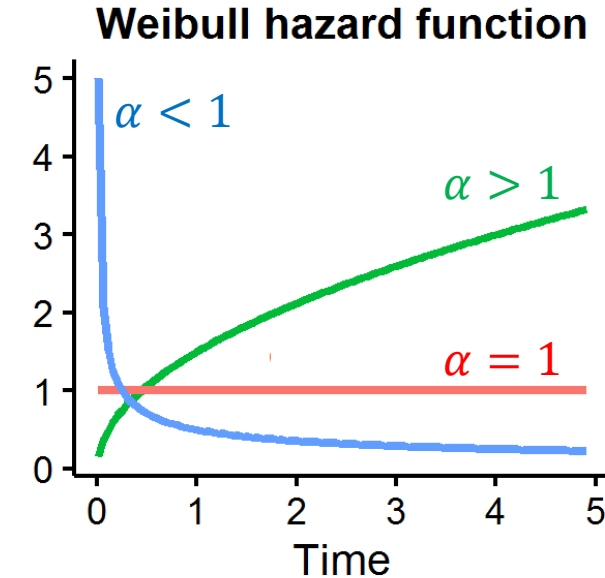
- For a Weibull distribution

$$F(t) = 1 - \exp\left(-\left(\frac{t}{\alpha}\right)^\beta\right)$$

$$f(t) = \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left(-\left(\frac{t}{\alpha}\right)^\beta\right)$$

$$HR(t) = \frac{f(t)}{1 - F(t)}$$

The hazard function (or hazard rate (HR), failure rate, risk of failure) specifies the instantaneous failure rate of death or failure at time  $t$ , given that the individual survives up to  $t$



<https://www.erikdrysdale.com/survival/>

# SFPOF By Lincoln



- The SFPOF introduced by Lincoln in 1985 can be defined as:

The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength  $\sigma_{RS}$  of the aircraft component

$$POF_{\text{no-surv}}(t) = P[S_{\text{Max}} > S_{RS}(t)] = \int_{\mathbf{x}} [1 - F_{EVD}(S_{RS}(t))] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

Without accounting for the fact that failure has not previously occurred

# SFPOF By Barrens (V2.0)



- The SFPOF (Lincoln) was updated in PROF V2.0

$$POF(t) = \underbrace{\int_0^{a_c} f(a) \int_0^{\infty} g(K_c) [1 - F_{EVD}(\sigma_{RS}(t))] dK_c da}_{\text{Crack is smaller than critical but max. stress exceeds residual strength}} + \underbrace{[1 - F(a_c)]}_{\text{Crack is bigger than critical}}$$

Crack is smaller than critical but max. stress exceeds residual strength

Crack is bigger than critical

# PROF V3.# - Lincoln



Failure is assumed to occur during a flight when a crack exceeds a critical size during the flight or when the largest stress in the flight exceeds the residual stress for the existing flaw size at the location

$$h_1(t) = \frac{f_{haz}(t)}{1 - F_{haz}(t)}$$

$$h_2(t) = POF(t) = \frac{1}{t_f} \int_{-\infty}^{\infty} \int_0^{a_{haz}} [1 - F_{EVD}(\sigma_{RS}(t))] \cdot f(a) da \cdot f(K_c) dK_c$$

$$SFPOF(t) = h_1(t) + h_2(t) \quad (\text{Lincoln})$$

$a_{haz}$  = crack size that produces the mean fracture toughness when encountering the average (the value at 63.2 percentile in Gumbel distribution) max stress in a flight

$t_f$  = hours per flight

# Freudenthal Method in PROF in PROF V3.#



- The conditional probability, given  $K_c$  and the initial flaw size  $a_0$ , of failing on flight  $n$  is the probability of surviving the first  $n-1$  flights and failing in flight  $n$ , and since peak loads are independent from flight to flight this is given by:

$$f_L(n|K_c, a_0) \cong \bar{H} \left( \frac{K_c}{\alpha(a(a_0, n))} \right) \prod_{i=1}^{n-1} H \left( \frac{K_c}{\alpha(a(a_0, i))} \right), \quad hz(n|K_c, a_0) = \frac{f_L(t)}{\bar{F}_L(t)} :$$

$$hz_{stress}(t) = \int_{-\infty}^{\infty} f_k(K_c) \int_0^{\infty} f_{a_0}(a_0) [1 - H \left( \frac{K_c}{\alpha(a(a_0, t))} \right)] da_0 dK_c$$

# SMART SFPOF Equations



The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength  $\sigma_{RS}$  of the aircraft component

$$POF_{\text{no-surv}}(t) = P[S_{\text{Max}} > S_{RS}(t)] = \int [1 - F_{EVD}(S_{RS}(t))] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

*Lincoln formulation  
(default in Smart/DT)*

$$POF_{\text{surv}}(t) = \int \left[ \prod_{i=1}^{t-1} F_{EVD}(\sigma_{RS}(t_i)) \right] [1 - F_{EVD}(\sigma_{RS}(t))] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

$$CTPOF(t) = \int \left[ 1 - \prod_{i=1}^t F_{EVD}(S_{RS}(t_i)) \right] f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x}$$

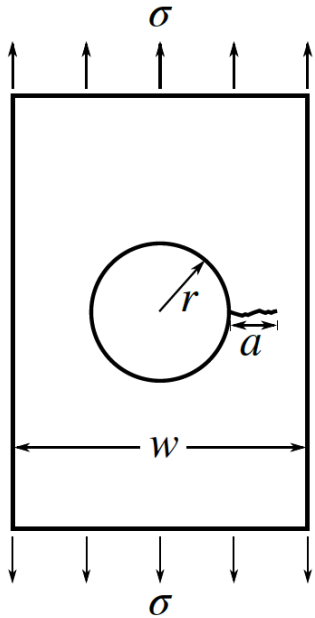
$$Hz(t) = \frac{POF_{\text{surv}}(t)}{1 - CTPOF(t)}$$

*Freudenthal  
formulation  
(Available in Smart/DT)*

$F_{EVD}$  – CDF of the maximum stress per flight (extreme value distribution)  
 $\sigma_{RS}(t)$  – residual strength



# Handbook Example



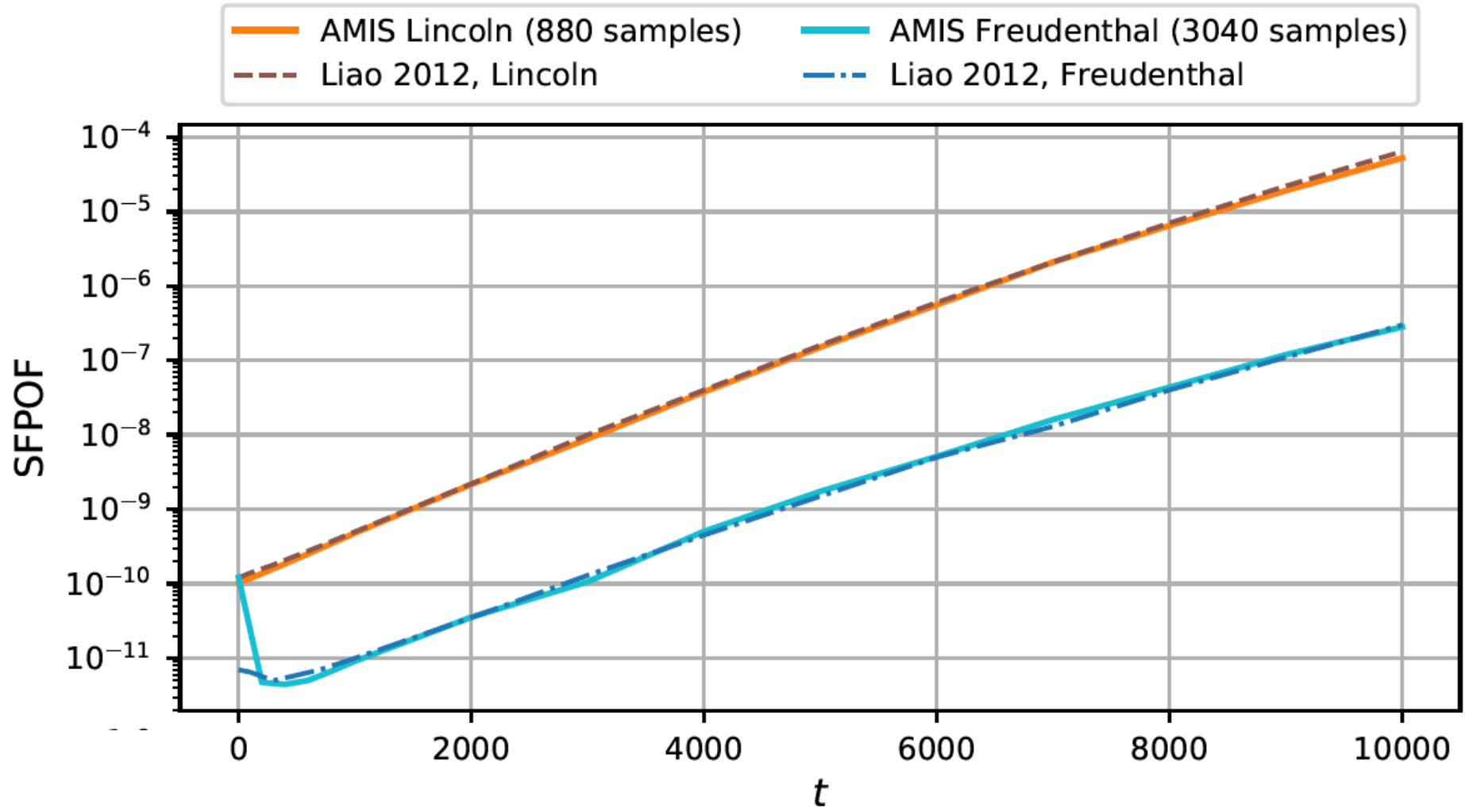
$$a(t) = a_0 \exp(t \cdot 2.93 \times 10^{-4})$$

$$\beta(a(t)) = \underbrace{\left(0.6762 + \frac{0.8734}{0.3246 + a(t)/r}\right)}_{\beta_{\text{hole}}} \times \underbrace{\left(\sqrt{\sec\left(\frac{\pi(r+a(t))}{w}\right)}\right)}_{\beta_{\text{width}}}$$

$$\sigma_{\text{rs}}(a(t)) = \frac{K_c}{\beta(a(t)) \sqrt{\pi a(t)}}$$

Random Variable	Distribution	Parameters
Initial crack size $a_0$	Lognormal	mean = 0.0030 in standard deviation = 0.0047 in
Fracture toughness $K_c$	Normal	mean = 34.8 ksi · in <sup>1/2</sup> standard deviation = 3.9 ksi · in <sup>1/2</sup>
Maximum stress $\sigma_{\text{EVD}}$	Weibull	location = 5.0 ksi scale = 10.0 ksi shape = 5.0
Width, $w$		10.0 in
Hole Radius, $r$		0.125 in

# Handbook Example Results



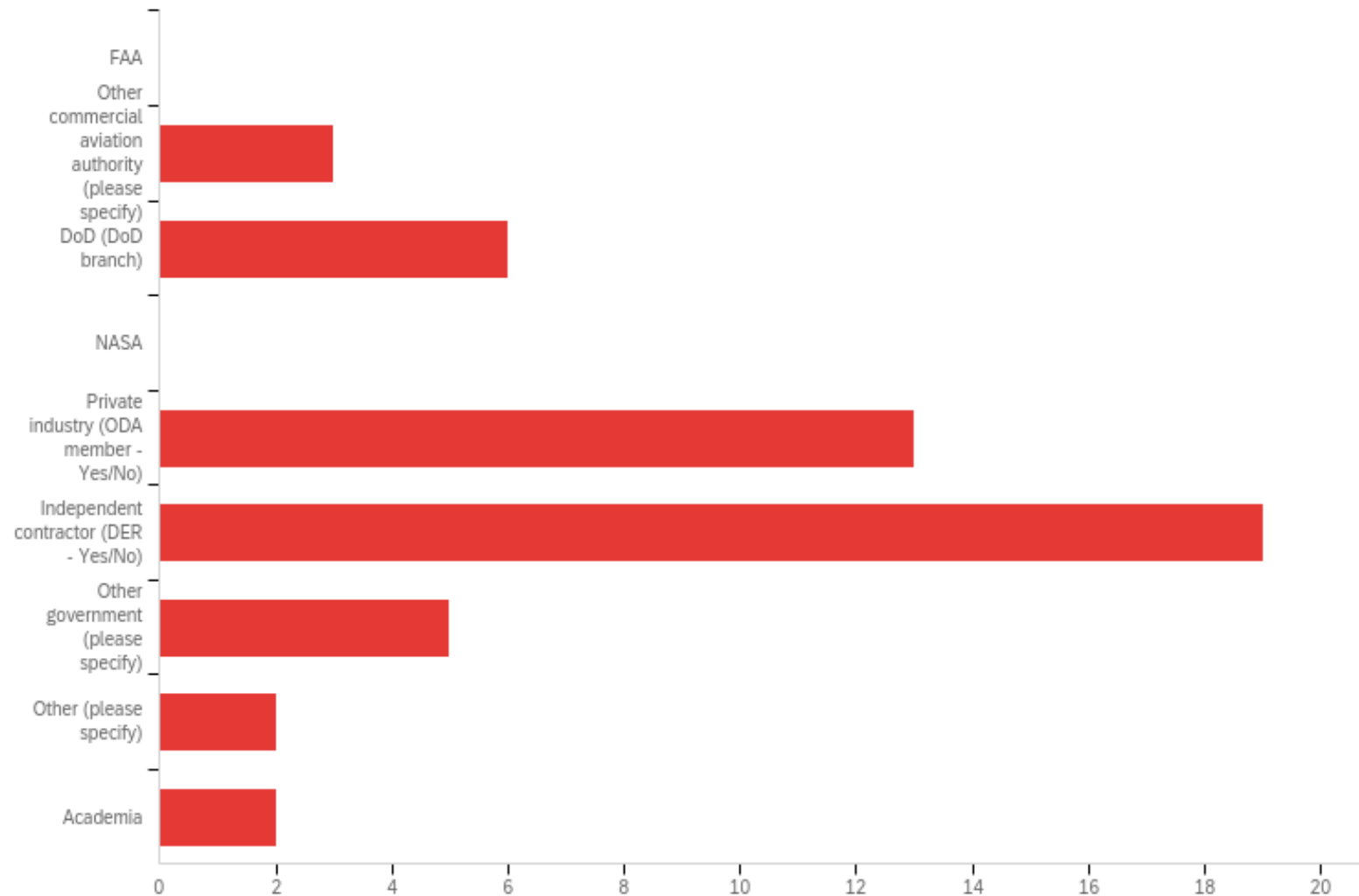


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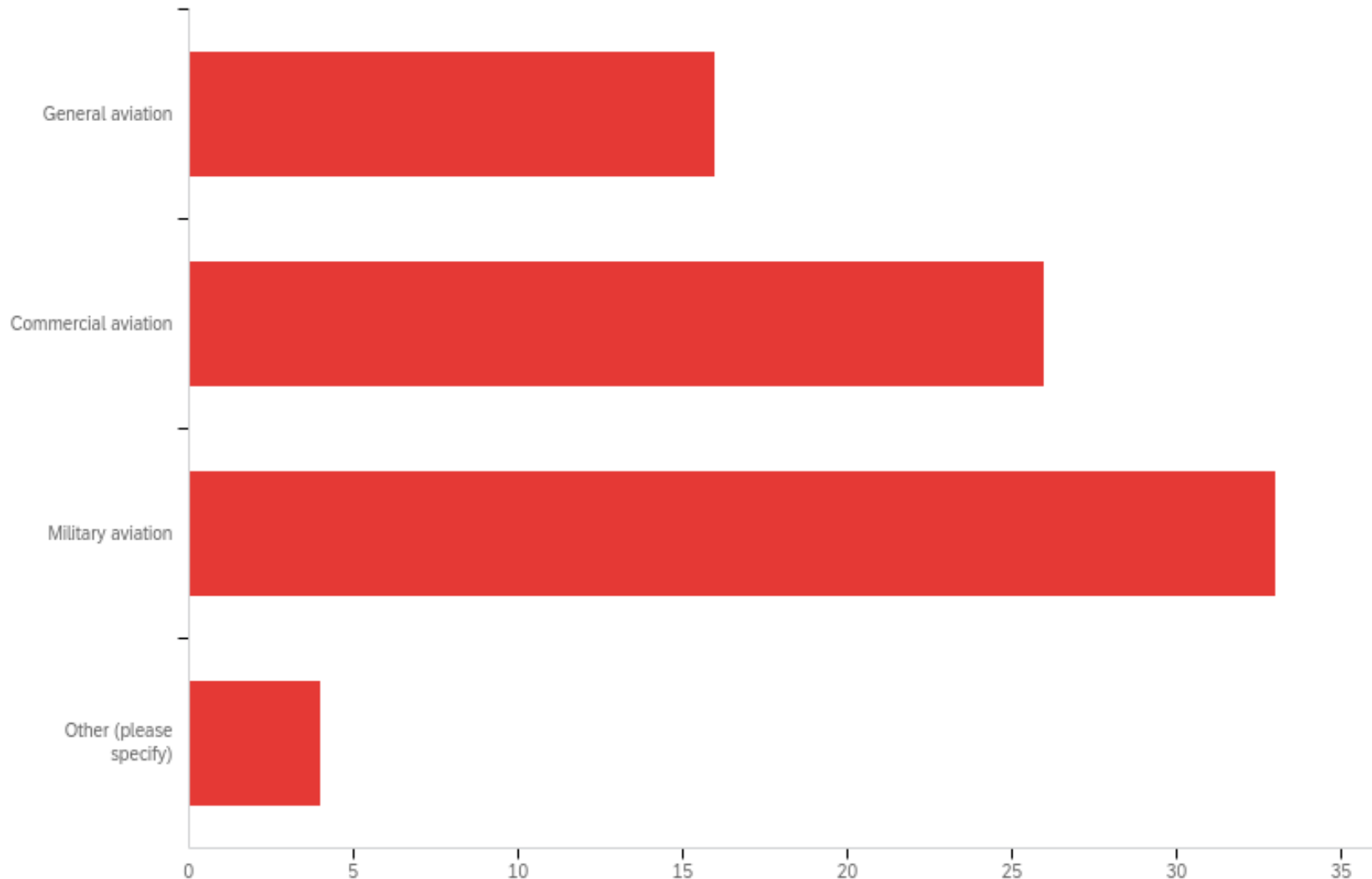
# Risk Assessment Survey

## Dec. 7<sup>th</sup> 2022

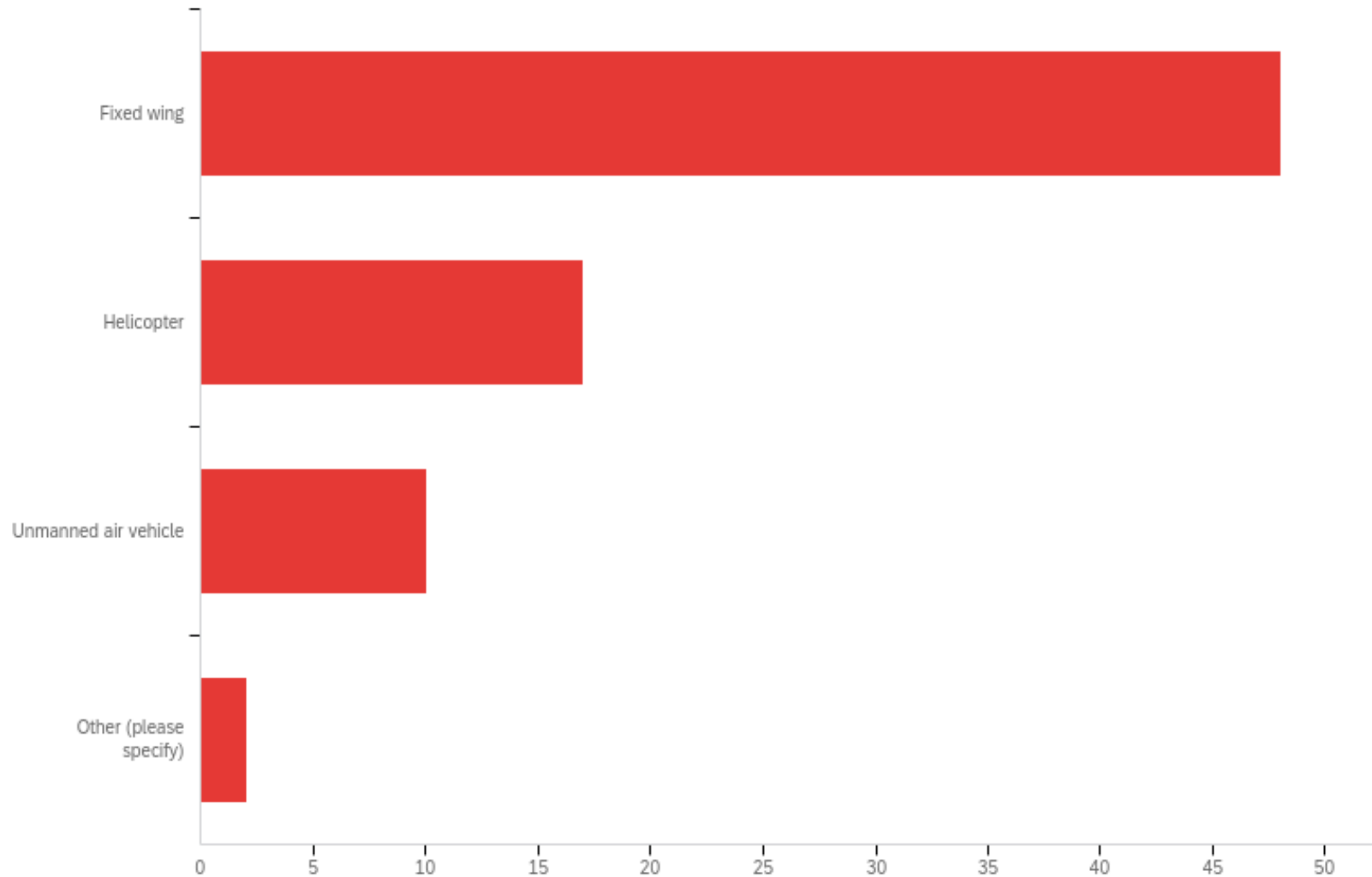
# Indicate your employment agency



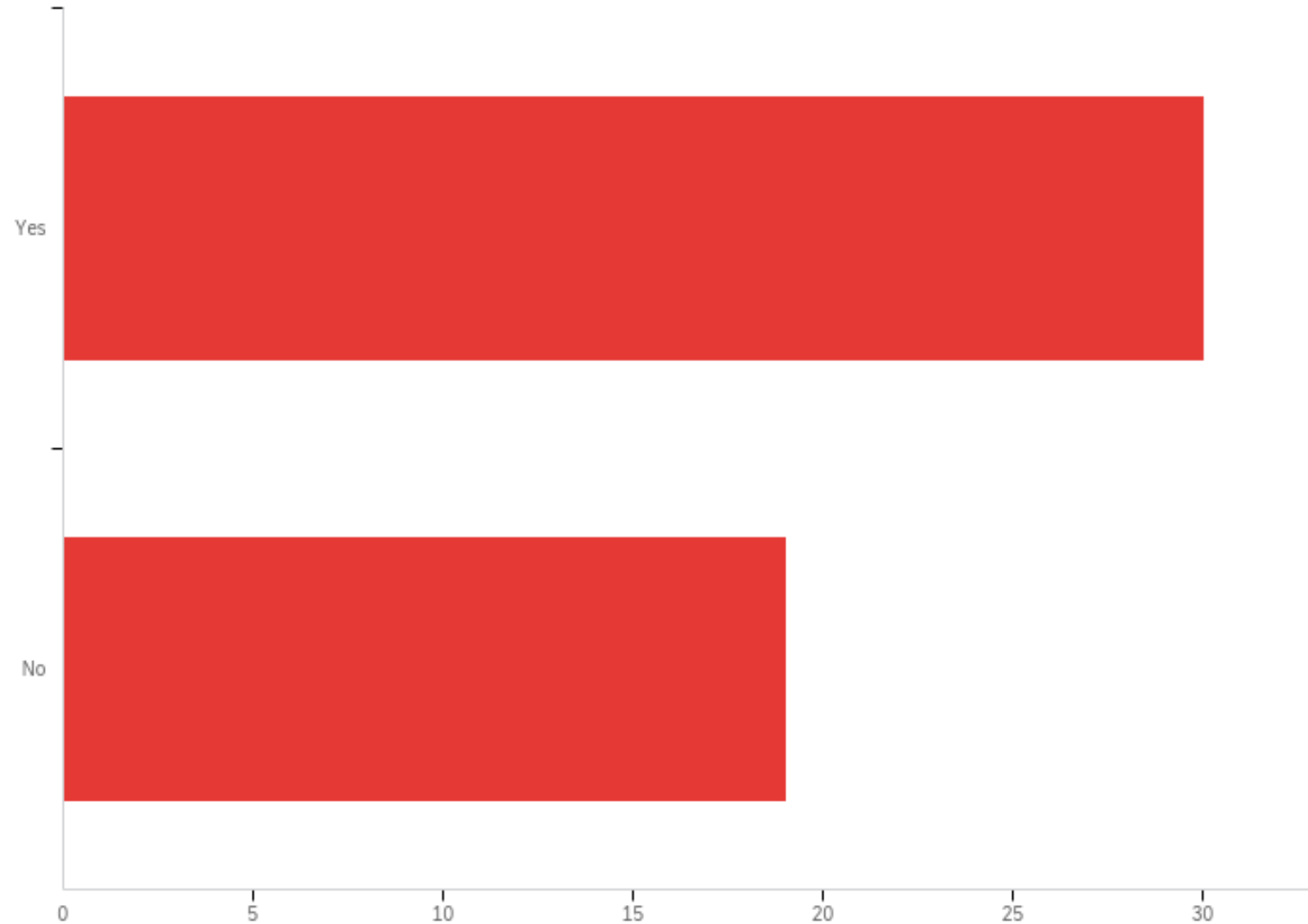
# What type of aircraft industry do you typically support?



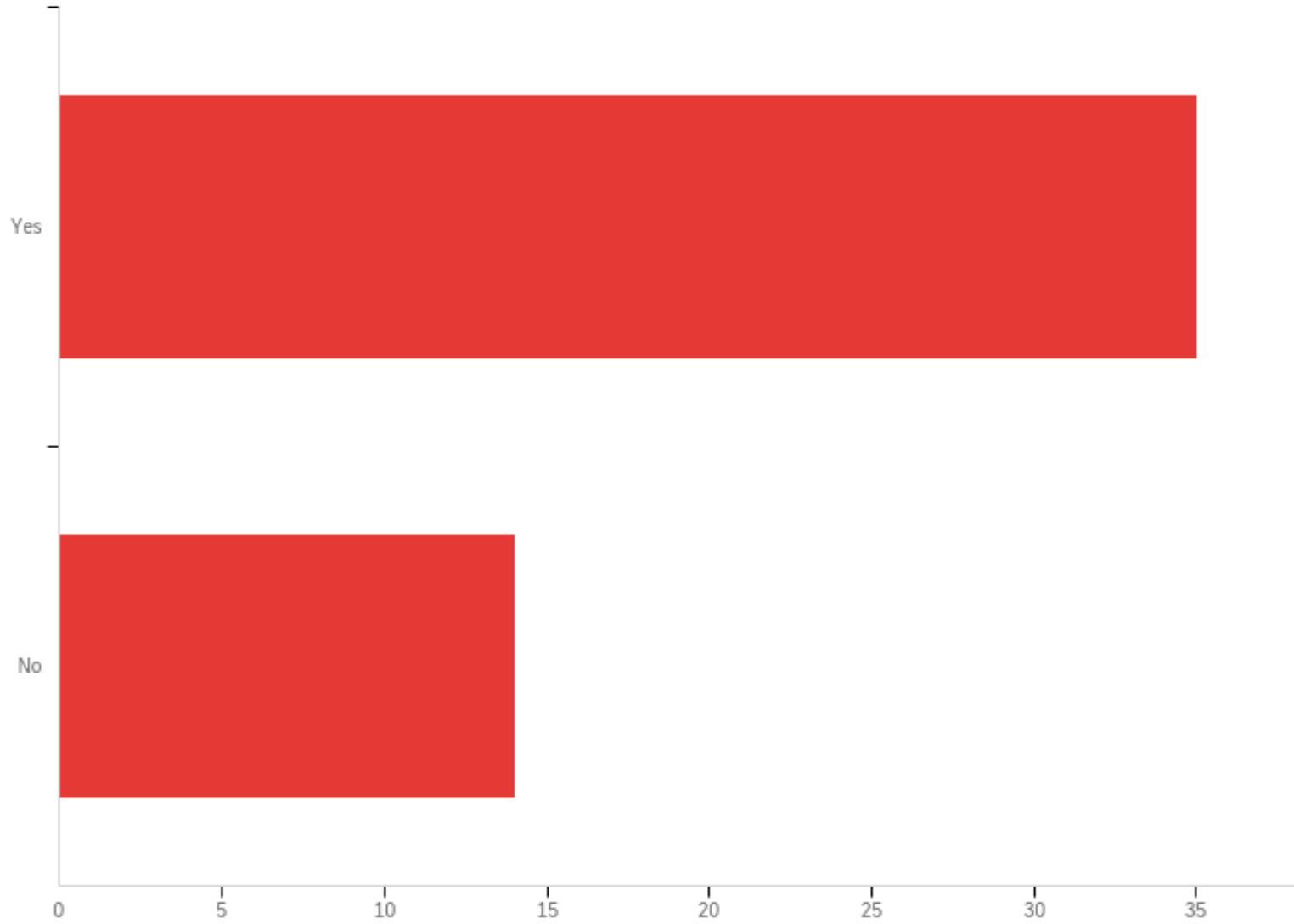
# What type of aircraft do you typically support?



# Have you used probabilistic risk assessment methods to calculate fatigue life (crack initiation)?


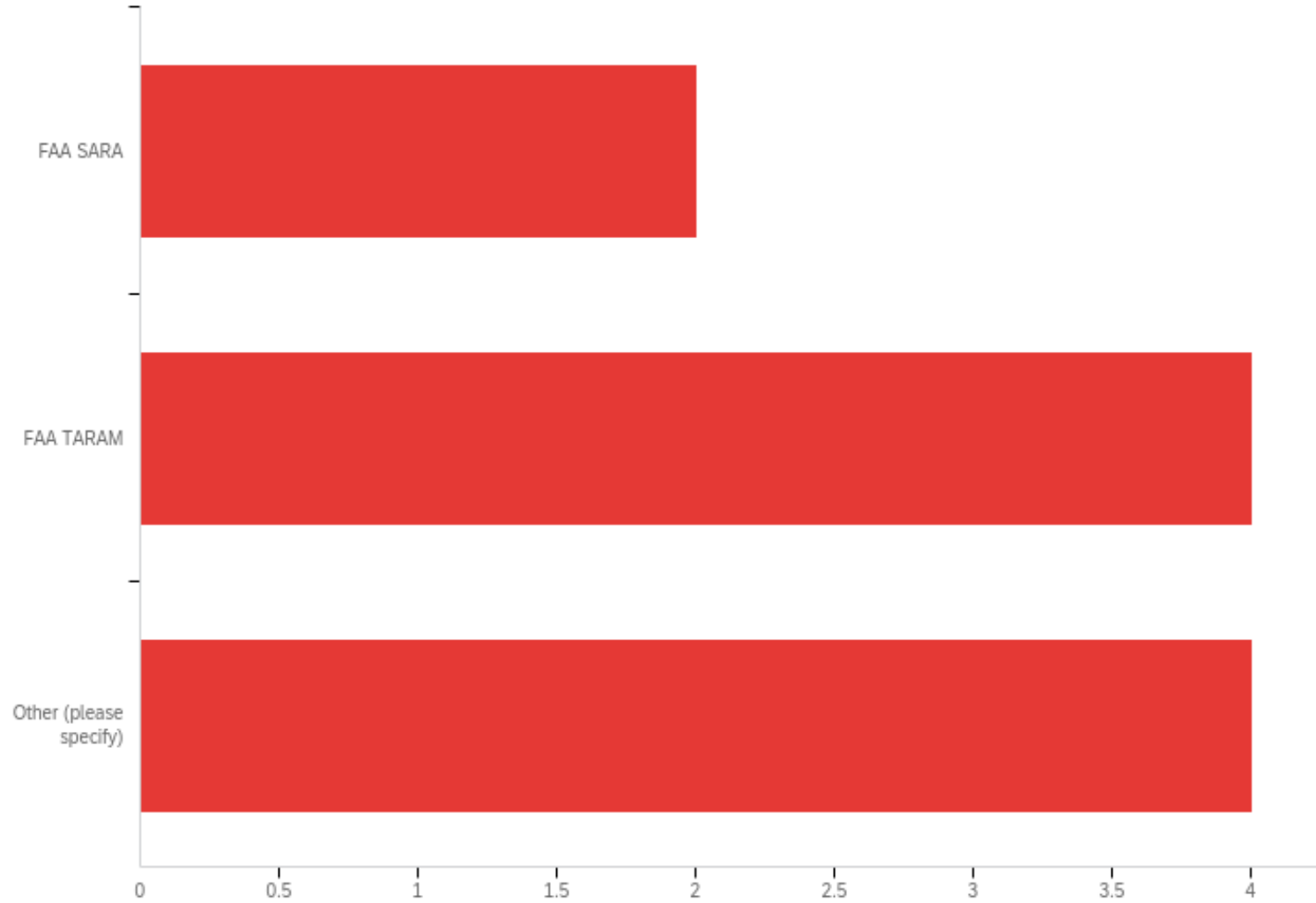


# Have you used probabilistic risk assessment methods for damage tolerance assessments (crack growth)?






# Have you used the FAA SARA or TARAM documents for risk assessment for Part 23 or 25 aircraft or other guidance documents?



**Federal Aviation Administration**  
Small Airplane Directorate

**Small Airplane Risk Analysis  
(SARA)  
Handbook**



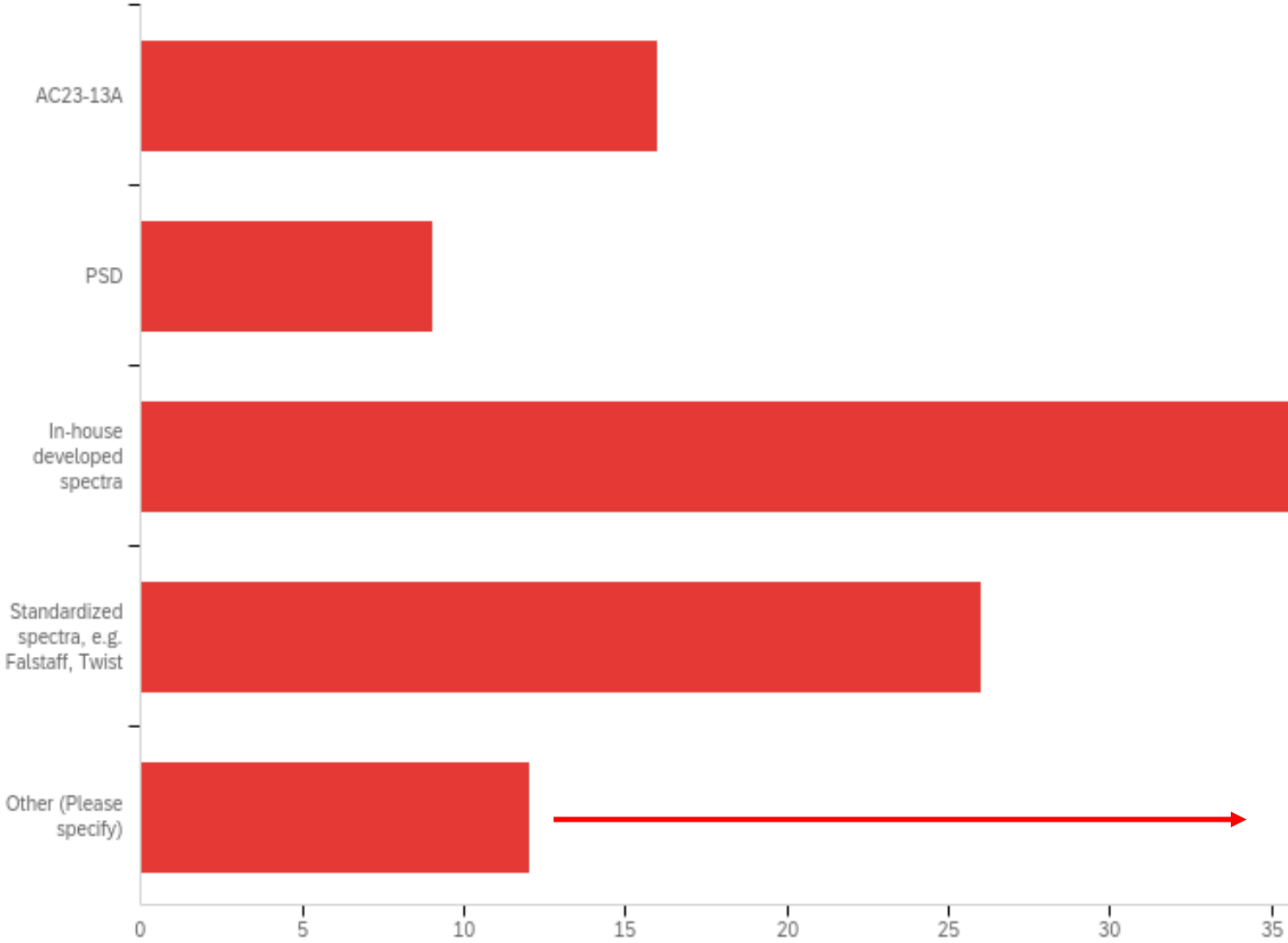
**U.S. DEPARTMENT OF TRANSPORTATION  
FEDERAL AVIATION ADMINISTRATION**  
Transport Airplane Risk Assessment Methodology  
Aviation Rulemaking Committee Charter

Effective Date: 6/22/2015

**SUBJECT: Transport Airplane Risk Assessment Methodology (TARAM) Aviation Rulemaking Committee (ARC)**

**1. PURPOSE.** This charter establishes the Transport Airplane Risk Assessment Methodology (TARAM) Aviation Rulemaking Committee (ARC), according to the Administrator's authority under Title 49 of the United States Code (49 U.S.C.) 106(p)(5). The sponsor of this ARC is the Manager of the FAA Transport Airplane Directorate and this charter outlines the committee's organization, responsibilities, and tasks.

# What source(s) do you use for generating load spectra for fatigue/damage tolerance analyses?



- UDRI / Tom Swift
- L/ESS
- from government
- OLM
- Patrick Safarian Method
- Loads & Environment Strain Survey
- Provided by MRO
- Spectra developed from in-service loads measurement program
- Measured spectra published by DOT/FAA, MIL-A-8860
- Usually combination of measured spectra form L/ESS
- supplied by client

# What source do you use for your equivalent initial defect/ flaw size distribution? (I)



Fleet history records
FAA Guidance - Patrick Safarian
Fractography data from FSFT
AF DT Handbook
DOT/FAA/CT-93/69
In house EIFS using damage-findings back-extrapolated using Master-Curve
internal, customer, other literature
Maintenance records, or testing
inspection data
Dod , AFGROW
Defects measure by micro CT, or microstructure variables, and occasionally classic LEM determined EIFS from coupon tests

data
FAA sourced data
In-service damage data, material database, published in references
JSSG-2006
Patrick Safarian
Teardown data when available.
In-house developed (some measured some computed)
.030
USAF
JSSG-2006; MIL-A-83444; NASA-STD-2009; AFFDL-TR-79-3021; EN-SB-08-002; EZ-SB-13-003; AC 25-24; Tom Swift
EIFS from in-service data, coupon data investigation

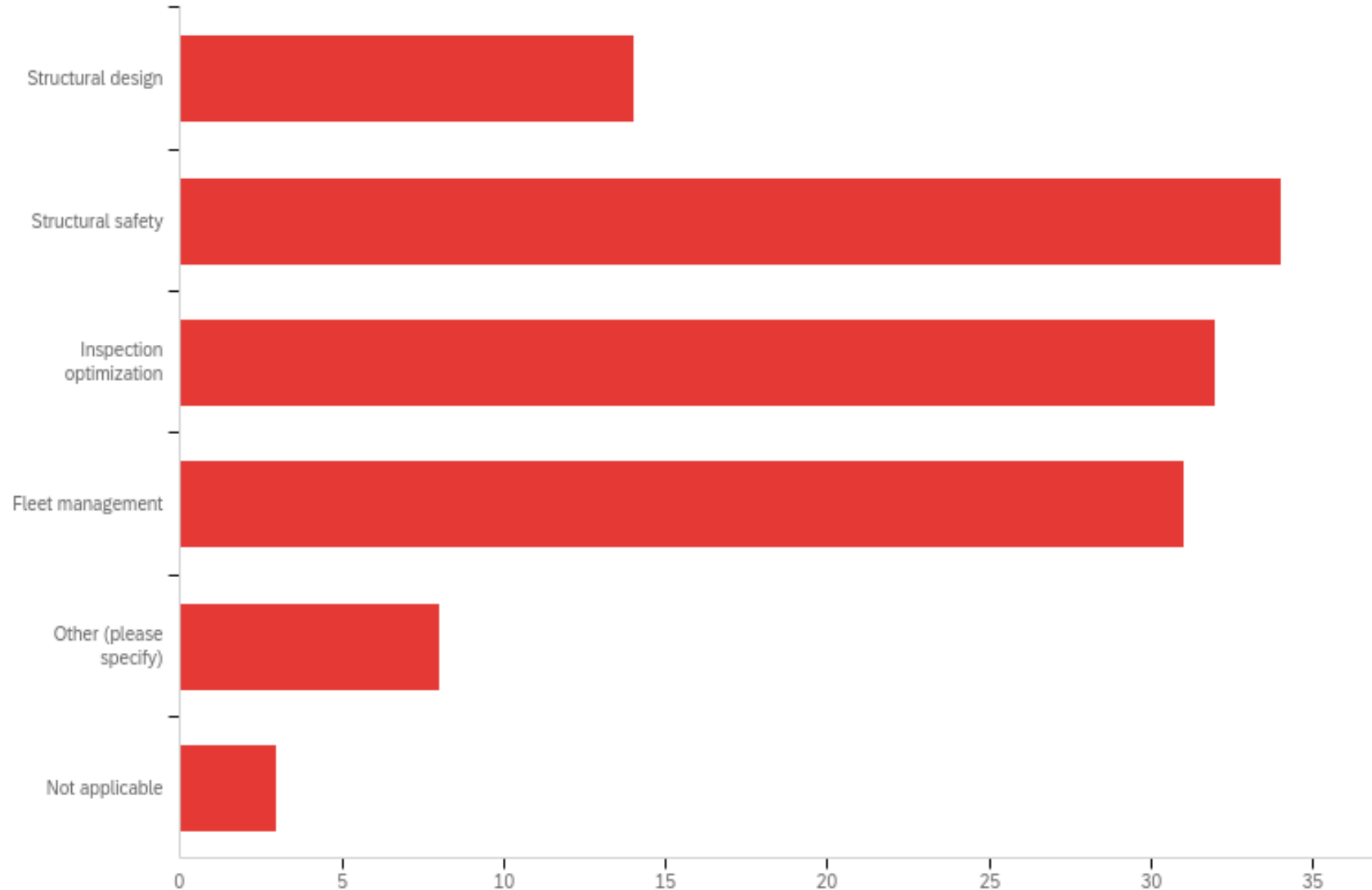
# What source do you use for your equivalent initial defect/ flaw size distribution? (II)



Industry standards, e.g. JSSG-2006
Seattle ACO unpublished guidance
experimental data
Aircraft tear down
Crack inspection data
inspection crack data
FSFT data
Military type flaw sizes
DOT/FAA/CT-93/69 II
Developed from fleet data

Field and/or FSDT data
Seattle ACO Guidance Published 10/99 (Dr. Safarian)
Wild ass guess (WAG)
non-destructive inspection
Historical Data

# For what purpose do you use risk assessment results?



# For what purpose do you use risk assessment results?



## Other:

We are currently working on using PRA as a relative assessment tool to justify short delays in inspections when they fall outside convenient maintenance opportunities

Research/Demo approaches for design, safety, management

Justification for Airworthiness Directives

WFD Assessment

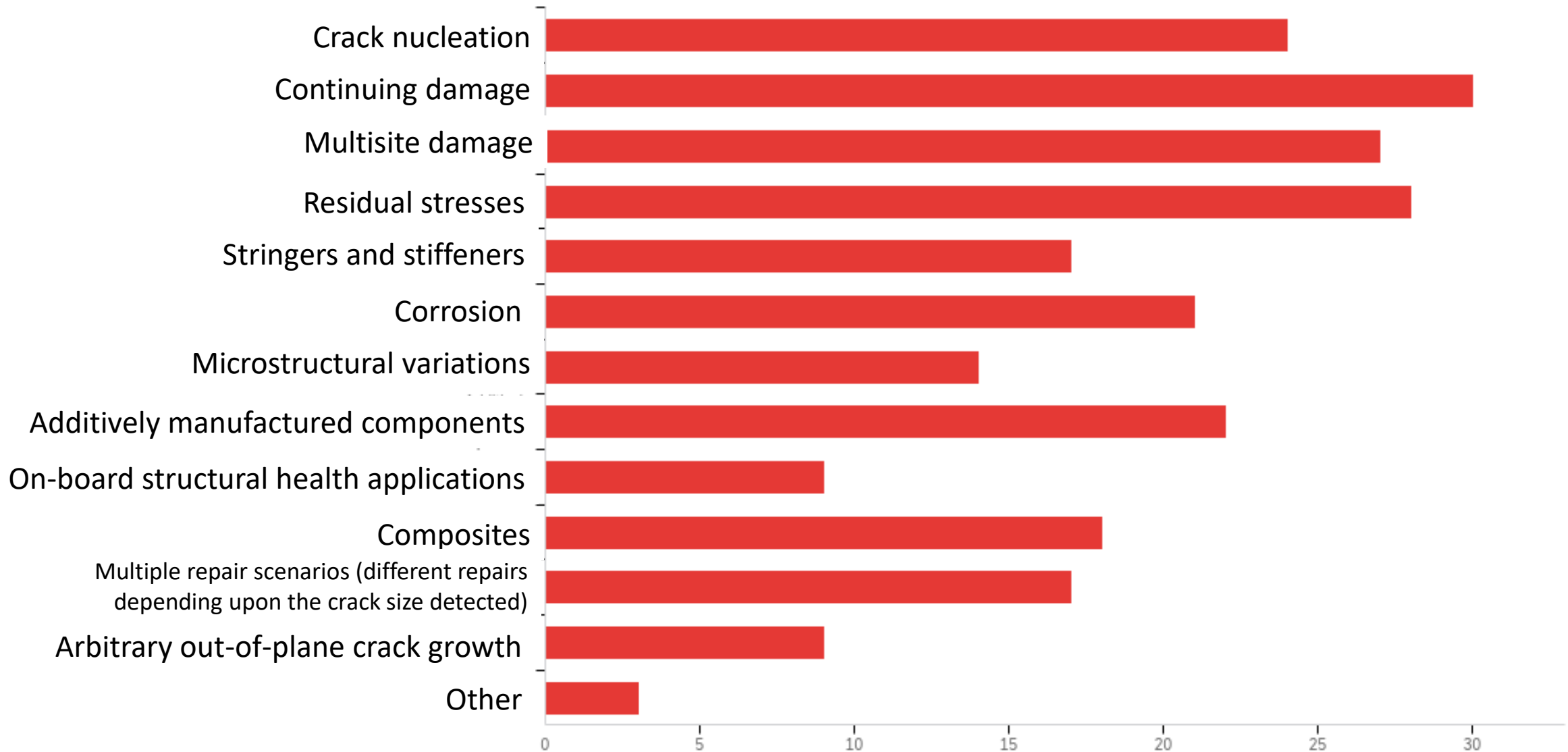
probabilistic methods have to be approved by the FAA for civil aviation. I have not heard of an FAA approval for probabilistic methods to certify airplanes (approve data).

Life of type (aircraft retirement prediction)

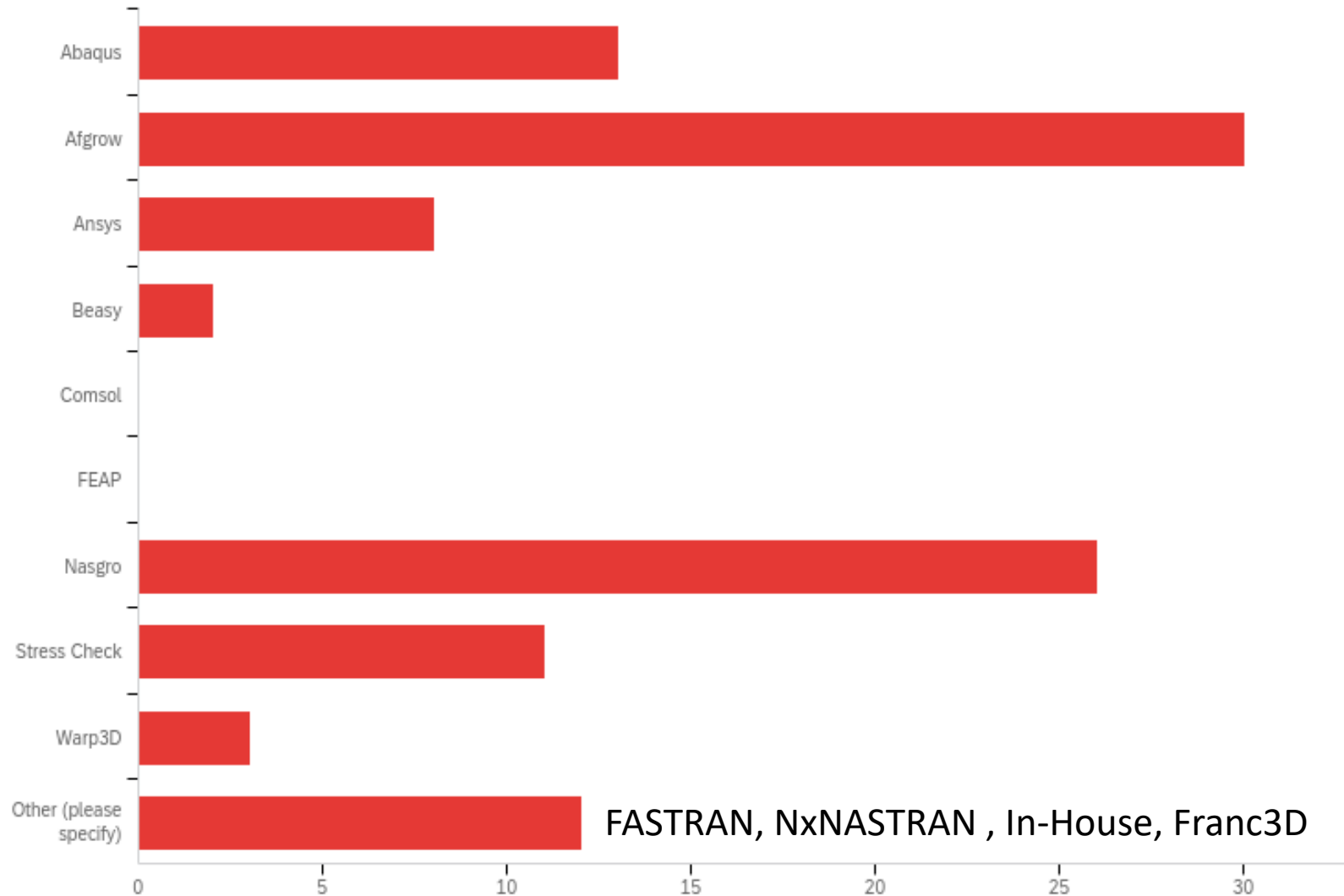
Defining structural Life limit as per MIL-A-1530D, also fleet risk when unexpected damage is found

Research

# Are there structural/fracture mechanics capabilities that you would like to use with risk assessment methods?

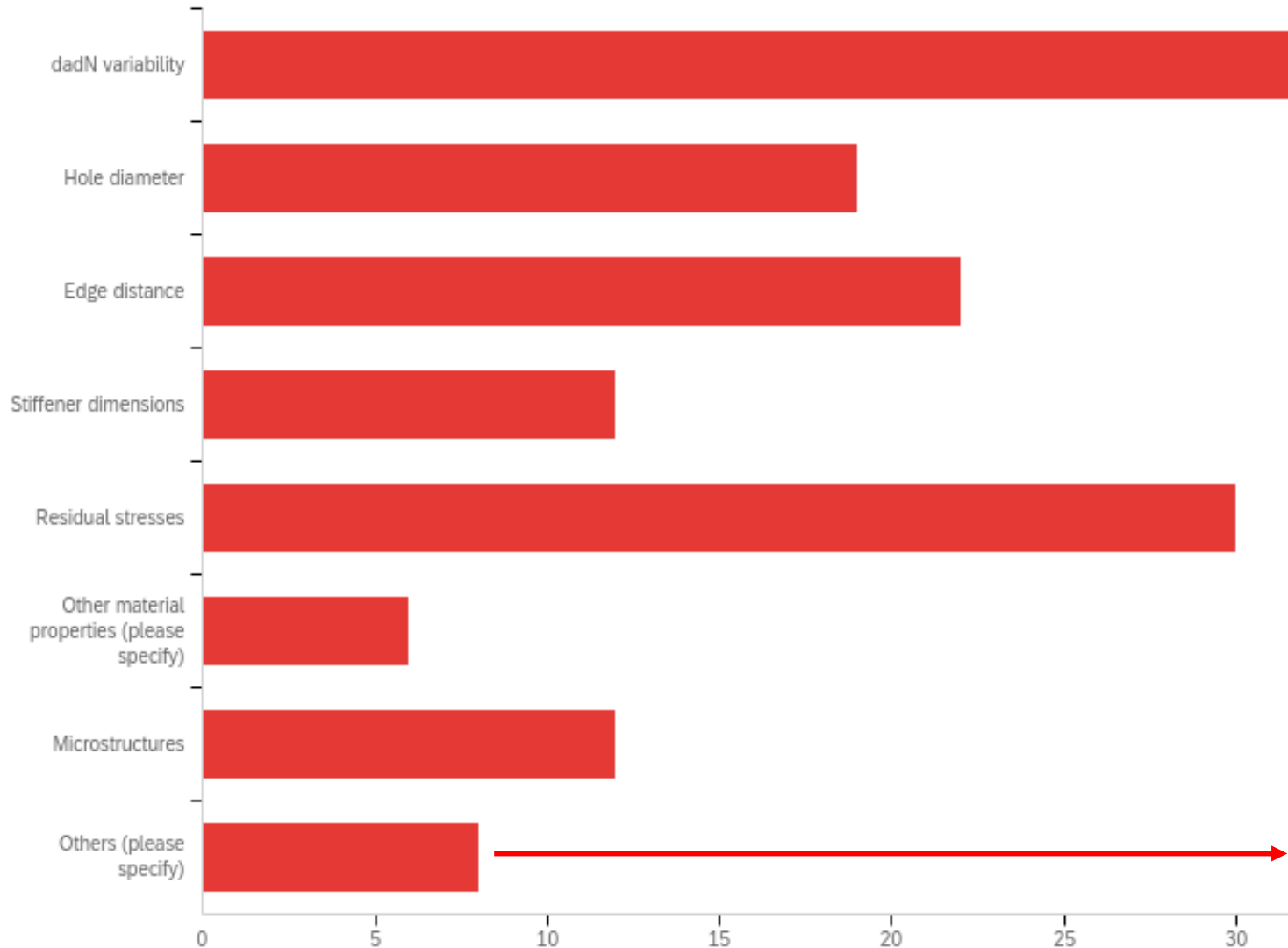


Are there software (commercial or open source) that you would like to use with a risk assessment analysis? (select all that apply)



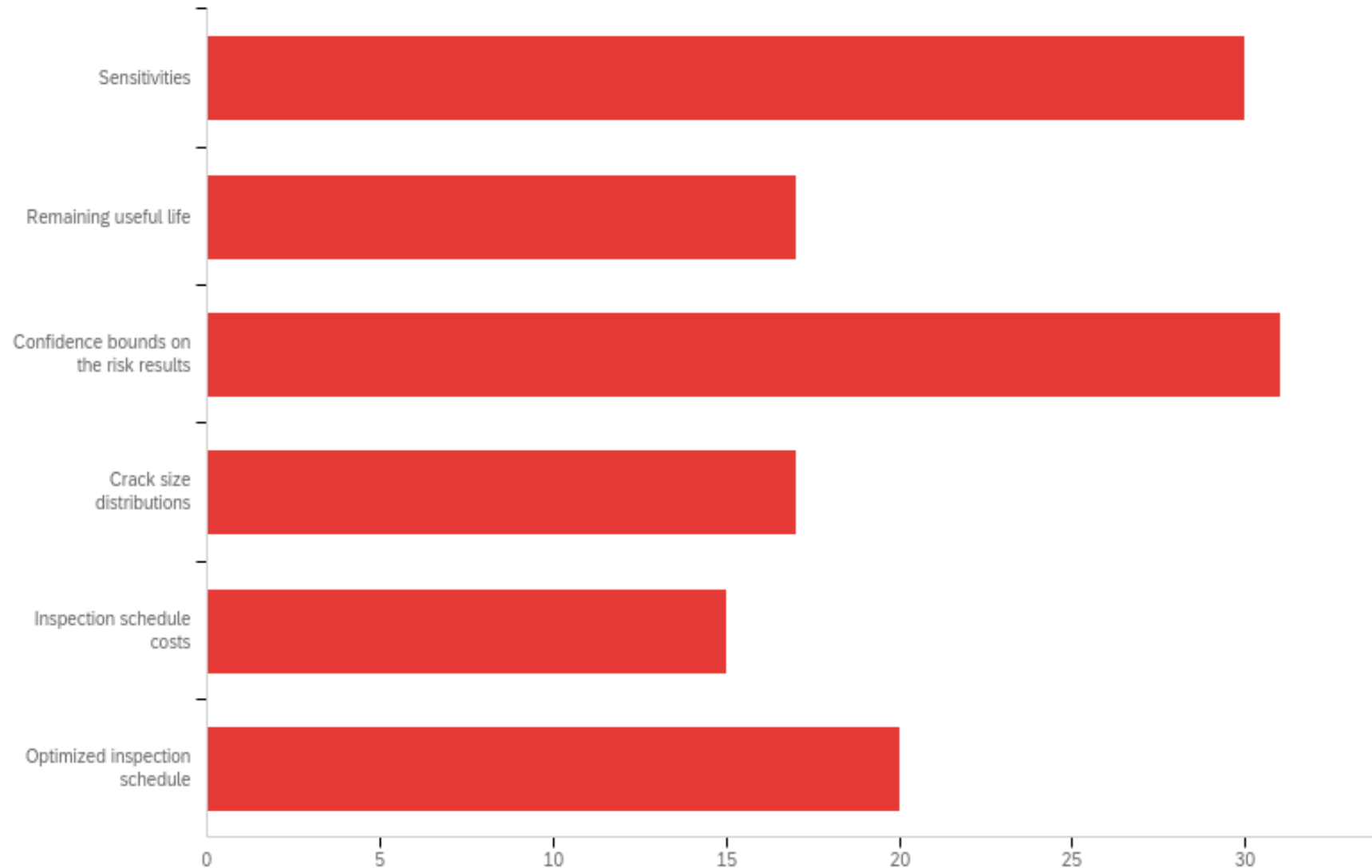


In addition to the equivalent initial flaw size (EIFS), fracture toughness and loading, are there additional random variables you would like to consider during a risk analysis and if so, what variables?

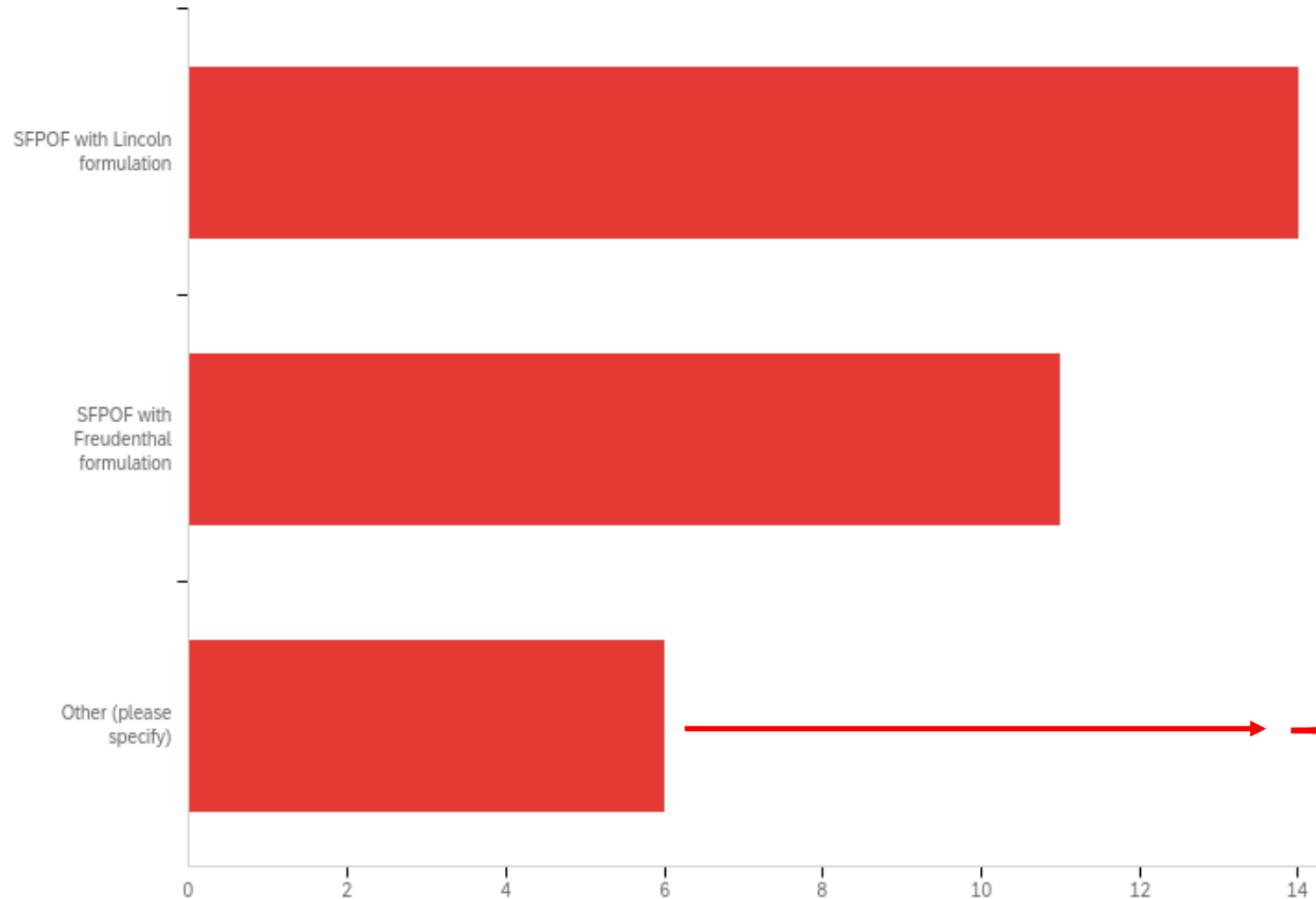


- Time from last inspection to failure
- None of the above
- NDI POD
- Interference
- See those parameters by Fawaz in 2001 and SwRI separately several years later.
- Fastener fit and fill
- Probability of detection (POD)
- uncertainties in SIF calc

# Are there probabilistic outputs that would be useful that you don't currently obtain?



# Do you use the Single Flight Probability of Failure (SFPOF) as your risk assessment measure?



To be honest I am not entirely sure - I think it is the top-most one as the software we used attempts to mimic PROF

Fleet level Weibull analysis

HR calculated from CPOF

the supporting data bases do not have a adequate pedigree for computational accuracy.

Survival analysis

This was discussed at some stage for Fracrisk, but not sure of the outcome

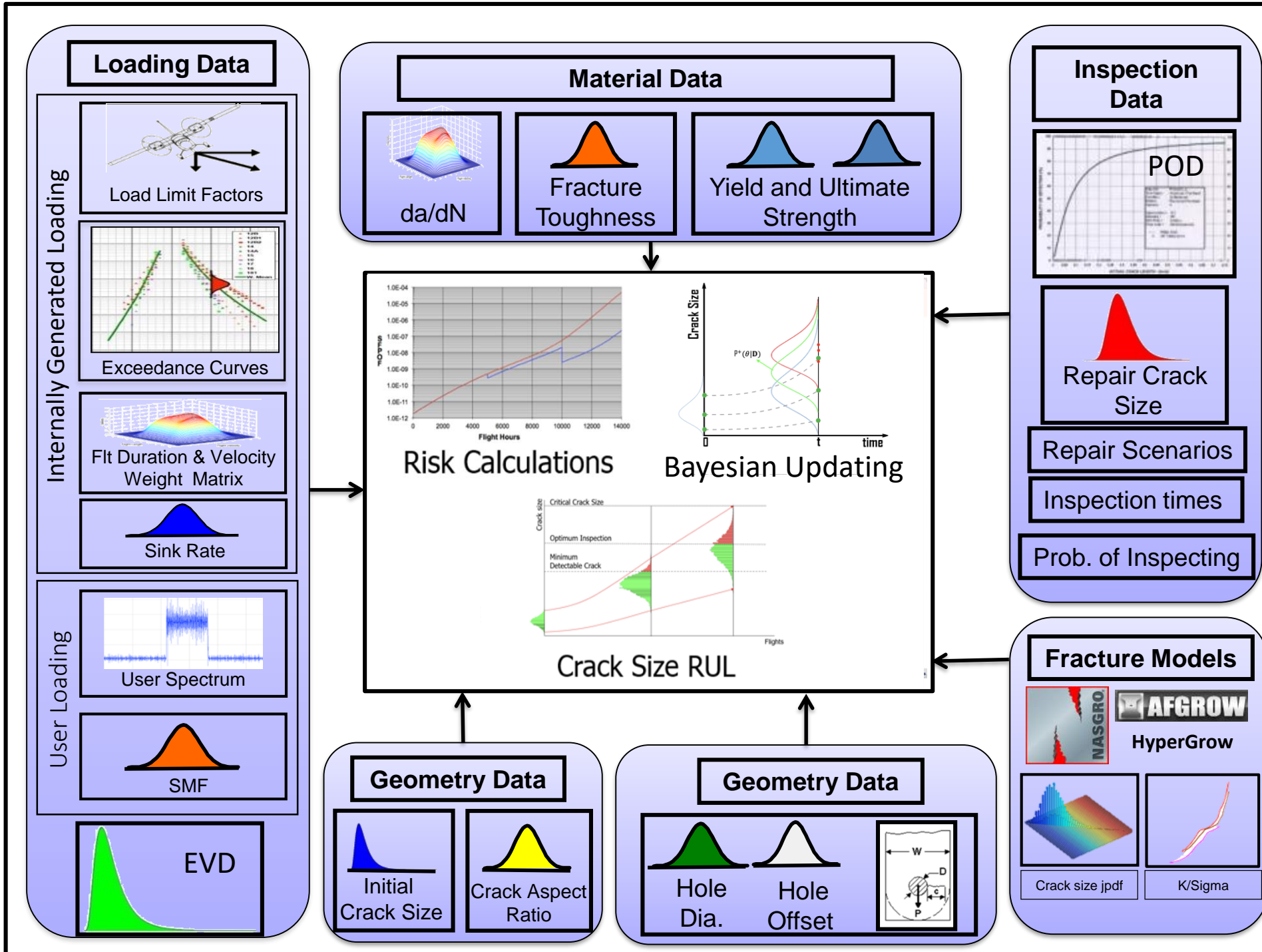
# Are there other desirable capabilities or features not previously mentioned?



Failsafe structure
Need to establish the means to define the parameter variation data bases.
In using PRA I get somewhat frustrated at the extreme sensitivities to particular inputs. Understanding these sensitivities and why they occur is not always straight-forward. For instance POI can cause huge changes in results in some cases but not others. And of course how do we know what a good POI number is? 0.95? 0.90?
crack shape effects
NDI updating using Bayesian, damage sensor results updating; load sensor data updating
assessment based on positive and negative findings of an operating fleet
Risk of rogue flaw from manufacturing process or accidental damage from inspection/maintenance
Updating of risk predictions using Bayesian Inference
Risk assessment methods only work with sufficient data for the random variables and there is not enough data to satisfy FAA or military certification officials to make absolute risk assessments. Risk assessments are always relative which is not much use for my customers.



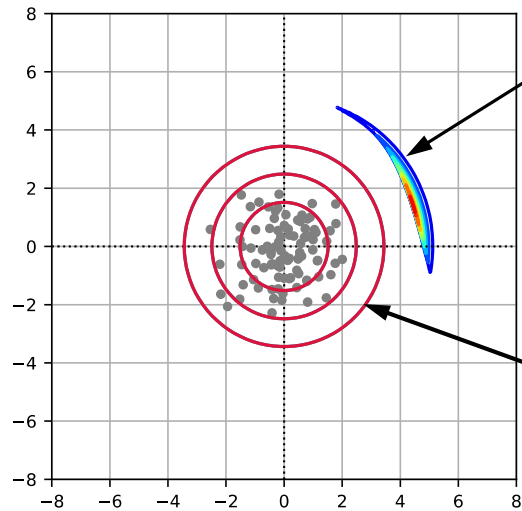
# The SMART Software



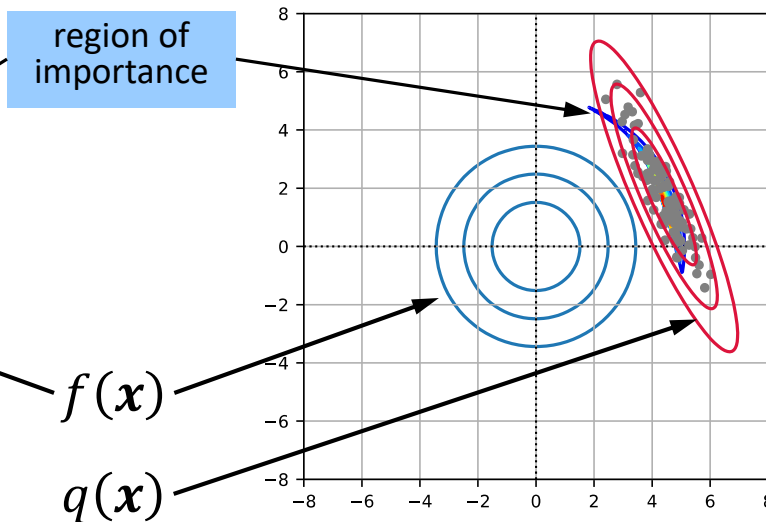
# Importance Sampling



Standard Monte Carlo Sampling



Importance Sampling



- Define  $H(\mathbf{x}; t) = 1 - F_{\text{EVD}}(\sigma_{\text{RS}}(\mathbf{x}; t))$

- From 
$$\text{var}(\hat{\mathbb{E}}) = \frac{1}{N^2} \sum_i \left( H(\mathbf{x}_i, t) \frac{f(\mathbf{x}_i)}{q(\mathbf{x}_i)} - \hat{\mathbb{E}} \right)^2$$

the estimator variance is 0 when

$$H(\mathbf{x}_i, t) \frac{f(\mathbf{x}_i)}{q(\mathbf{x}_i)} - \hat{\mathbb{E}} = 0$$

so the optimal estimated density is

$$\hat{g}(\mathbf{x}_i) = \frac{H(\mathbf{x}_i, t) f(\mathbf{x}_i)}{\hat{\mathbb{E}}}$$

$$\mathbb{E}[H(\mathbf{x}, t)] = \int H(\mathbf{x}, t) f(\mathbf{x}) d\mathbf{x}$$

$$\hat{\mathbb{E}}[H(\mathbf{x}, t)] = \frac{1}{N} \sum_i H(\mathbf{x}_i, t)$$

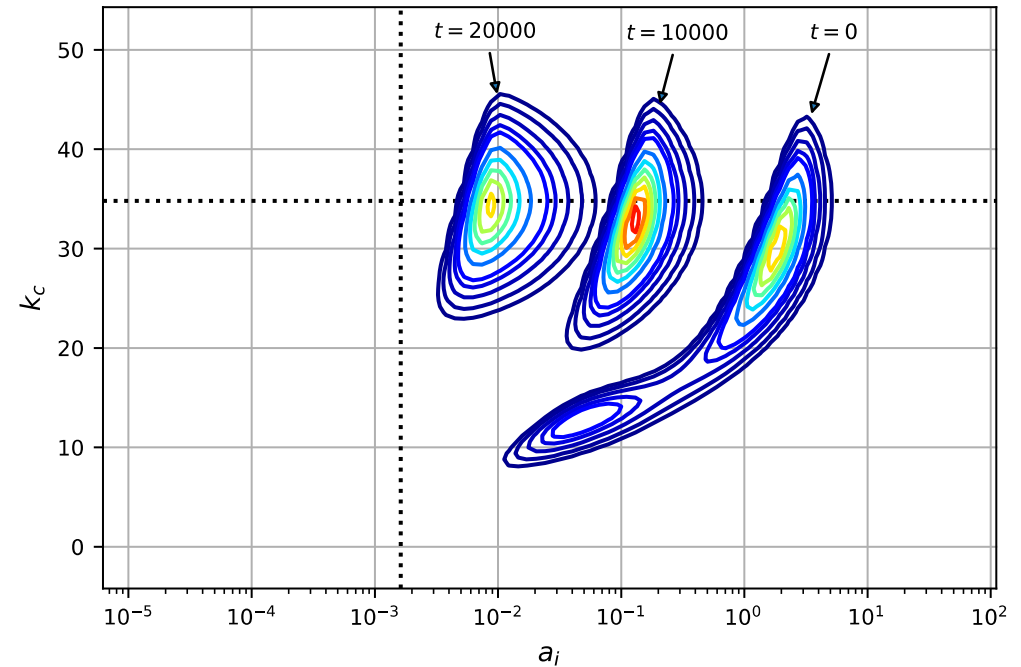
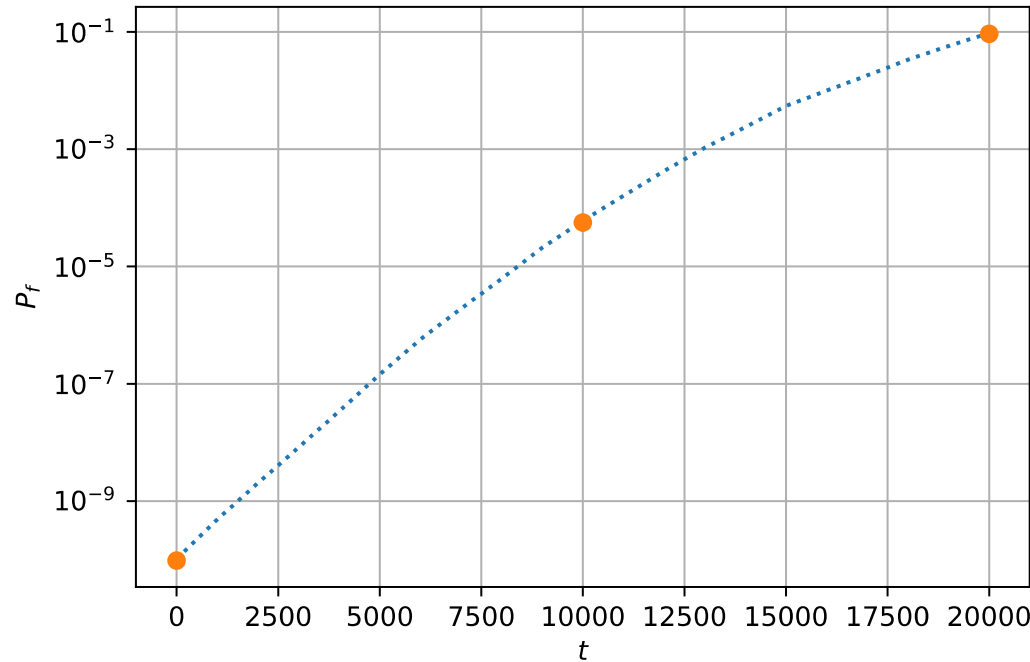
$$\mathbb{E}[H(\mathbf{x}, t)] = \int H(\mathbf{x}, t) \frac{f(\mathbf{x})}{q(\mathbf{x})} q(\mathbf{x}) d\mathbf{x}$$

$$\hat{\mathbb{E}}[H(\mathbf{x}, t)] = \frac{1}{N} \sum_i H(\mathbf{x}_i, t) w(\mathbf{x}_i)$$

Importance weight

$$w(\mathbf{x}_i) = f(\mathbf{x}_i) / q(\mathbf{x}_i)$$

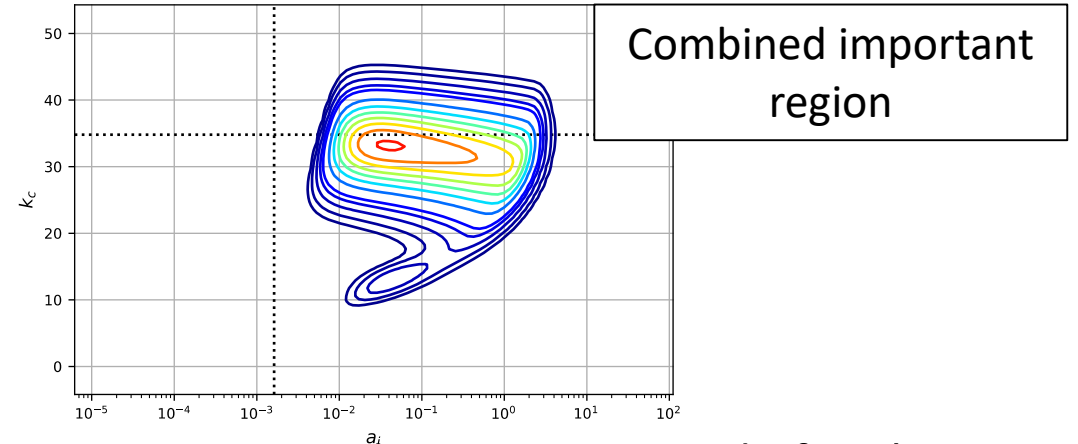
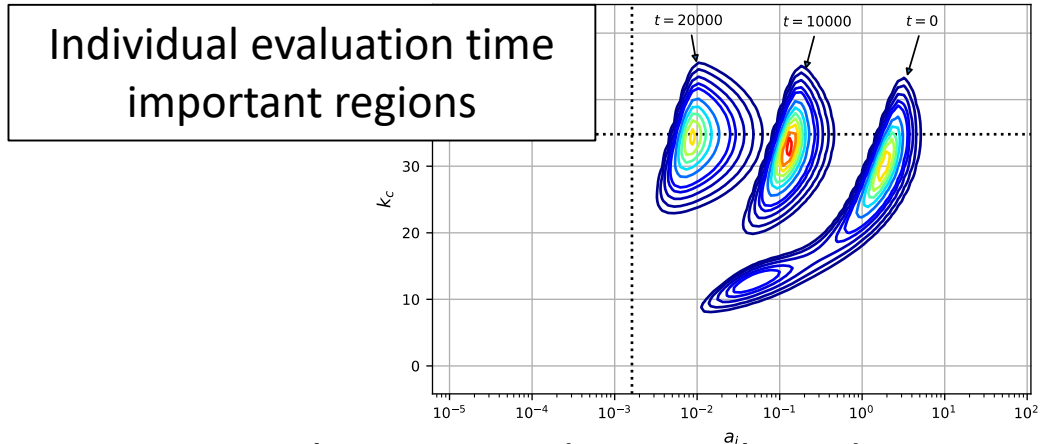
# Standard Adaptive Importance Sampling Approach



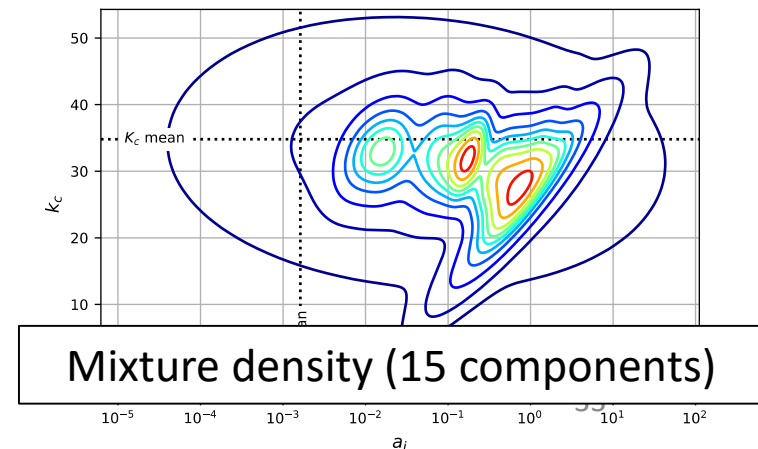
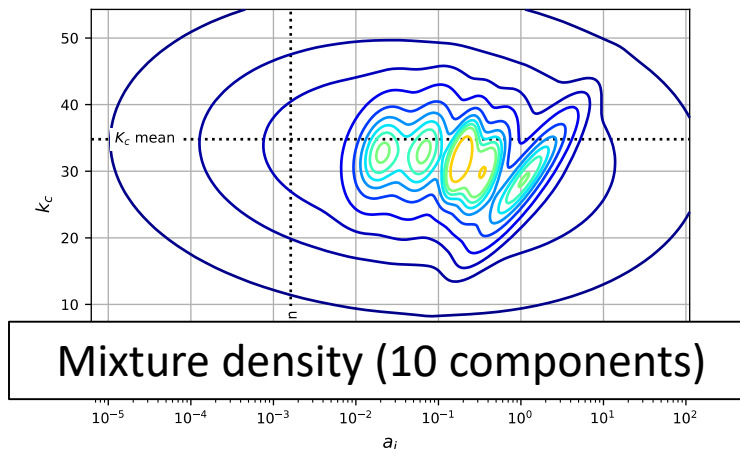
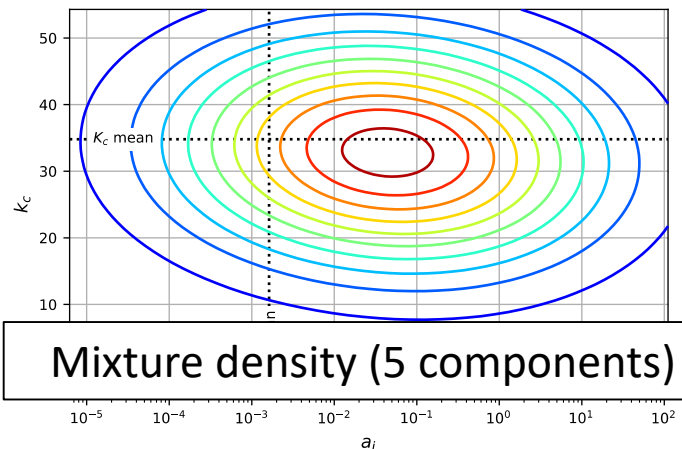
- Adapt a sampling density to the important region for each evaluation time,  $t$ 
  - Regions move as  $t$  changes
  - Regions can be multimodal
- Adaptation process require several iterations to converge for each  $t$  using small sample sizes



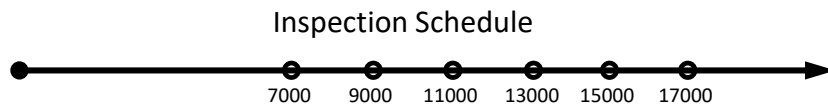
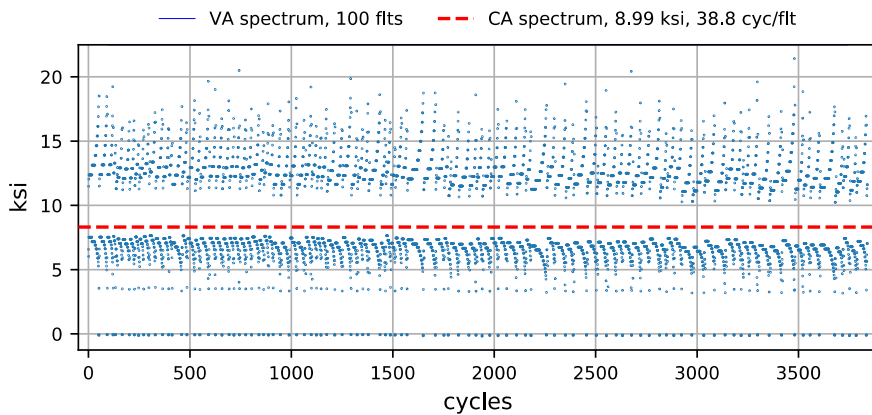
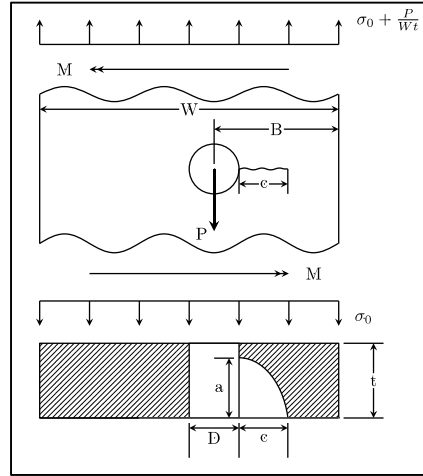
# Adaptive Multiple Importance Sampling Approach



- Approximate the averaged or combined important region using a mixture density composed of multivariate normal sampling densities optimized for individual evaluation times
- Key advantage is that samples can be used for more than one important region where regions overlap



# NASGRO Example with Inspections and Repairs



Parameter	Value
Width	Deterministic 2.5 in
Thickness	Deterministic 0.25 in
Initial Crack Size	$LN(0.005, 0.002)$ in
Aspect Ratio (A/C) <sup>1</sup>	$N(1.5, 0.14)$
Fracture Toughness	$N(34.8, 3.90)$ ksi $\sqrt{\text{in}}$
Log Paris Constant	$N(-8.777, 0.08)$
Paris Exponent	Deterministic 3.273
Hole Diameter	Deterministic 0.1562 in
Hole Offset <sup>2</sup>	$N(0.5, 0.05)$ in
Maximum Stress per Flight	$EVD(16.74, 2.08, 0.0)$ ksi
Probability of Detection	$LN(0.021, 0.028)$ in



<sup>1</sup> Random A/C values were clipped to Nasgro CC16 stress intensity factor limits

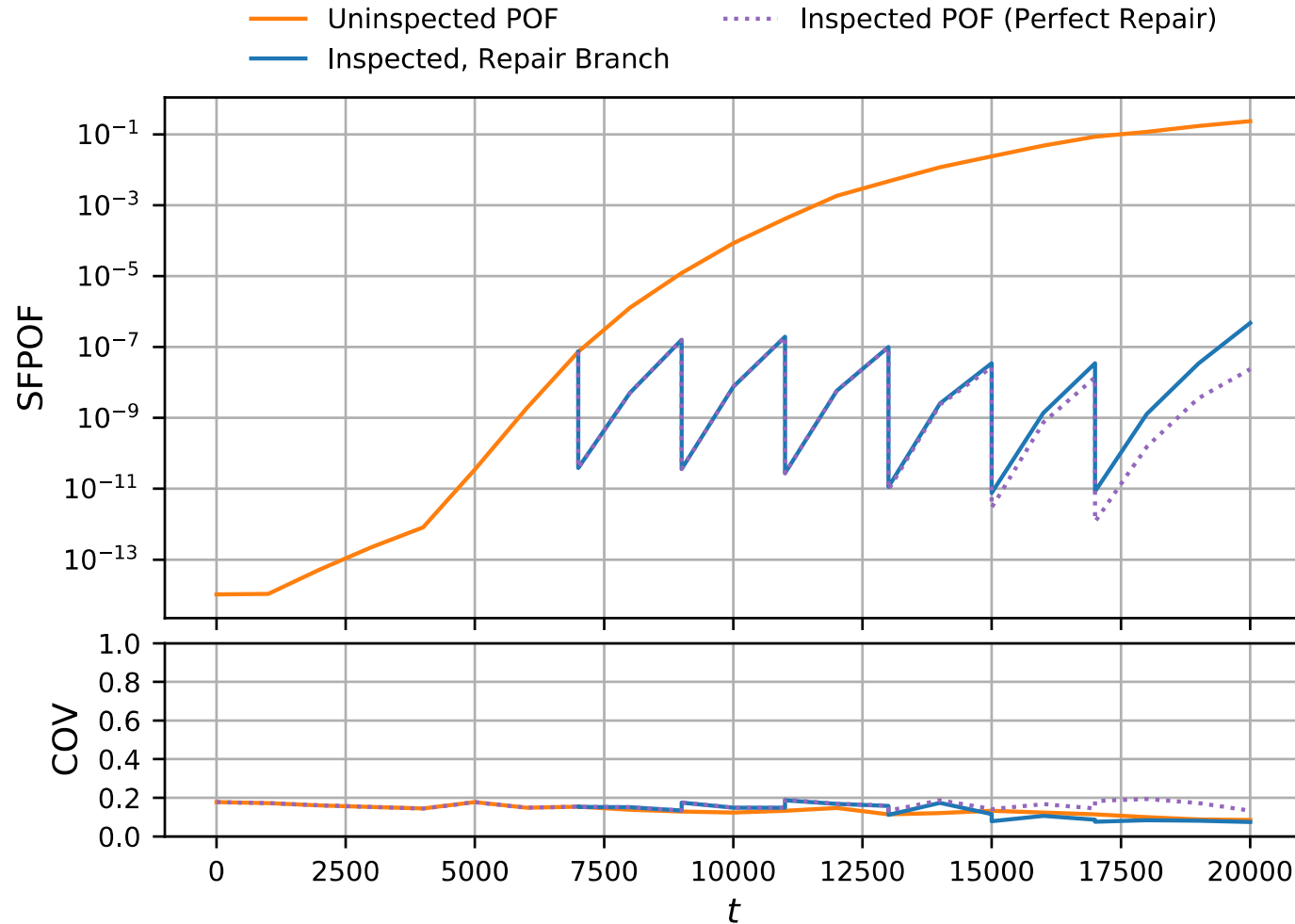
$$0.1 \leq A / C \leq 10$$

<sup>2</sup> Random Hole Offset values outside Nasgro CC16 stress intensity factor limit

$$\frac{D + C}{2B + C} \leq 0.7$$

were treated as immediate fracture

# POF Results with Repairs



- PDTA AMIS
  - Inspected POF: 4060 samples
  - Uninspected POF: +0 samples
  - Percent Cracks Det: +140 samples
  - Repairs Branch POFs: 4060 samples
- 8260 total samples
- COV for the total POF including repairs decreases because the combined POF is increasing by an order of magnitude
- Total run time: ~28min

# Bayes theorem



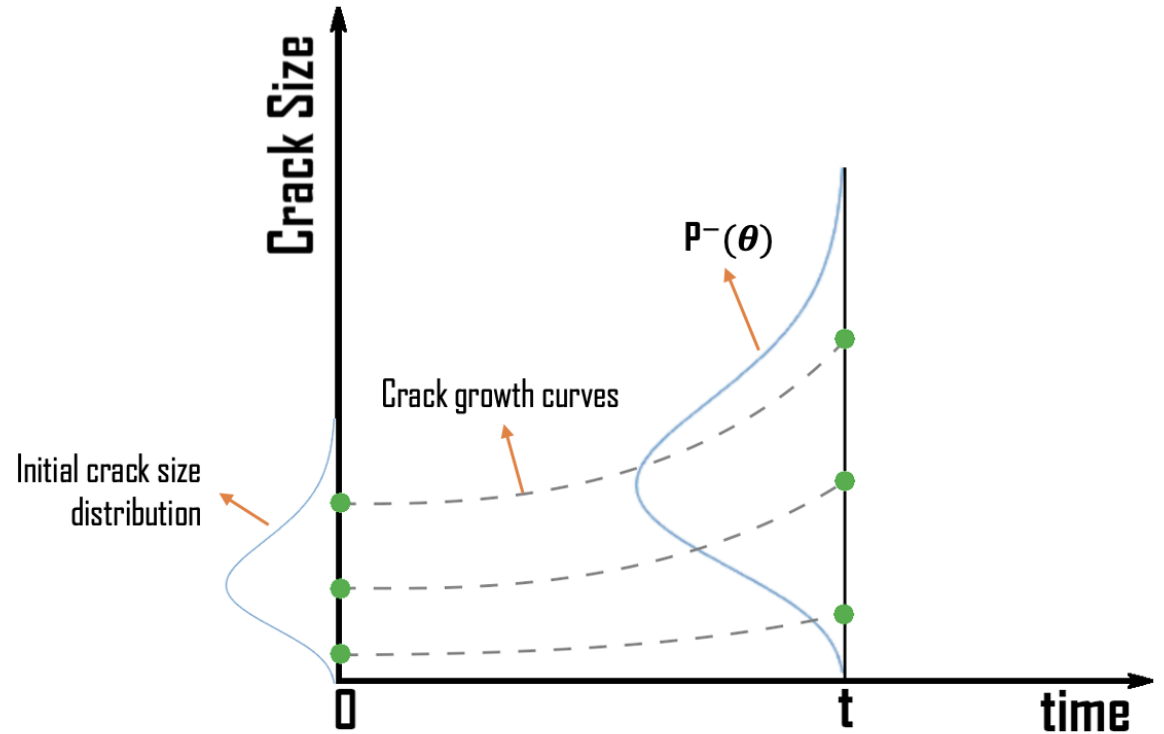
$$\underbrace{P^+(\theta|\mathbf{D})}_{\text{Posterior distribution}} = \frac{\overbrace{L(\mathbf{D}|\theta)}^{\text{Likelihood}} \cdot \overbrace{P^-(\theta)}^{\text{Prior distribution}}}{\underbrace{NF}_{\text{Normalization factor}}}$$

- $\theta$  represents the parameters mean( $\mu$ )  $\rightarrow$  independent variable and standard deviation( $\sigma$ )  $\rightarrow$  assumed, it will be fixed,
- $\mathbf{D}$  represents the vector of the measurements (or inspections),
- $P^-$  represents the prior distribution  $\rightarrow$  Distribution of crack size at the time,
- $L(\mathbf{D}|\theta)$  represents the likelihood function of the parameters.
- **NF** Normalization Factor, used to get a probability density function.
- $P^+$  represents the posterior distribution given the detected crack sizes.

# Bayesian Updating



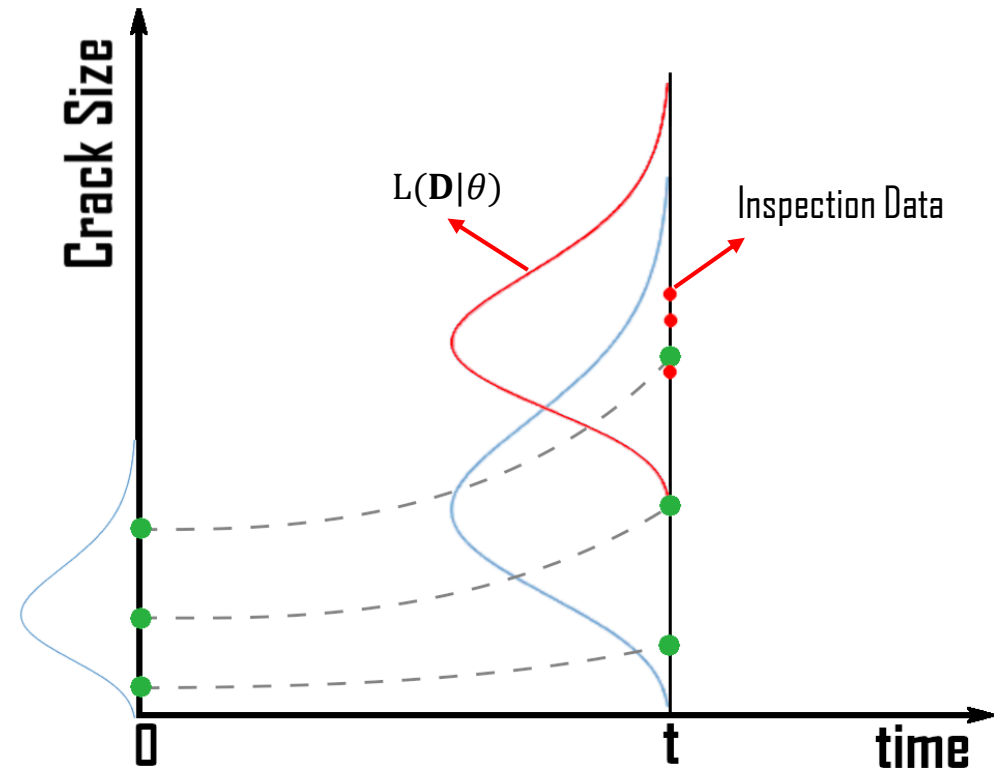
$$\underbrace{P^+(\theta | \mathbf{D})}_{\text{Posterior distribution}} = \frac{\overbrace{L(\mathbf{D} | \theta)}^{\text{Likelihood}} \cdot \overbrace{P^-(\theta)}^{\text{Prior distribution}}}{\underset{\substack{\text{NF} \\ \downarrow \\ \text{Normalization factor}}}{NF}}$$



# Bayesian Updating



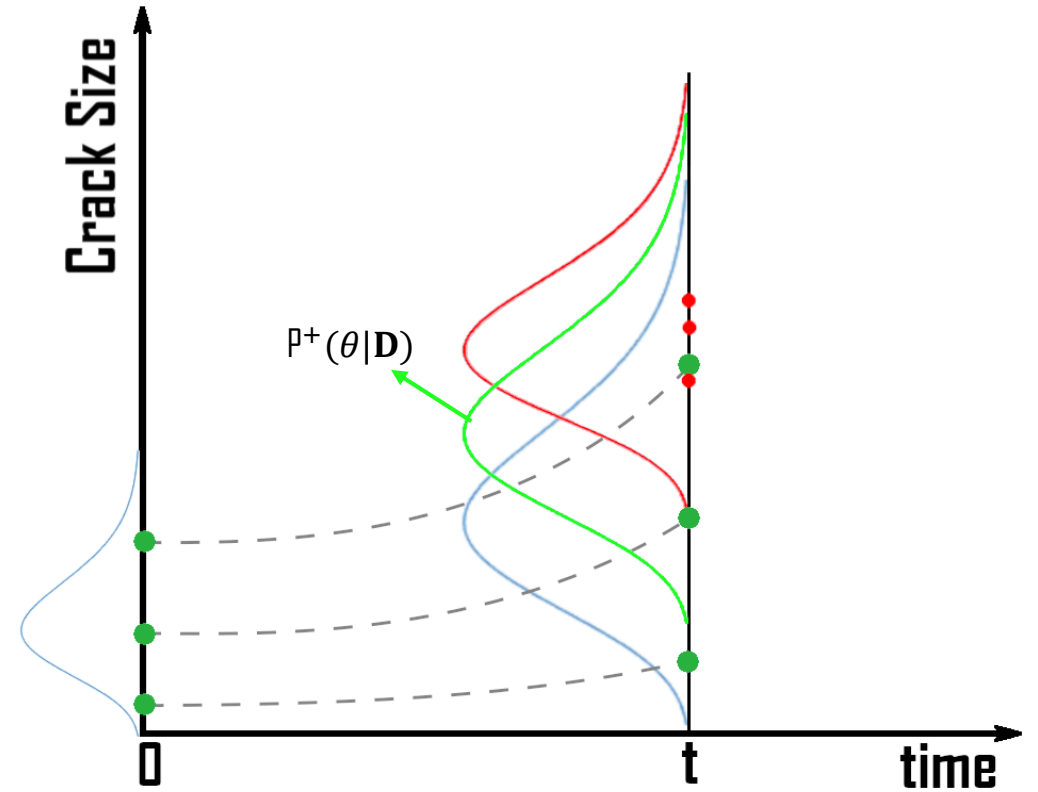
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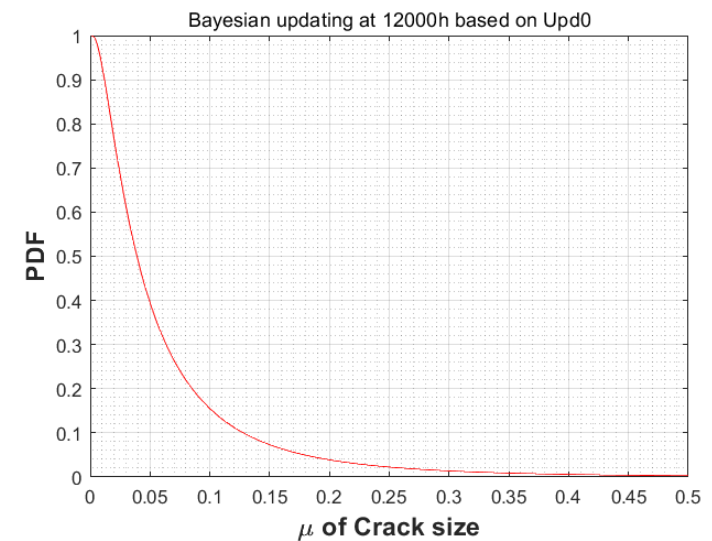
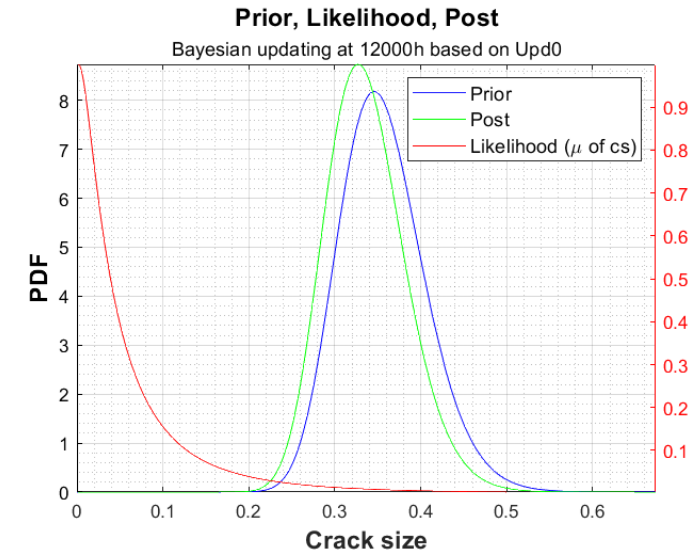
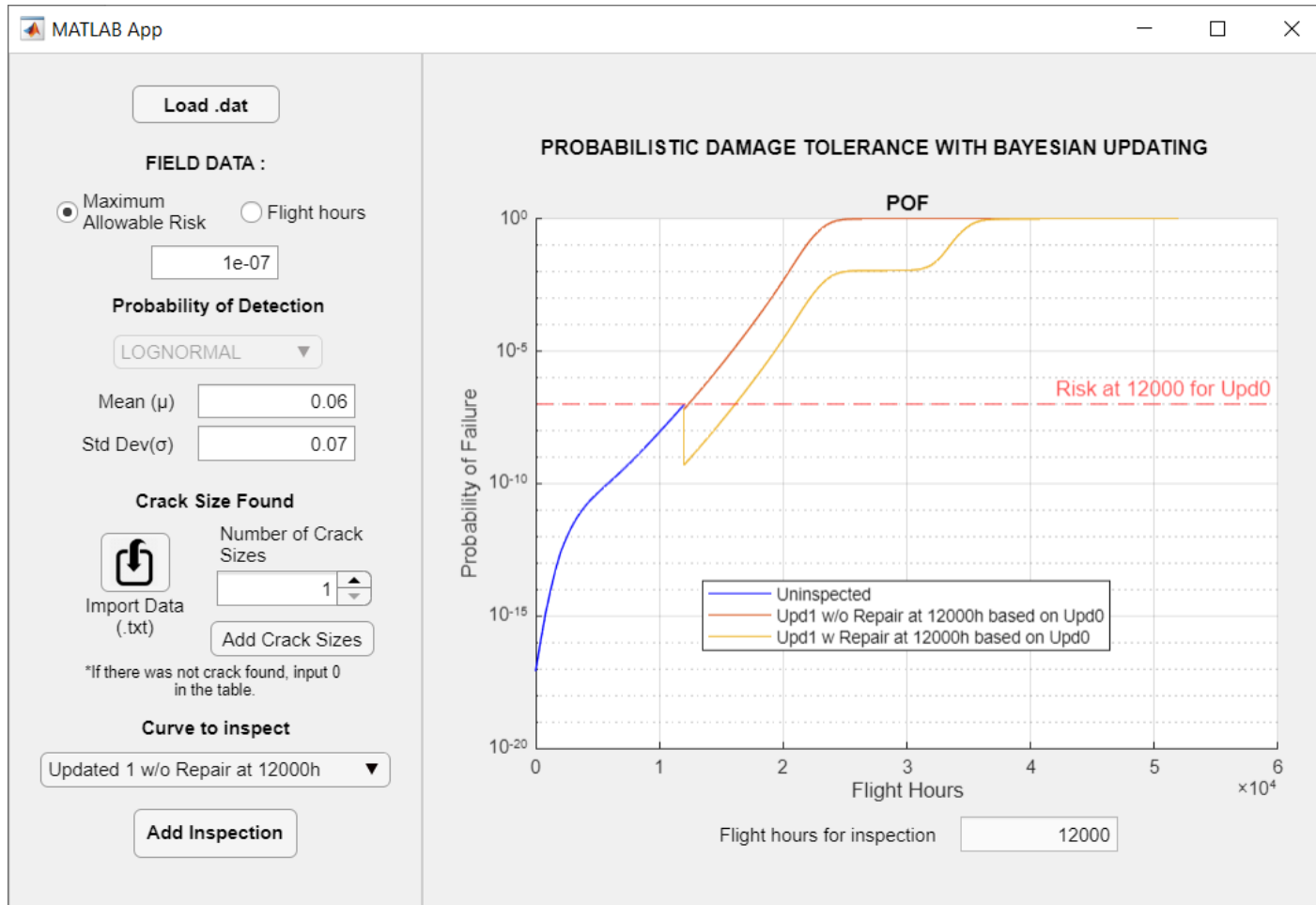
# Bayesian Updating



$$\underbrace{P^+(\theta | \mathbf{D})}_{\text{Posterior distribution}} = \frac{\underbrace{L(\mathbf{D} | \theta)}_{\text{Likelihood}} \cdot \underbrace{P^-(\theta)}_{\text{Prior distribution}}}{\underbrace{\text{NF}}_{\text{Normalization factor}}}$$

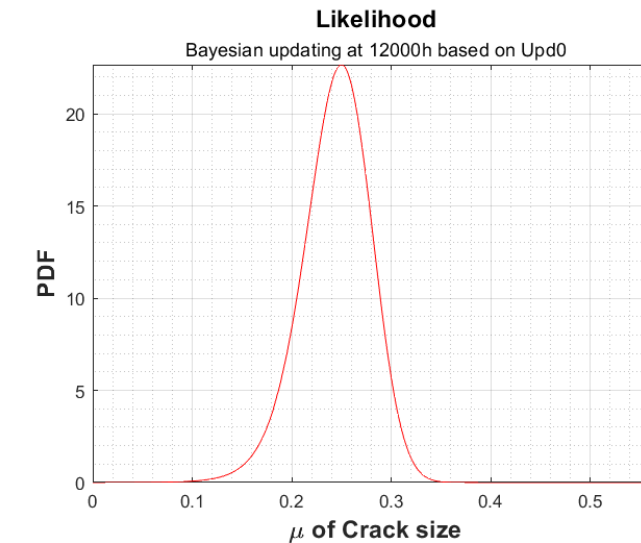
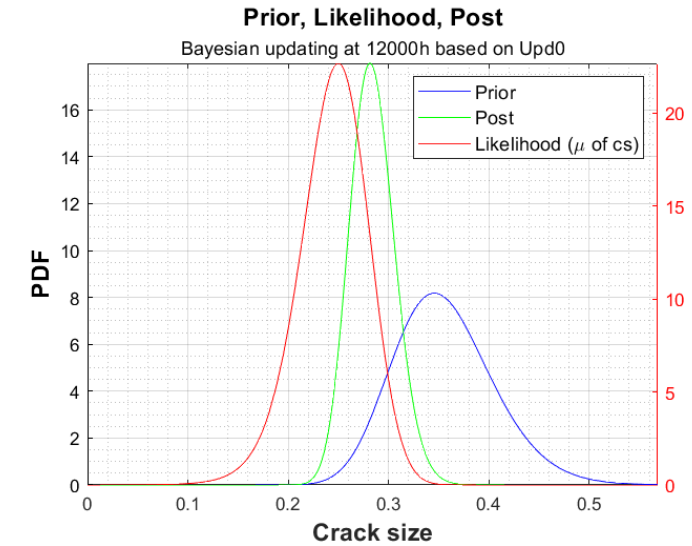
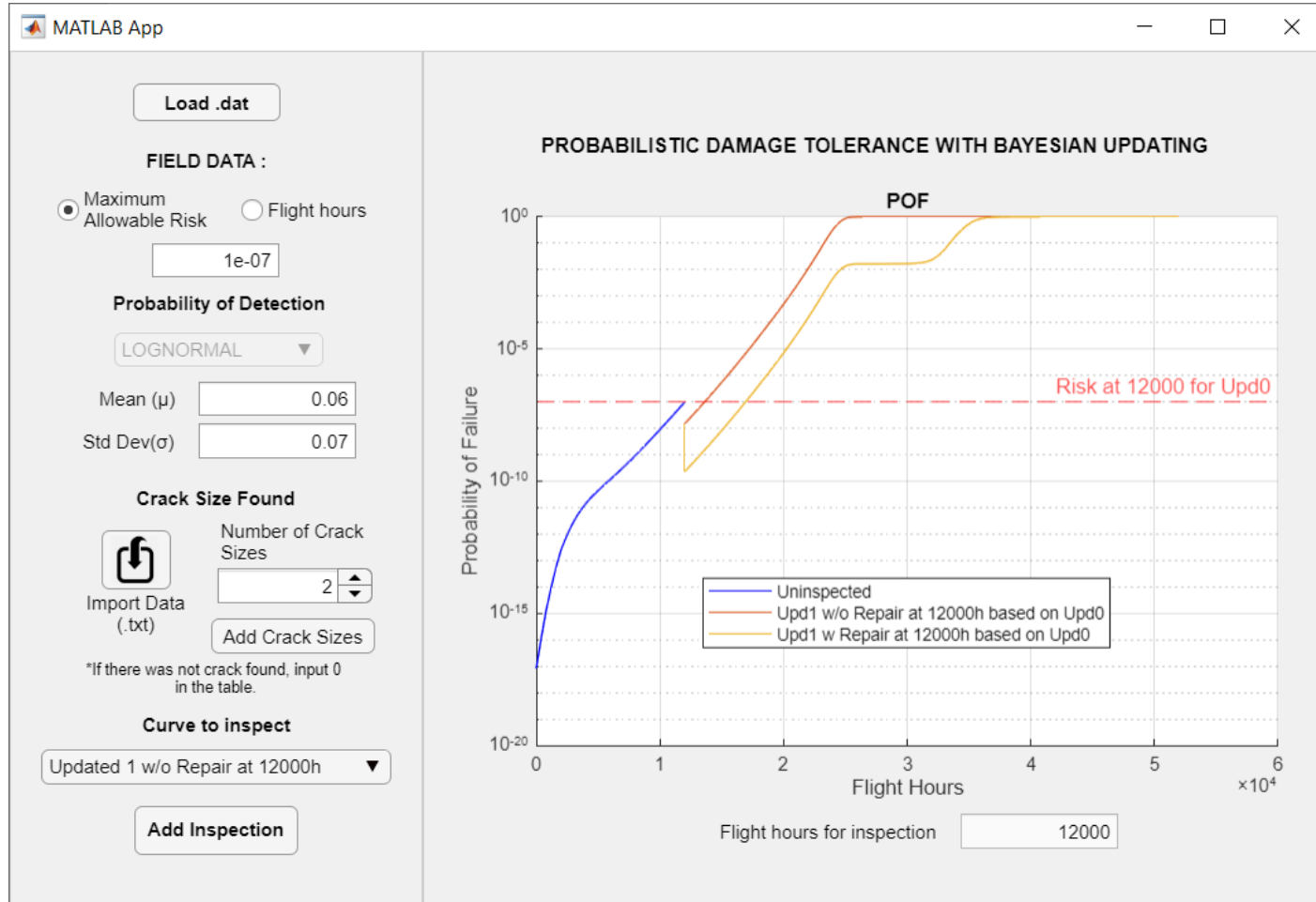


# Results Crack size No detection

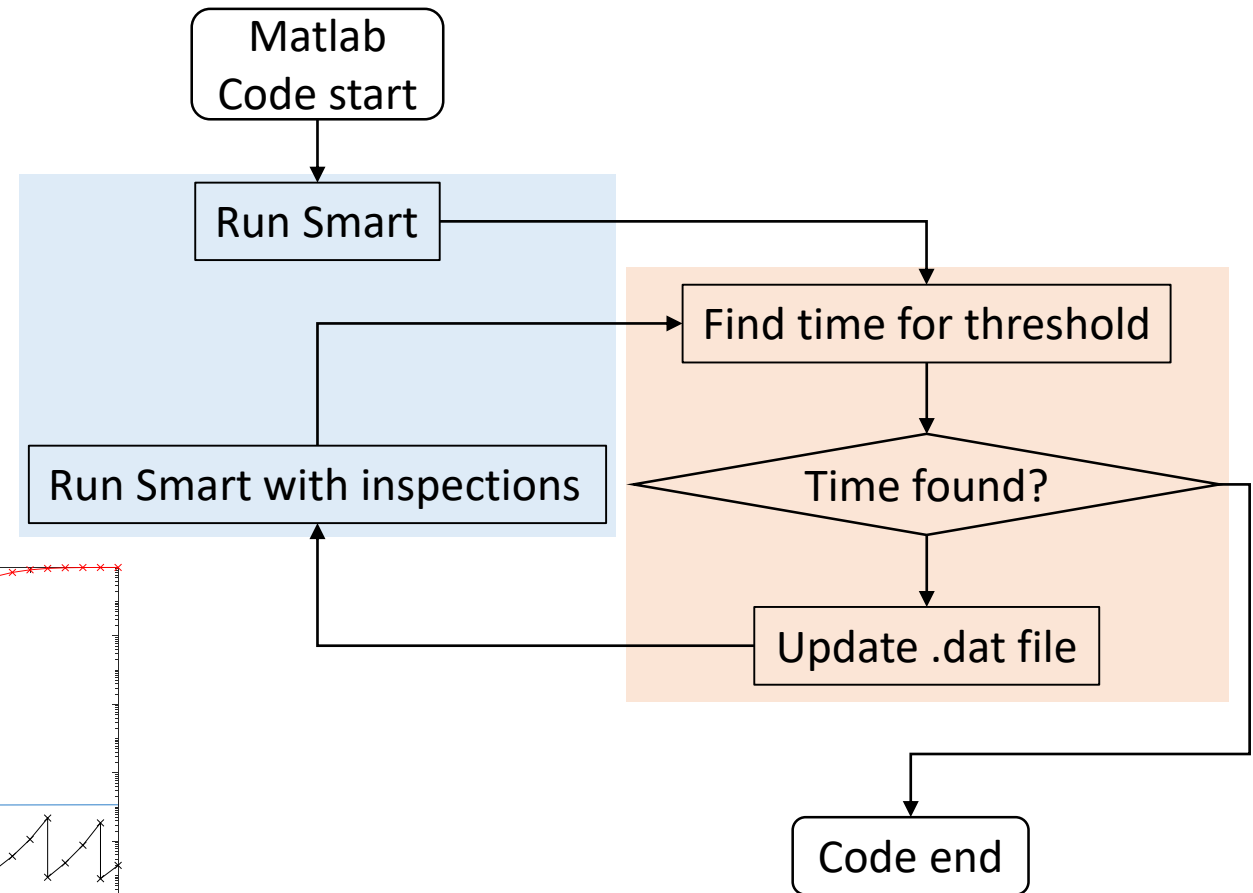
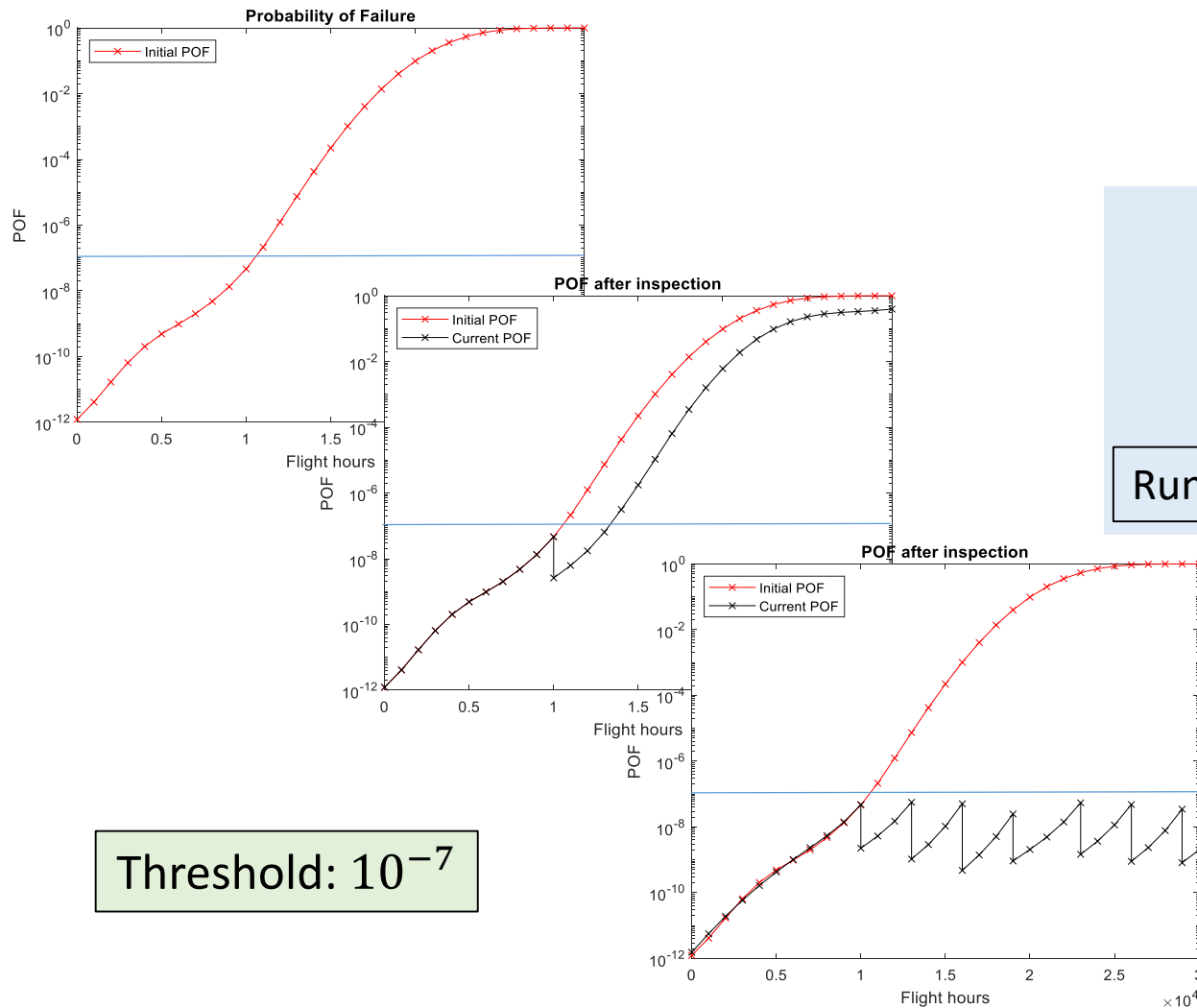




# Results Crack size det.= 0.3 and 0.2 in



# Optimized Inspections - Constant risk threshold



# Optimized Inspections - Shortest Path Method

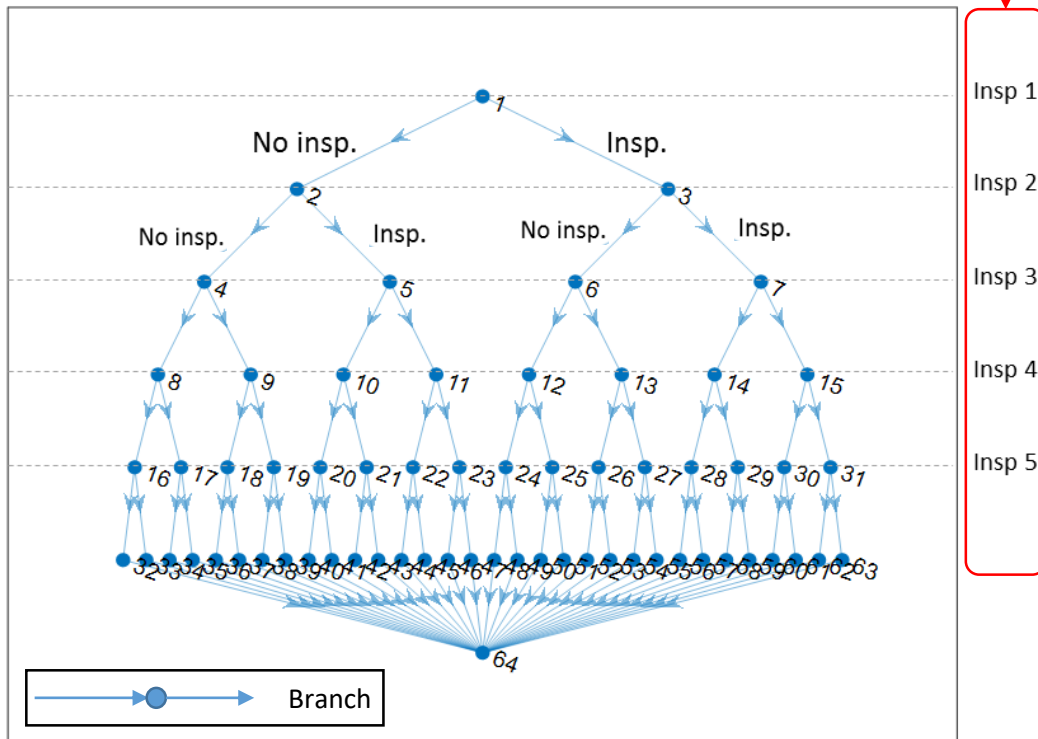


User defined candidate inspection times

Matlab Code start

Generate neural web with all possible branch combinations

Neuronal Web



Branch to evaluate?

Run Smart with inspections

POF under the threshold?

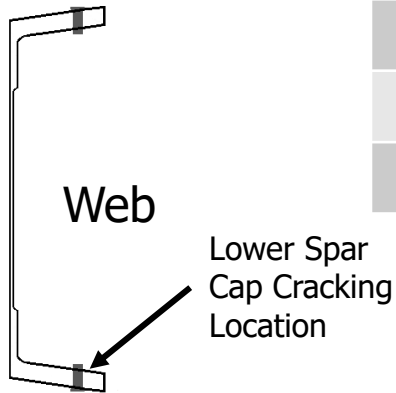
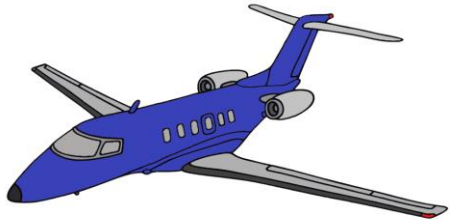
Reject branch

Skip branches to evaluate

Find the shortest path

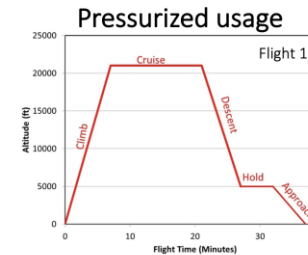
Code end

# Optimized Inspections - Input Data



Wing Forward Spar

Random Variable	Distribution	Parameters
Paris m	Binormal	Mean = 2.586 Standard Deviation = 0.0
Paris c (log)	Binormal	Mean = -7.888 Standard Deviation = 0.04
Correlation	-	0
Walker Exponent	-	0.82
Ultimate Stress	Normal	Mean = 69.0 ksi Standard Deviation = 0.0 ksi
Yield Stress	Normal	Mean = 58.0 ksi Standard Deviation = 0.0 ksi
Hole Offset	Normal	Mean = 0.9000 in Standard Deviation = 0.0 in



Parameter	Value
Design Load Limit Factors	Man (-1.50 3.60) Gust (-3.00 5.00)
Ground Stress	-100 psi
One-g Stress	3800 psi
Average Velocity	300 knots

Segment	Weight	KEAS	% Duration
CLIMB	14511	246	0.19
CRUISE	13709	221	0.55
DESCENT	13100	300	0.14
HOLD	12932	250	0.06
APPROACH	12803	150	0.06

$V_c = 300$  KEAS      MTOW = 17200 lb

Average Speed During Flight, % Max Takeoff Weight

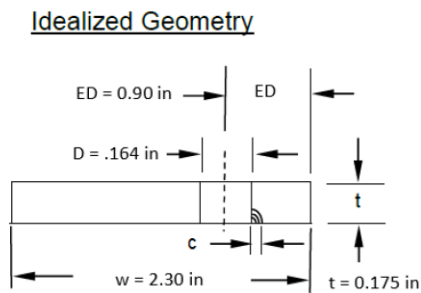
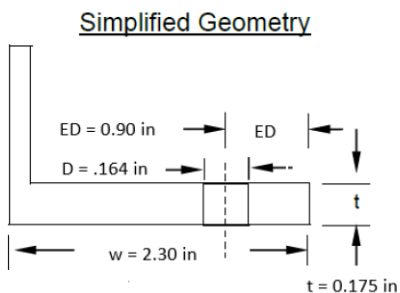
Flight Time (hrs)	% of Flights	0.500	0.826	0.833	0.957	1.00
0.62	1.0	0.14	0.19	0.13	0.38	0.16

Average Speed During Flight, % Design Velocity

Flight Time (hrs)	% of Flights	0.803	0.810	0.816	0.834	0.855
0.62	1.0	0.14	0.13	0.16	0.38	0.19

Sort matrix in ascending order for speed & weight

Variable	Dist. Type	Mean	St. Dev.	Notes
Initial Crack Size	Lognormal	0.00248 in	0.00129	Reamed Fastener Hole
Repair Crack Size	Lognormal	0.00248 in	0.00129	Assuming Repair is Replacement of Part
Fracture Toughness	Normal	26.0 ksi	2.0	7050-T651 Plate
EVD	Gumbel	14.5 ksi	0.8	

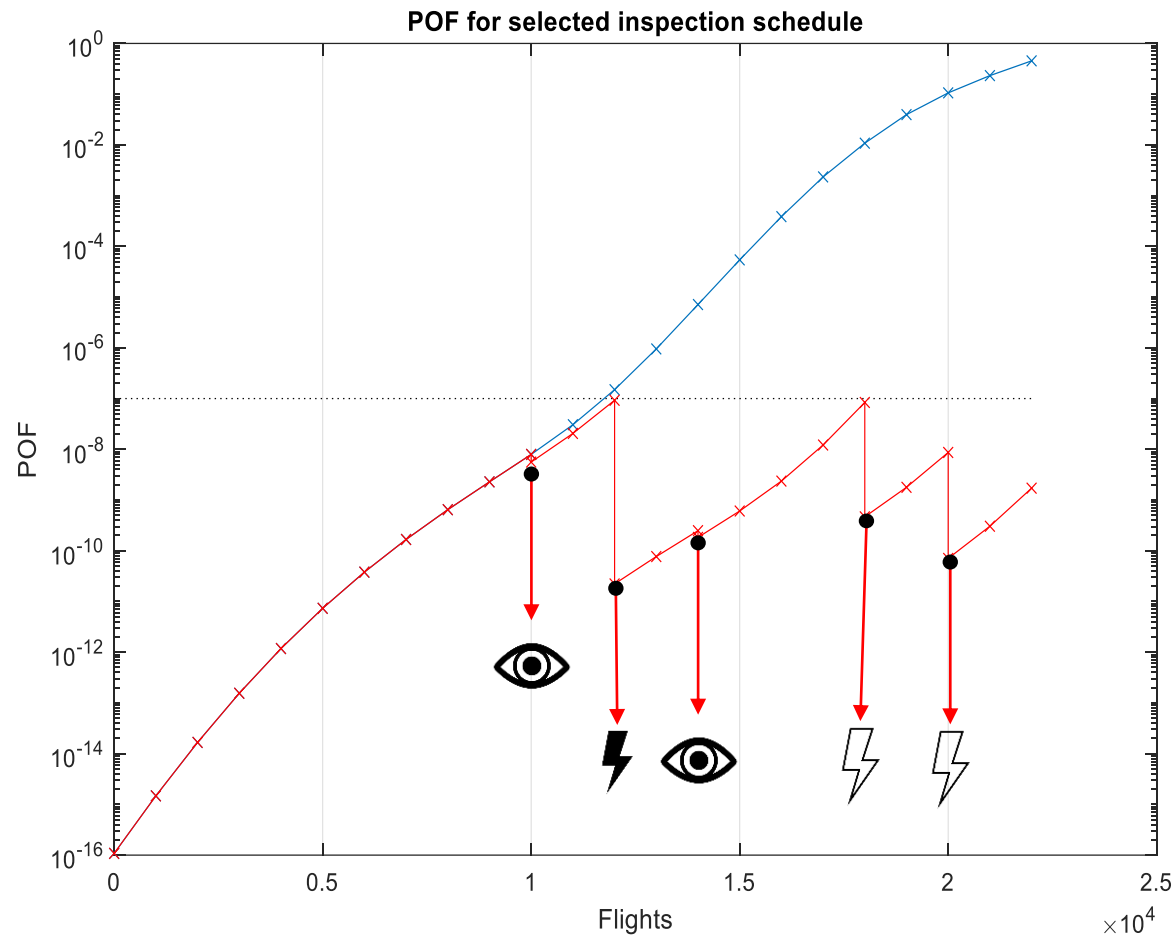
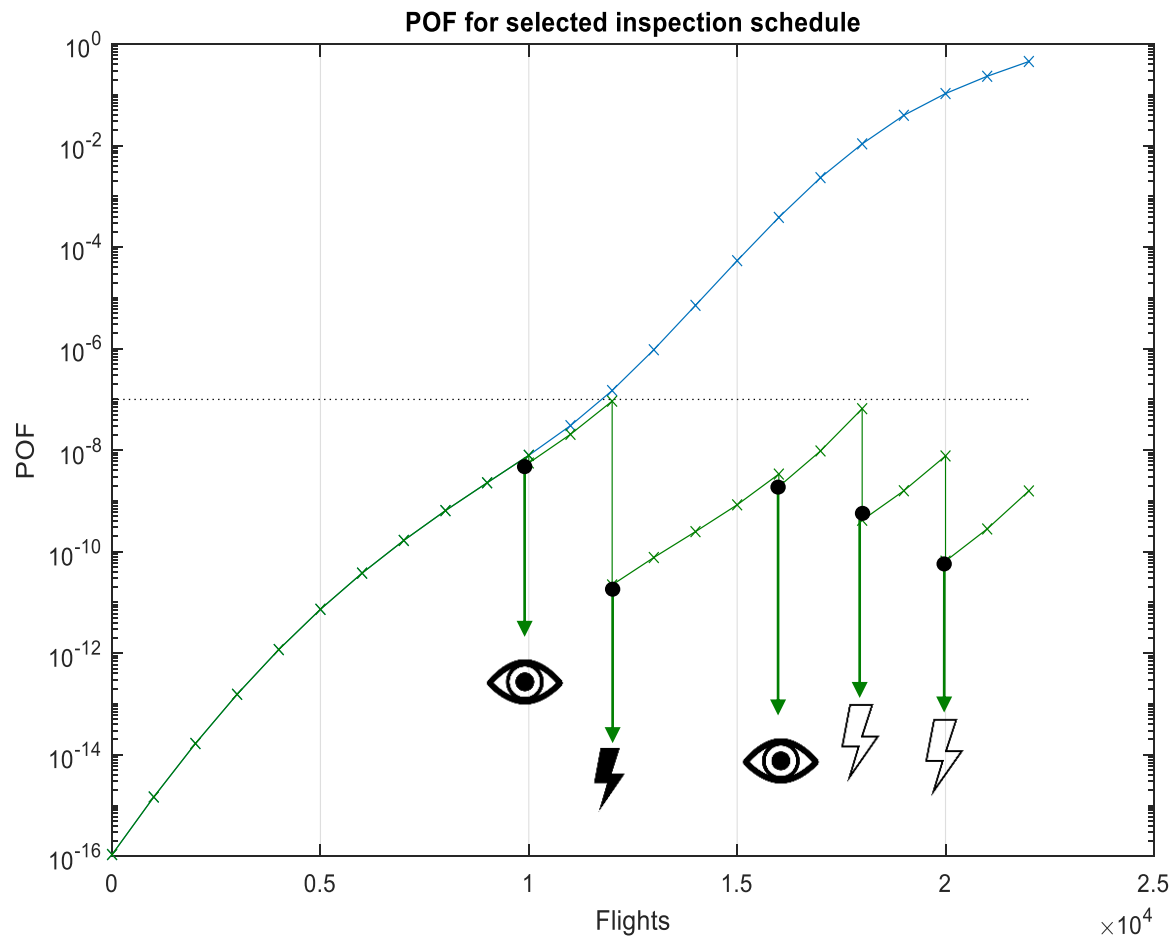


Inspections	Inspection Type	Material	Crack Type	Dist. Type	Mean [in]	St. Dev. [in]	Source	Cost
⚡ POD 1	Automated bolt hole eddy current	Aluminum	T	Lognormal	0.0180	0.0109	Aeronautical Applications of Non-destructive	50x
⚡ POD 2	Eddy current sliding probe	Aluminum	Overall	Lognormal	0.0788 +0.0625	0.0302	NDE Capabilities Book	10x
👁️ POD 3	Visual	Aluminum		Lognormal	0.99714 +0.0625	3.66907	NDE Capabilities Book	1x

# Optimized Inspections - Results



Possible inspection times = [2000, 4000, 6000, 8000, 10000, 12000, 14000, 16000, 18000, 20000]



Operations = 1,048,576  
SMART-DT runs= 1,287

# How it looks in SMART



SMART|DT HndbkOptInsp\_RiskThresh.smdt

SMART|DT

Information Analysis Material Geometry Loading **Inspections** Run Results

Inspection Schedule Type

- Risk Threshold
- User Specified
- Risk Threshold**
- Minimal Cost

Inspection Presets

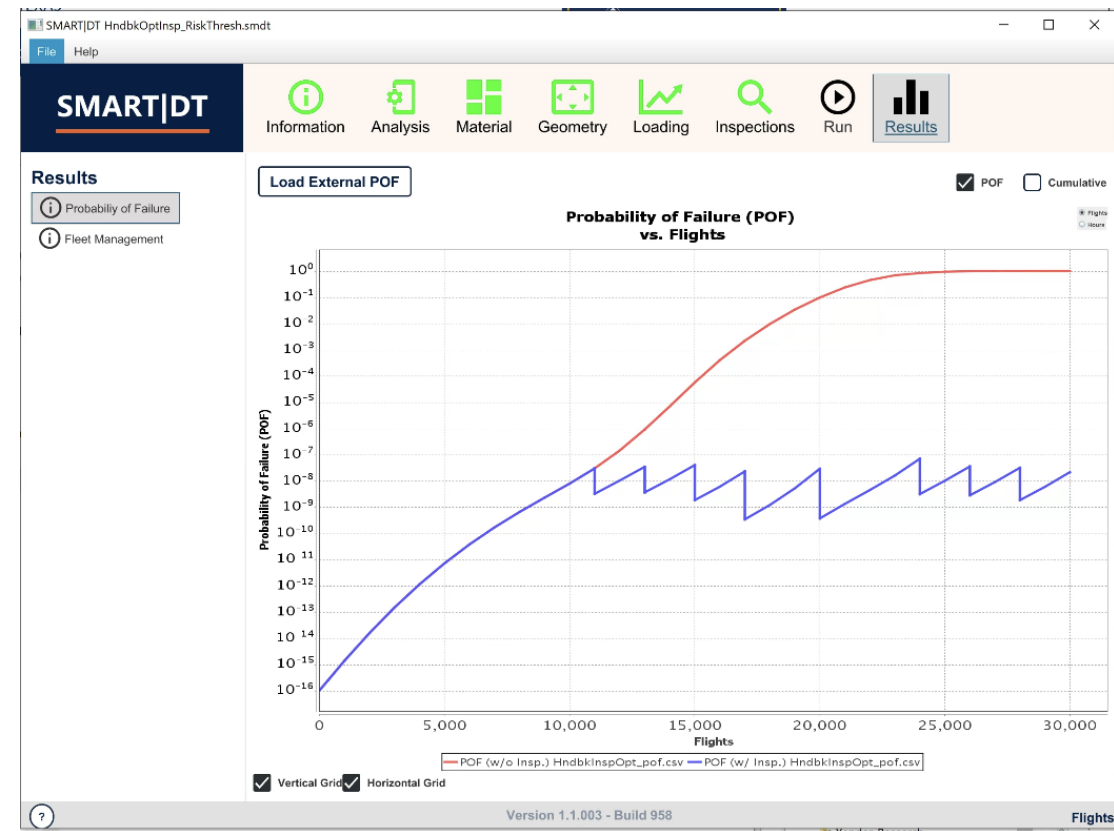
Name	Type	Inspection Prob.	Detection Prob.	Repaired Crack
EddyCurrent		1.0	$\mu 0.0788 \sigma 0.0302$ LN	Same as Original
AutBHEddyCurrent		1.0	$\mu 0.0179 \sigma 0.0108$ LN	Same as Original

Delete Edit Add

Risk Level: 1e-7

Preset: EddyCurrent

Version 1.1.003 - Build 958 Flights



# Fleet Management



Airplane number	Time in service
1	1,053
2	5,350
3	3,947
4	3,850
5	7,500
6	12,300
7	17,683
8	6,356
9	8,540
10	7,640

SMART|DT Untitled.smdt
— □ ×

File Help

SMART|DT

Information
 Analysis
 Material
 Geometry
 Loading
 Inspections
 Run
 Results

**Results**

Probability of Failure

Fleet Management

Load External POF
 POF  Cumulative

### Probability of Failure (POF) vs. Flights

Flights  
 Hours

POF (w/o Insp.) HndbkInspOpt\_pof.csv
POF (w/ Insp.) HndbkInspOpt\_pof.csv

Vertical Grid
  Horizontal Grid

Current Time in Service	No. Aircraft	Expected Future Hours (dt)	Hz(t)*dt	Hz(t)
1,053	1			
5,350	1			
3,947	1			
3,850	1			

Load
Save
Compute

Total Hazard:

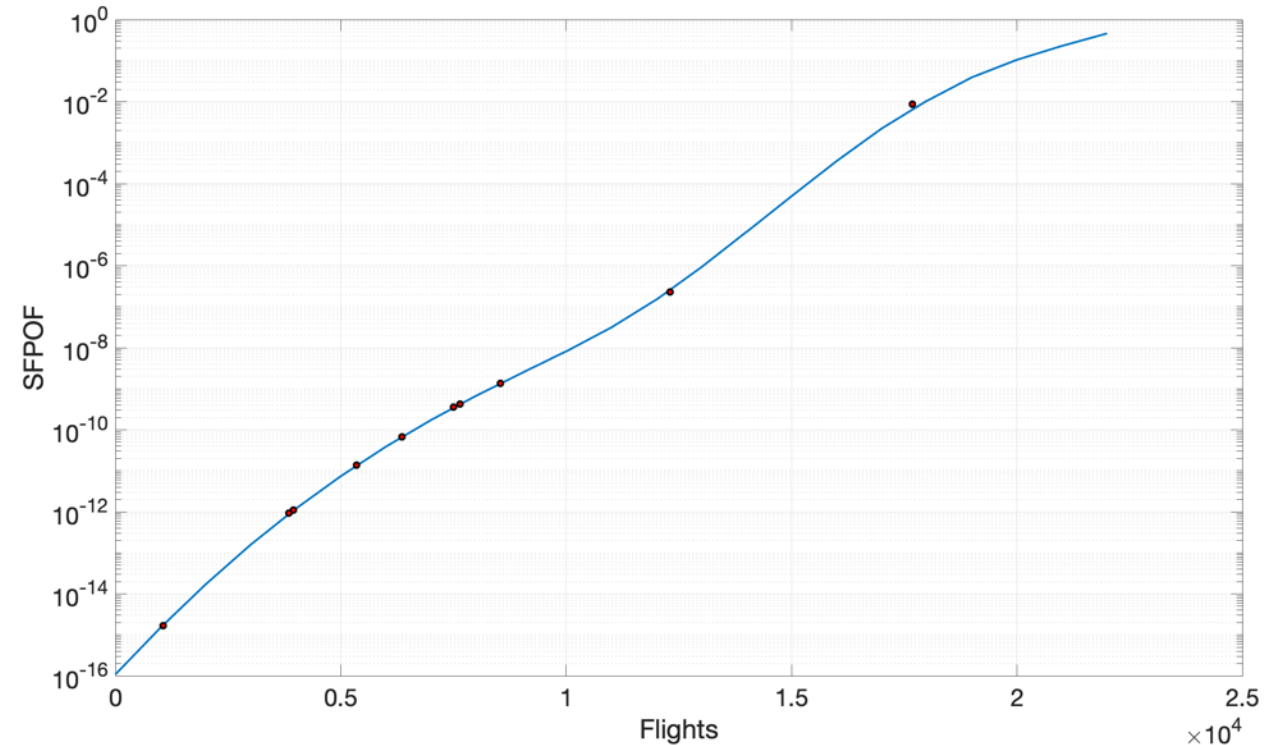
Add
Delete

# Fleet Management



- Scenario 1 – without inspections

Airplane number	Time in service	Hazard Rate	Probability of Failure For Expected Future Hours		
			100	500	1,000
1	1,053	2.32E-15	3.10E-13	3.10E-12	1.02E-11
2	5,350	1.84E-11	2.00E-09	1.31E-08	4.04E-08
3	3,947	1.14E-12	1.24E-10	1.21E-09	3.98E-09
4	3,850	1.04E-12	1.09E-10	9.64E-10	3.43E-09
5	7,500	4.24E-10	4.49E-08	2.74E-07	8.25E-07
6	12,300	3.79E-07	4.18E-05	2.87E-04	9.94E-04
7	17,683	7.88E-03	8.29E-01	5.29E+00	1.67E+01
8	6,356	8.66E-11	9.33E-09	6.01E-08	1.77E-07
9	8,540	1.60E-09	1.68E-07	1.01E-06	3.05E-06
10	7,640	4.94E-10	5.19E-08	3.21E-07	9.91E-07
<b>Total Hazard</b>			<b>8.29E-01</b>	<b>5.29E+00</b>	<b>1.67E+01</b>



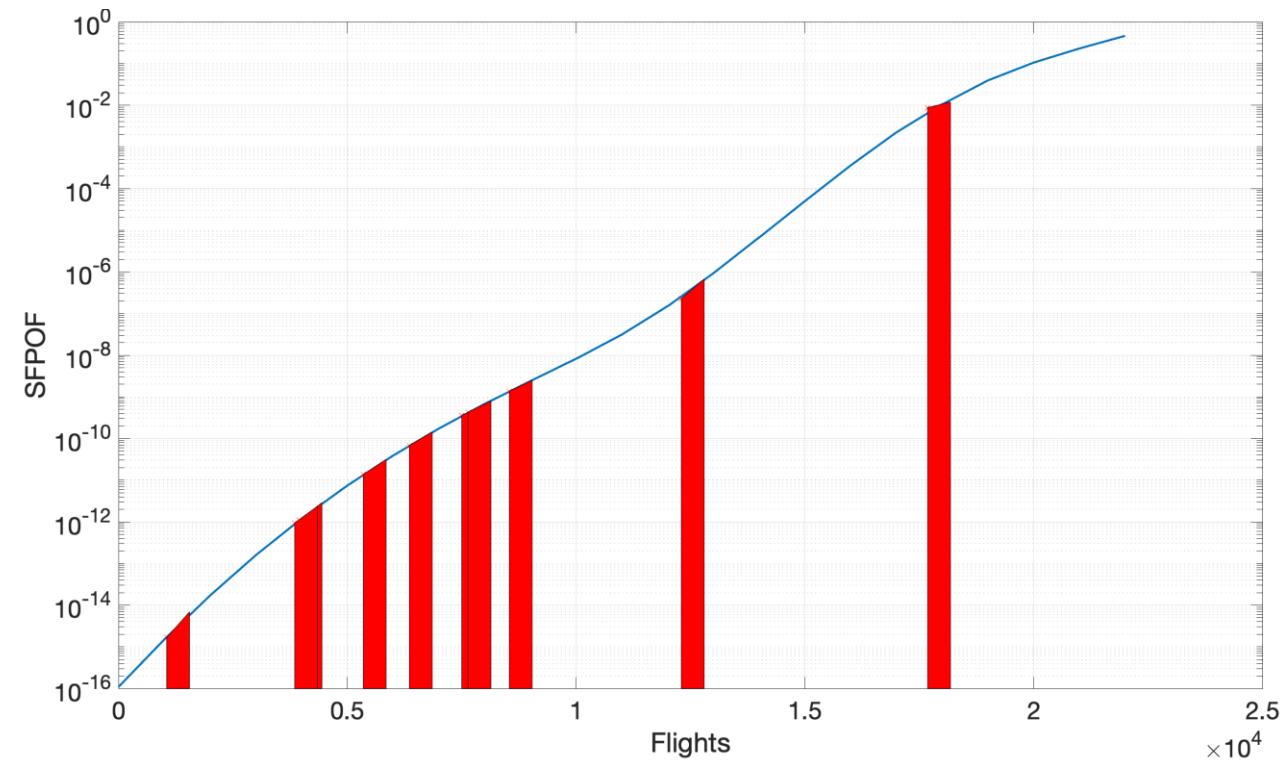


# Fleet Management



- Scenario 1 – without inspections

Airplane number	Time in service	Hazard Rate	Probability of Failure For Expected Future Hours		
			100	500	1,000
1	1,053	2.32E-15	3.10E-13	3.10E-12	1.02E-11
2	5,350	1.84E-11	2.00E-09	1.31E-08	4.04E-08
3	3,947	1.14E-12	1.24E-10	1.21E-09	3.98E-09
4	3,850	1.04E-12	1.09E-10	9.64E-10	3.43E-09
5	7,500	4.24E-10	4.49E-08	2.74E-07	8.25E-07
6	12,300	3.79E-07	4.18E-05	2.87E-04	9.94E-04
7	17,683	7.88E-03	8.29E-01	5.29E+00	1.67E+01
8	6,356	8.66E-11	9.33E-09	6.01E-08	1.77E-07
9	8,540	1.60E-09	1.68E-07	1.01E-06	3.05E-06
10	7,640	4.94E-10	5.19E-08	3.21E-07	9.91E-07
<b>Total Hazard</b>			<b>8.29E-01</b>	<b>5.29E+00</b>	<b>1.67E+01</b>

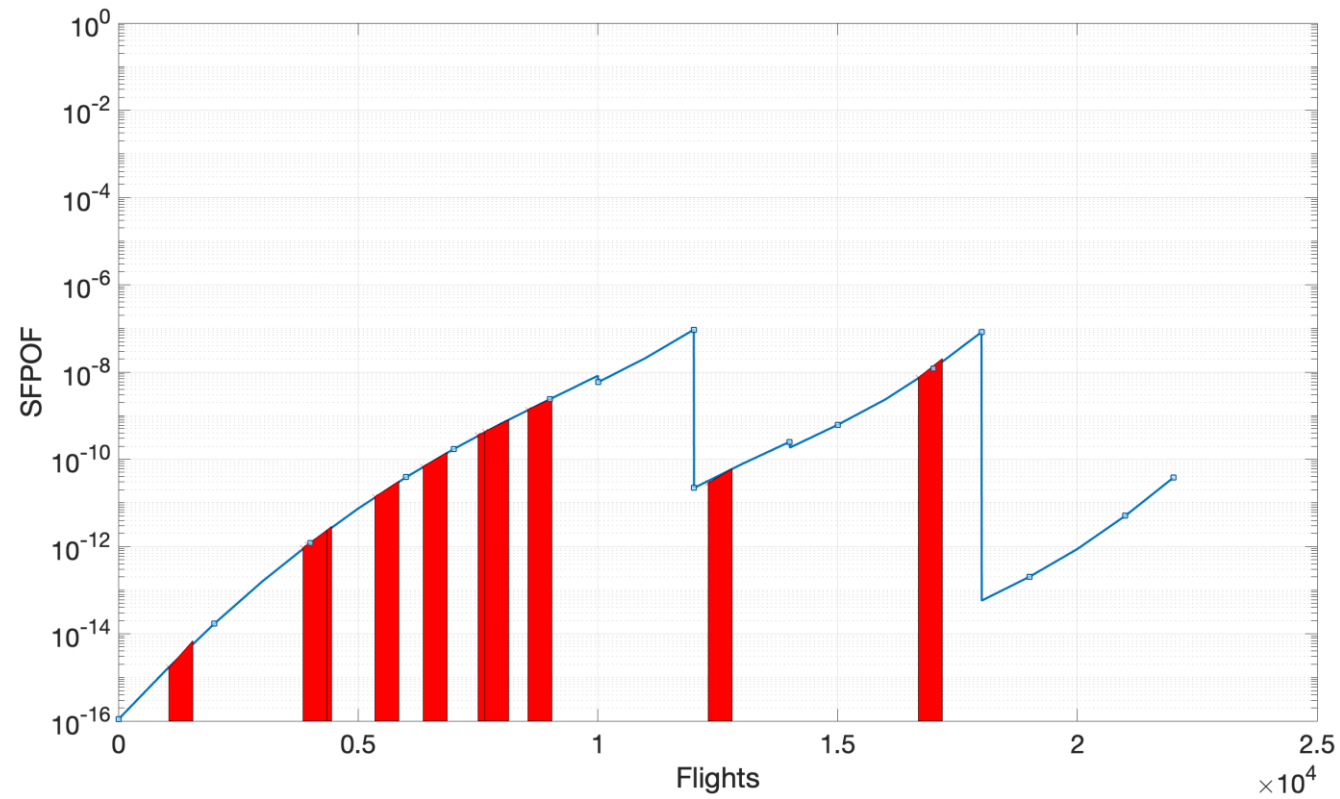


# Fleet Management



- Scenario 2 - With inspections

Airplane number	Time in service	Hazard Rate	Probability of Failure For Expected Future Hours		
			100	500	1,000
1	1,053	1.67E-15	3.10E-13	3.10E-12	1.02E-11
2	5,350	1.37E-11	2.00E-09	1.31E-08	4.04E-08
3	3,947	1.10E-12	1.24E-10	1.21E-09	3.98E-09
4	3,850	9.40E-13	1.09E-10	9.64E-10	3.43E-09
5	7,500	3.53E-10	4.49E-08	2.74E-07	8.25E-07
6	12,300	2.85E-11	4.11E-09	2.61E-08	7.14E-08
7	17,683	7.46E-09	9.50E-07	6.73E-06	2.80E-05
8	6,356	6.79E-11	9.33E-09	6.01E-08	1.77E-07
9	8,540	1.36E-09	1.68E-07	1.01E-06	3.05E-06
10	7,640	4.29E-10	5.19E-08	3.21E-07	9.91E-07
<b>Total Hazard</b>			<b>1.23E-06</b>	<b>8.45E-06</b>	<b>3.31E-05</b>



# Discussion and Where to Go Next



- Lincoln Vs. Freudenthal?
- Benchmark problems for the community
- Additional Capabilities?
- Additional Random Variables?
- Risk Assessment for Composite Structures?
  - NDI
  - Damage Tolerance
- Additive Manufacturing
- We are looking for parallel analyses between SMART and PROF for problems of use (non-academic) to the community.
- Recommendations from a review of the FAA's TARAM risk assessment process. (Next Slide)

# Discussion and Where to Go Next



- **Recommendation 12:** Within 18 months of receipt of this report, the Federal Aviation Administration should develop and maintain a technical training program for aviation safety engineers and their management who conduct and review Transport Airplane Risk Assessment Methodology analysis. The training should include the concepts of probabilistic risk analysis and the use of risk assessment results in the continued operational safety (COS) decision-making, similar in scope to those used in other federal agencies, to ensure the assumptions and limitations of the probabilistic risk analysis techniques are applied to the COS of commercial airplane operations.
- **Recommendation 13:** Within 6 months of receipt of this report, the Federal Aviation Administration should initiate research and continuous improvement programs in probabilistic risk analysis, including the use of risk assessment results in continued operational safety decision-making.

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**DETAILS**

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