Overview of Probabilistic Fatigue And Damage Tolerance Analysis for General Aviation

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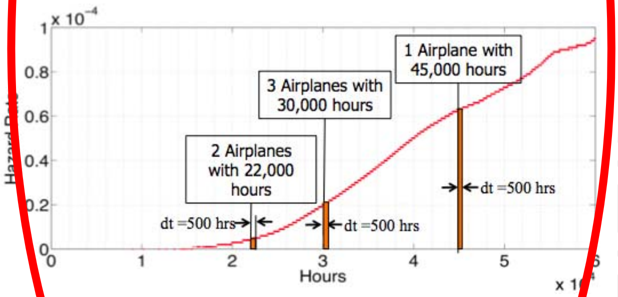
Program Overview

2007-2011

Phase I

Probabilistic Fatigue Analysis for Small Airplanes (SMART_{LD})

Safe-life Approach

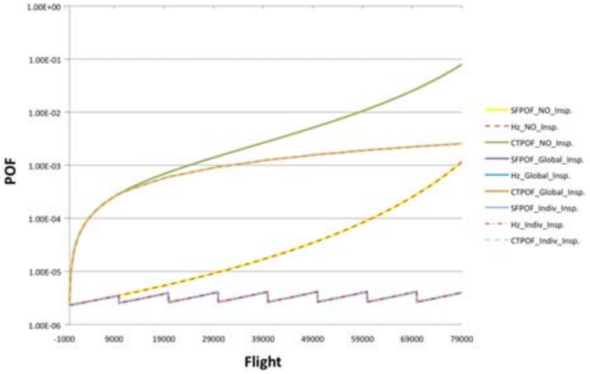


- Prob. Life distribution
- Hazard Rate
- Sensitivity Analysis

2009-2013

Phase II

Probabilistic Damage Tolerance Analysis for Small Airplane (SMART_{DT})



- SFPOF, Hz, CTPOF
- Inspection/Repair Effect
- Sensitivity Analysis

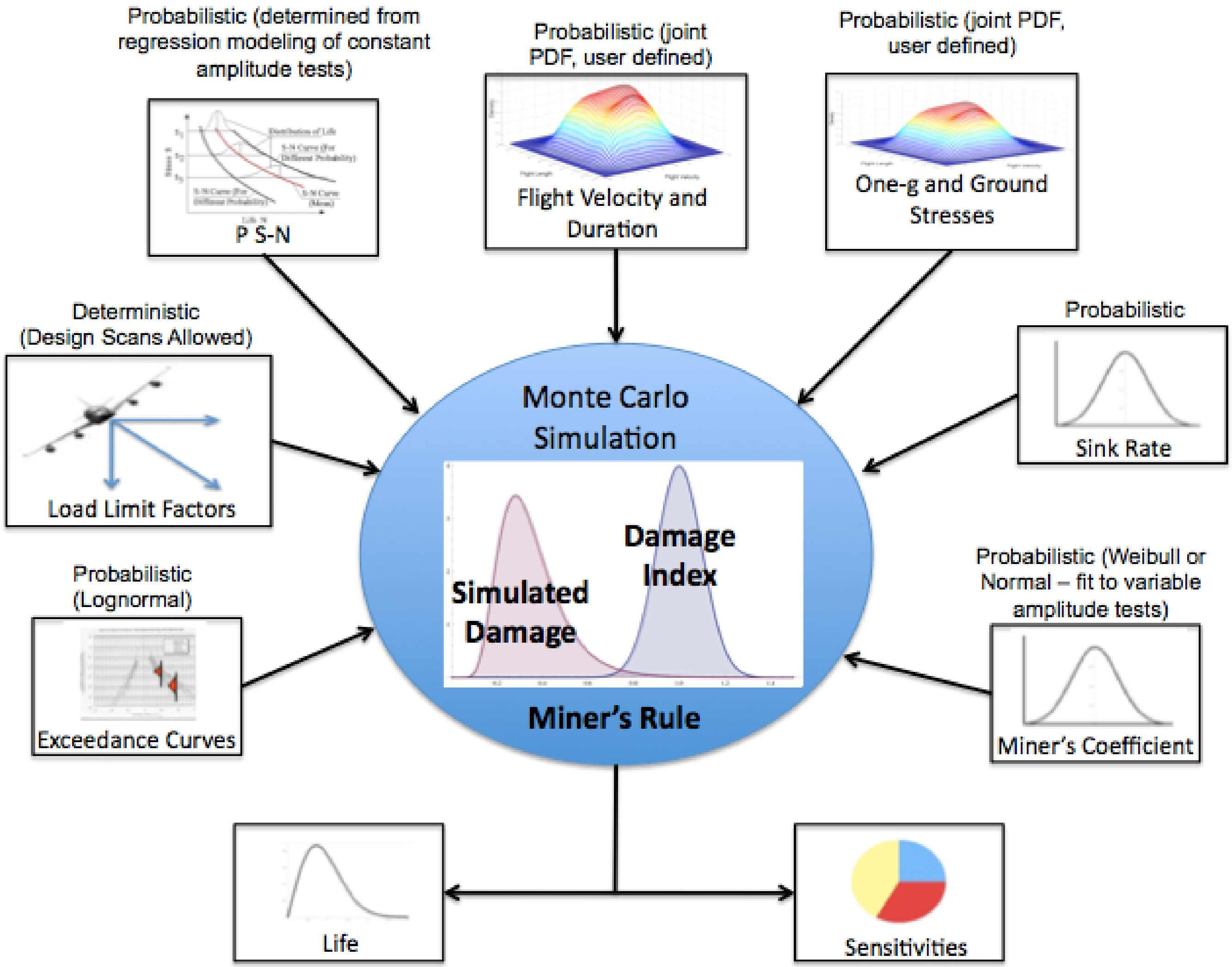
2012-2016

Phase III

Probabilistic Fatigue Management Program for General Aviation

- Develop experience and familiarity with probabilistic approaches within engineering personnel that design, manufacture and maintain general aviation aircraft.
- Verification with in-service findings.
- Develop a Probabilistically-based fatigue management plan (PFMP) for general aviation

Risk Methodology



Probabilistic (determined from regression modeling of constant amplitude tests)

P S-N

Probabilistic (joint PDF, user defined)

Flight Velocity and Duration

Probabilistic (joint PDF, user defined)

One-g and Ground Stresses

Deterministic (Design Scans Allowed)

Load Limit Factors

Probabilistic

Sink Rate

Probabilistic (Lognormal)

Exceedance Curves

Probabilistic (Weibull or Normal – fit to variable amplitude tests)

Miner's Coefficient

Monte Carlo Simulation

Simulated Damage

Damage Index

Miner's Rule

Life

Sensitivities

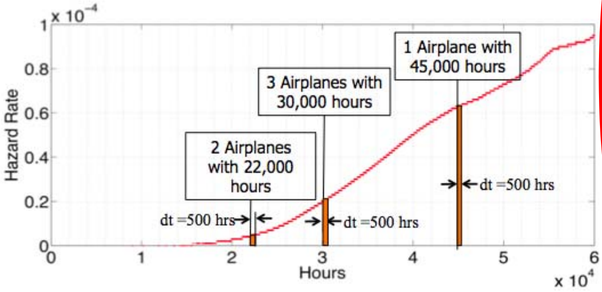
Program Overview

2007-2011

Phase I

Probabilistic Fatigue Analysis for Small Airplanes (SMART$_{LD}$)

Safe-life Approach



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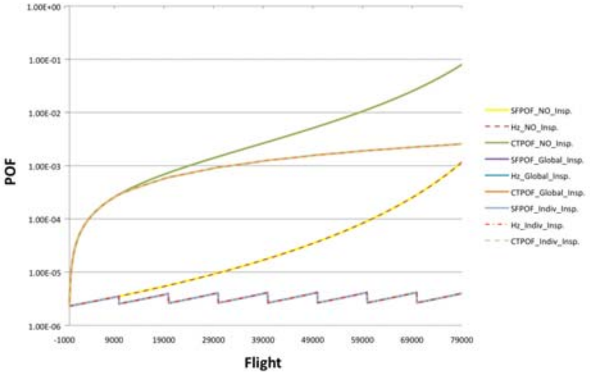
2009-2013

Phase II

Probabilistic Damage Tolerance Analysis for Small Airplane (SMART$_{DT}$)



- SFPOF, Hz, CTPOF
- Inspection/Repair Effect
- Sensitivity Analysis

2012-2016

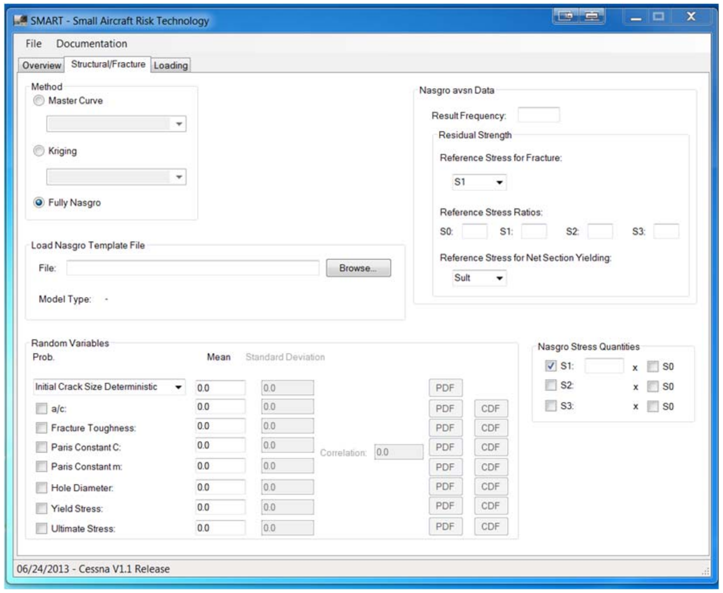
Phase III

Probabilistic Fatigue Management Program for General Aviation

UTSA
Cessna
Nuss Sol.

Smart|DT Capabilities

- Loading Generation
  - Computed from exceedance curves (Internal library and user exceedance option) – Weighted usage available.
  - Flight Duration and weight matrices, Design load limit factors, one-g stress, and ground stress as user input.
  - Stresses and/or flights randomizations
  - Spectrum editing option (Rainflow, rise/fall, Dead band)
  - User-defined spectra (Afgrow format)
- Extreme Value Distribution
  - User input, e.g., Gumbel, Frechet , and Weibull.
  - Ultimate/Limit load (deterministic)
  - Computed from exceedance curves, weight matrix, etc. (Gumbel, Frechet , and Weibull)
- Probability calculations
  - SFPOF (no survival term)
  - Hazard fn. (with survival term)
  - Cumulative (with survival term)
- Crack growth
  - Direct Nasgro link (for all computations – as an option)
  - Extension to Afgrow (Current Work)
  - Through, Corner, Surface crack growth geometry options
  - Master curve for 2D (ai and Kc) interpolation (user input or developed from Nasgro/Afgrow)
  - Kriging for efficient probabilistic fracture analysis
- Probabilistic methods
  - Standard Monte Carlo
  - Numerical integration
- Inspection capabilities
  - Any number of inspections (arbitrary limit set to 15)
  - Arbitrary repair crack size distribution (lognormal, tabular, deterministic)
  - Arbitrary POD (lognormal, tabular)
  - Deterministic POD
  - User defined probability of inspection
- Random variables
  - ai, Kc, Evd – all cases
  - Crack growth parameters, hole diameter, crack aspect ratio
- Computational implementation
  - Standard Fortran 95/03 (ifort) - Unix, Windows
  - GUI (Windows)
  - High Performance Computing



SMART - Small Aircraft Risk Technology

Method

○ Master Curve

○ Kriging

● Fully Nasgro

Load Nasgro Template File

File:

Model Type: -

Nasgro avsn Data

Result Frequency:

Residual Strength

Reference Stress for Fracture: S1

Reference Stress Ratios: S0: S1: S2: S3:

Reference Stress for Net Section Yielding: Sult

Random Variables

Prob.	Mean	Standard Deviation
Initial Crack Size Deterministic	0.0	0.0
☐ a/c:	0.0	0.0
☐ Fracture Toughness:	0.0	0.0
☐ Paris Constant C:	0.0	0.0
☐ Paris Constant m:	0.0	0.0
☐ Hole Diameter:	0.0	0.0
☐ Yield Stress:	0.0	0.0
☐ Ultimate Stress:	0.0	0.0

Correlation: 0.0

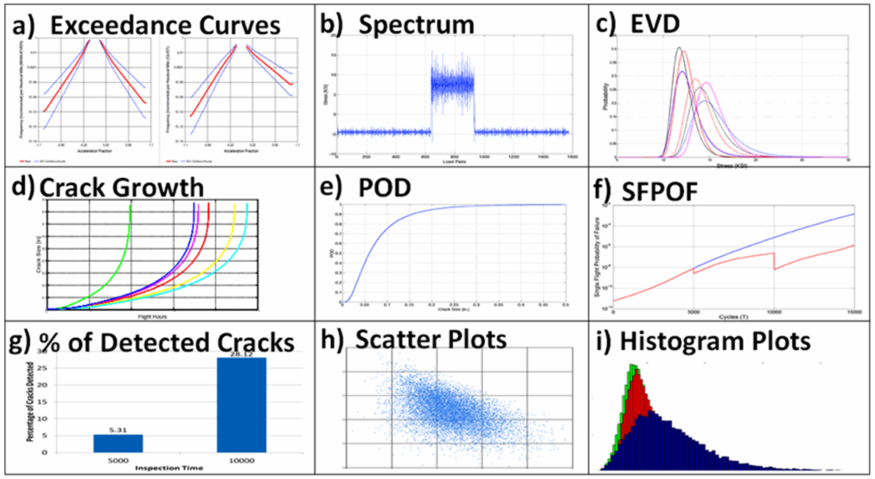
Nasgro Stress Quantities

☑ S1: x ☐ S0

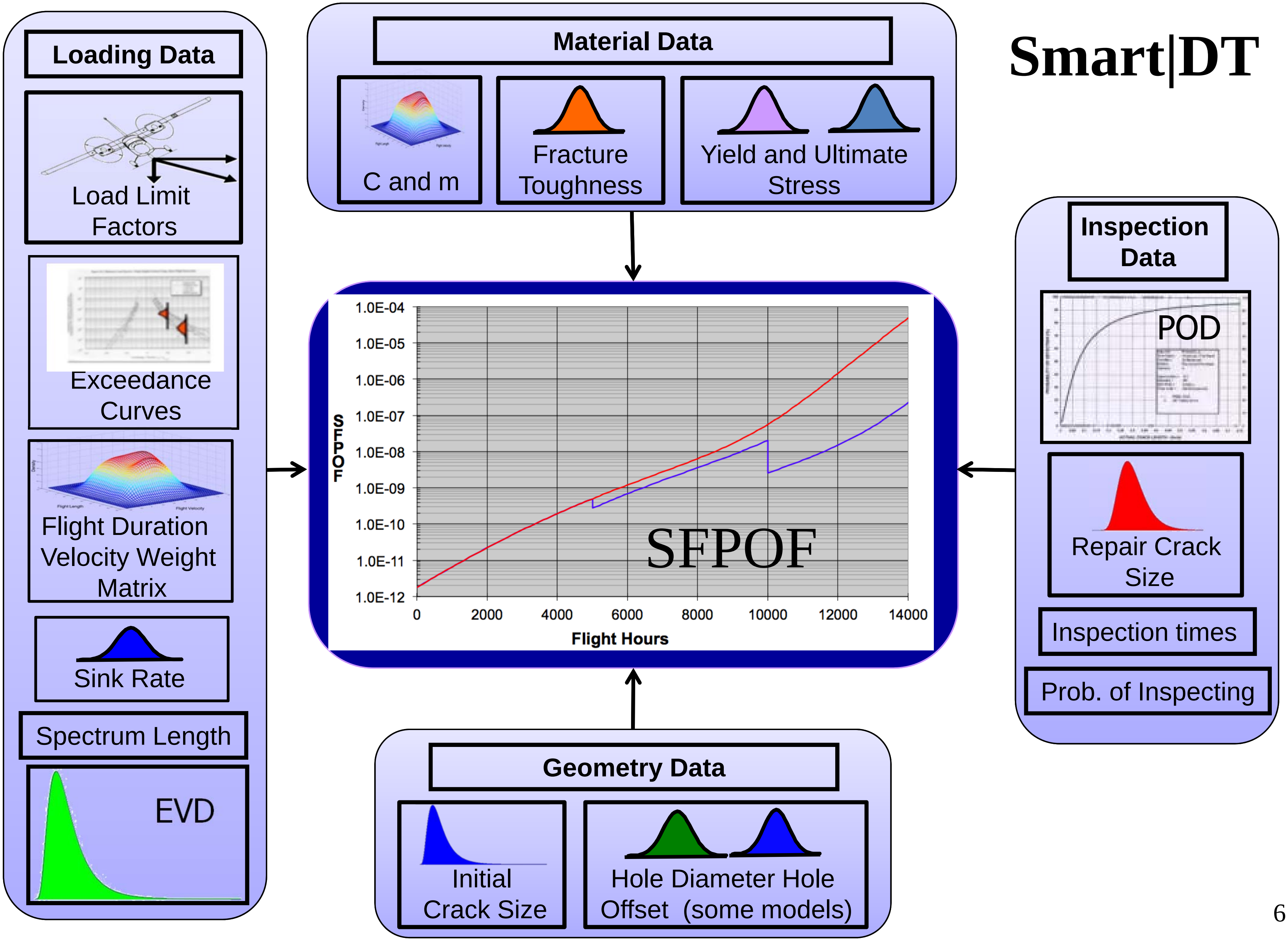
☐ S2: x ☐ S0

☐ S3: x ☐ S0

06/24/2013 - Cessna V1.1 Release



Smart|DT



Loading Data

Load Limit Factors

Exceedance Curves

Flight Duration Velocity Weight Matrix

Sink Rate

Spectrum Length

EVD

Material Data

C and m

Fracture Toughness

Yield and Ultimate Stress

Inspection Data

POD

Repair Crack Size

Inspection times

Prob. of Inspecting

Geometry Data

Initial Crack Size

Hole Diameter Hole Offset (some models)

POF Calculations

The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength σ_{RS} of the aircraft component

$$P_f = P\left[\sigma_{Max} > \sigma_{RS}\right]$$

$$POF(t) = \int\int\int_{\sigma_{RS}<\sigma_{Max}} f_{EVD}(evd)\, f_{a_0}(a_0)\, f_{K_c}(K_c)\, da_0\, dK_c$$

Integrate EVD random variable analytically (conditional expectation)

$$POF(t) = \int_{0}^{\infty}\int_{-\infty}^{\infty} [1 - F_{EVD}(\sigma_{RS}(a(a_0, t))]\, f_{a_0}(a_0)\, f_{K_c}(K_c)\, dK_c\, da_0$$

POF Calculations

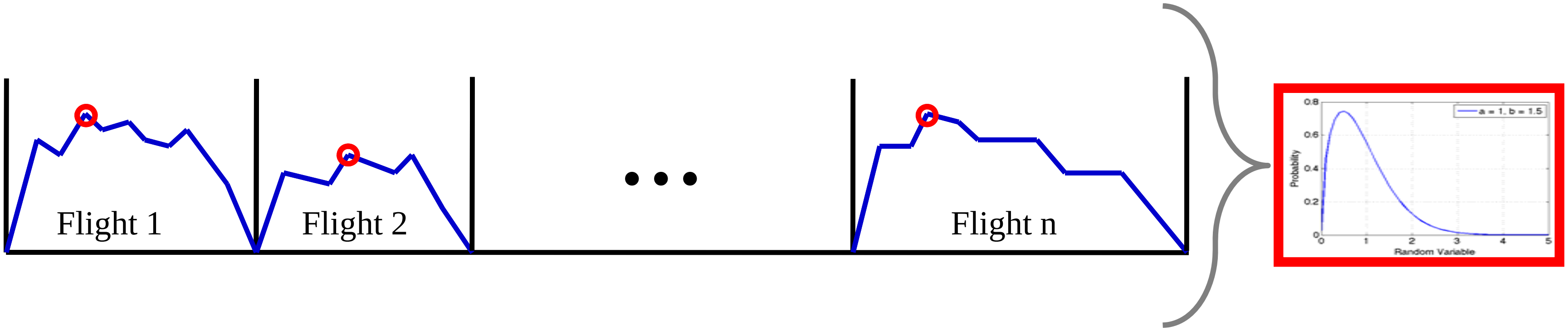
Residual strength defined in terms of fracture, yielding, critical crack size.

$$POF(t) = \int_{0}^{\infty} \int_{-\infty}^{\infty} \left[1 - F_{EVD} \left(Min[\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}, NSY, a_{Cr}] \right) \right] f_{a_0}(a_0) f_{K_c}(K_c) da_0 \, dK_c$$

$$POF(t) = \int_{0}^{\infty} \int_{-\infty}^{\infty} \left[1 - F_{EVD} \left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}} \right) \right] f_{a_0}(a_0) f_{K_c}(K_c) da_0 \, dK_c$$

EVD Generation

- A critical component is the extreme load per flight. This extreme load is (should be) determined from the same spectrum used for the crack growth.



$$F_{EVD}(x) = \exp\left\{ -\left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}$$

$\xi = 0$ Gumbel

$\xi > 0$ Frechet

$\xi < 0$ Weibull

Survival Term or Not?

$$POF(t) = \int_{0}^{\infty} \int_{-\infty}^{\infty} \left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}} \right) \right] f_{a_0}(a_0)\, f_{K_c}(K_c)\, da_0\, dK_c$$

$$POF(t) = \int_{-\infty}^{\infty} \int_{0}^{\infty} \boxed{\left[\prod_{t=1}^{T-1} F_{EVD}\left(\frac{K_C}{\beta(a_o,t)\sqrt{\pi a(a_o,t)}} \right) \right]} \left[1 - F_{EVD}\left(\frac{K_C}{\beta(a_o,T)\sqrt{\pi a(a_o,T)}} \right) \right] f_{a_0}(a_0)\, f_{K_c}(K_c)\, da_0\, dK_c$$

Hazard Fn.

$$Hz(T) = \frac{1}{R(T)} \int_{-\infty}^{\infty} \int_{0}^{\infty} \left[\prod_{t=1}^{T-1} F_{EVD}\left(\frac{K_C}{\beta(a_o,t)\sqrt{\pi a(a_o,t)}} \right) \right] \left[1 - F_{EVD}\left(\frac{K_C}{\beta(a_o,T)\sqrt{\pi a(a_o,T)}} \right) \right] f_{a_0}(a_0)\, f_{K_c}(K_c)\, da_0 dK_c$$

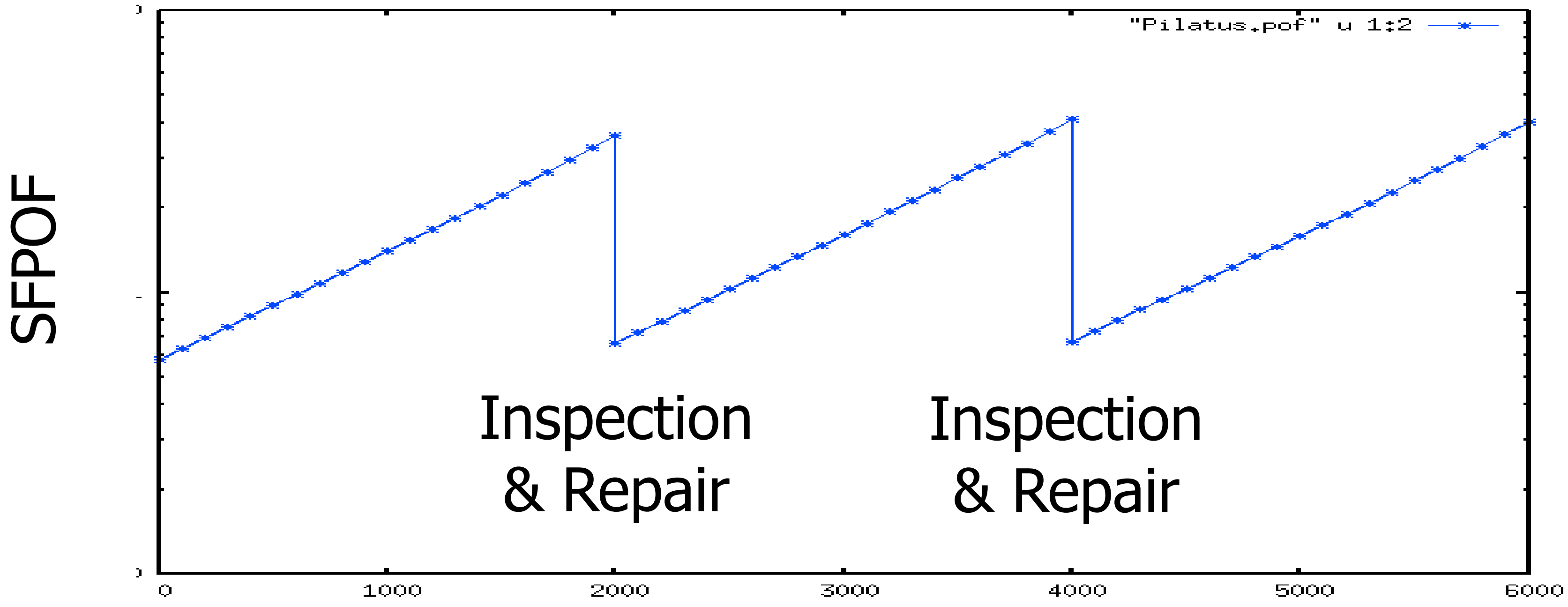
Difficult Integral?

$$POF(t)=\int_{0}^{\infty}\int_{-\infty}^{\infty}\left[1-F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right]f_{a_0}(a_0)\,f_{K_c}(K_c)\,da_0\,dK_c$$

Small probabilities: (1E-14 - 1E-5)

Time dependent: multiple integrals

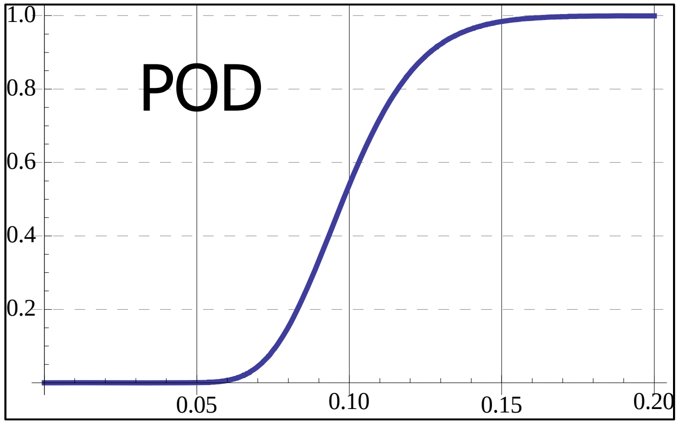
Inspections

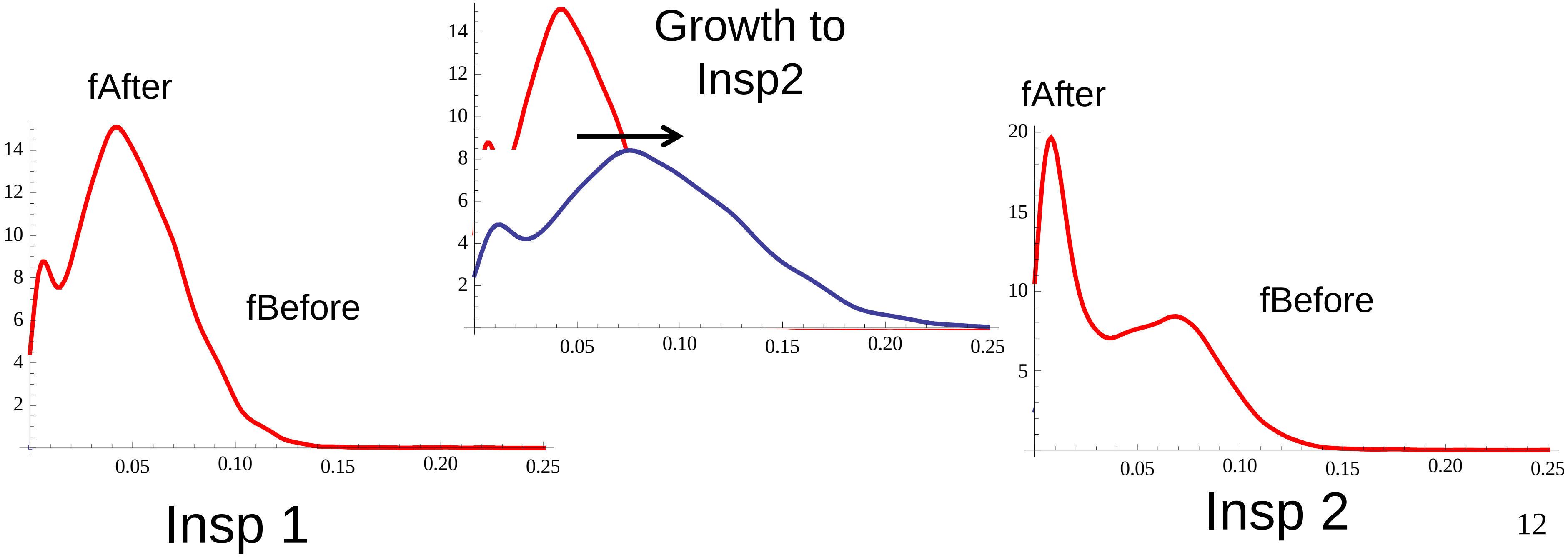


Inspections

$$f_{After}(a) = p_\infty f_R(a) + (1 - POD(a)) f_{Before}(a)$$

A percentage of cracks are detected and repaired. This leads to multi-modal PDFs.





Solution Methods

Sampling
- Robust but too many samples may be required

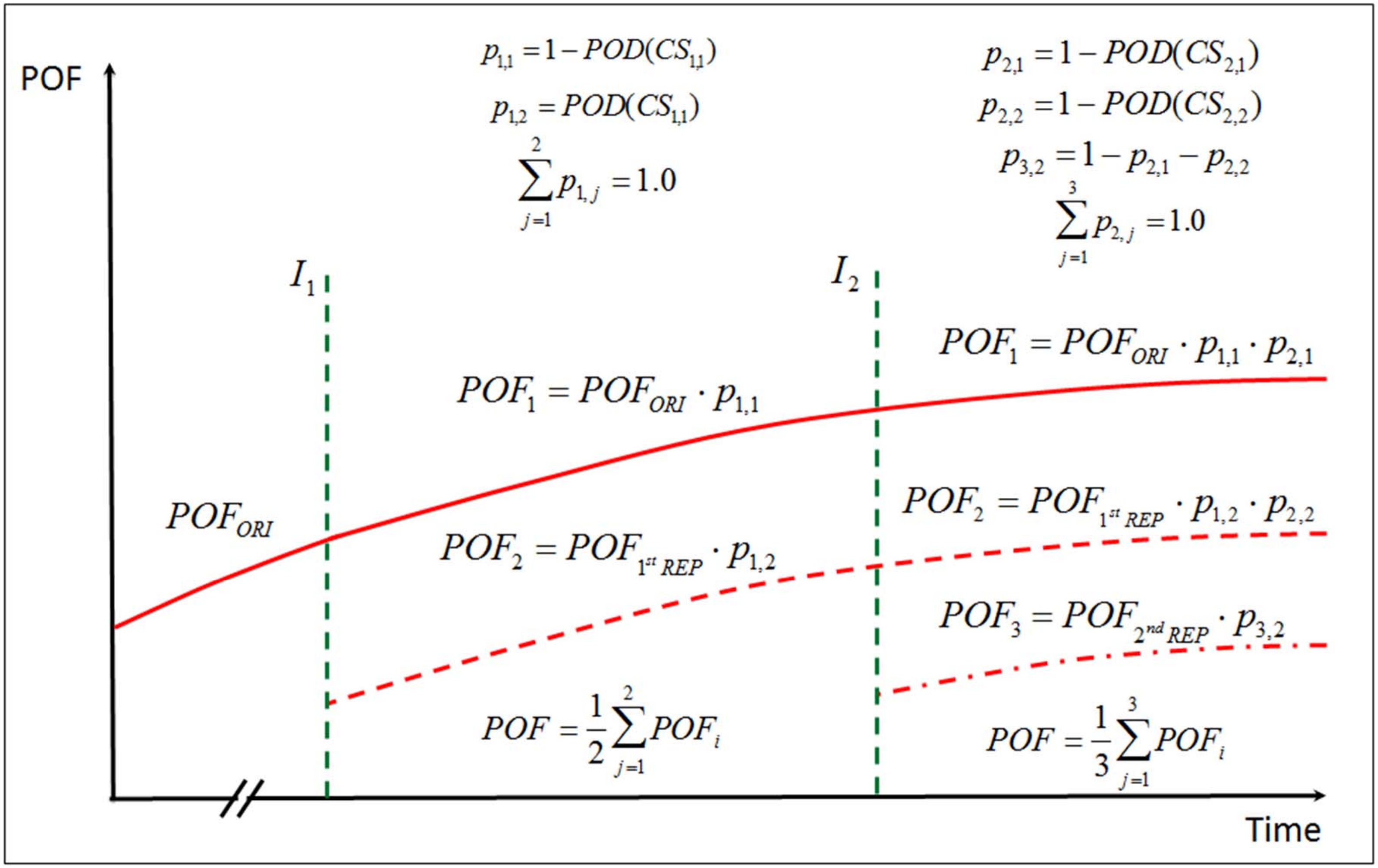
Importance Sampling
- More efficient
- Must scale to high dimensions of random variables
- Must be robust given time dependent integrals and inspection

Numerical integration
- Fast up to 3 dimensions or so.
- Sophisticated methods may allow for higher dimensions.
- Multi-mode crack size distribution may cause difficulties

First Order Reliability Method (FORM/SORM)
- Very good and efficient for small probabilities
- Sensitivities computed as a byproduct
- Multi-mode crack size distribution may cause inaccuracies

Monte Carlo Sampling

$$POF(t) \approx E[1-F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)]$$



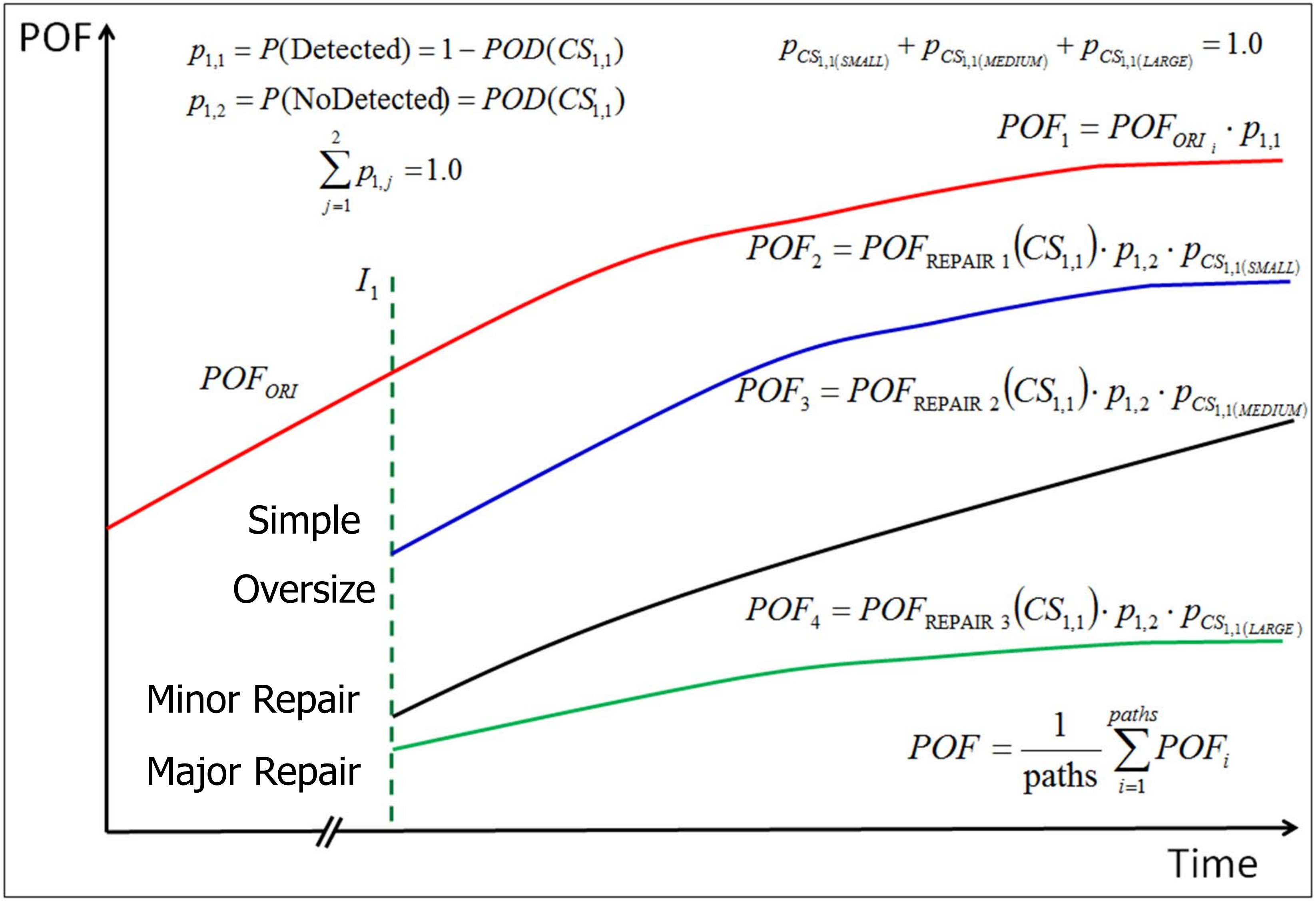
$$p_{1,1} = 1 - POD(CS_{1,1})$$
$$p_{1,2} = POD(CS_{1,1})$$
$$\sum_{j=1}^{2} p_{1,j} = 1.0$$

$$p_{2,1} = 1 - POD(CS_{2,1})$$
$$p_{2,2} = 1 - POD(CS_{2,2})$$
$$p_{3,2} = 1 - p_{2,1} - p_{2,2}$$
$$\sum_{j=1}^{3} p_{2,j} = 1.0$$

$$POF_1 = POF_{ORI} \cdot p_{1,1}$$
$$POF_2 = POF_{1^{st}REP} \cdot p_{1,2}$$
$$POF = \frac{1}{2}\sum_{j=1}^{2} POF_i$$

$$POF_1 = POF_{ORI} \cdot p_{1,1} \cdot p_{2,1}$$
$$POF_2 = POF_{1^{st}REP} \cdot p_{1,2} \cdot p_{2,2}$$
$$POF_3 = POF_{2^{nd}REP} \cdot p_{3,2}$$
$$POF = \frac{1}{3}\sum_{j=1}^{3} POF_i$$

Different Repair Scenarios



$$p_{1,1} = P(\text{Detected}) = 1 - POD(CS_{1,1})$$

$$p_{1,2} = P(\text{NoDetected}) = POD(CS_{1,1})$$

$$\sum_{j=1}^{2} p_{1,j} = 1.0$$

$$p_{CS_{1,1(SMALL)}} + p_{CS_{1,1(MEDIUM)}} + p_{CS_{1,1(LARGE)}} = 1.0$$

$$POF_1 = POF_{ORI_i} \cdot p_{1,1}$$

$$POF_2 = POF_{\text{REPAIR 1}}(CS_{1,1}) \cdot p_{1,2} \cdot p_{CS_{1,1(SMALL)}}$$

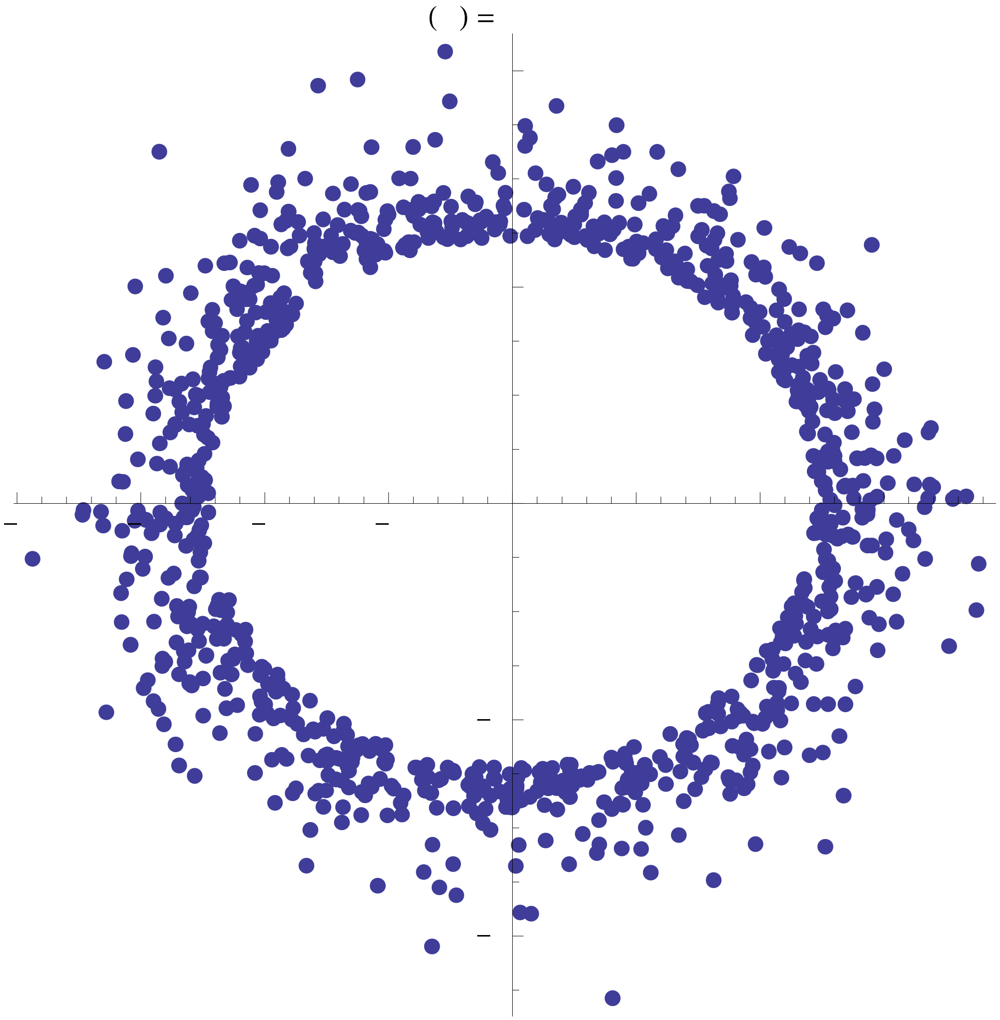
$$POF_3 = POF_{\text{REPAIR 2}}(CS_{1,1}) \cdot p_{1,2} \cdot p_{CS_{1,1(MEDIUM)}}$$

$$POF_4 = POF_{\text{REPAIR 3}}(CS_{1,1}) \cdot p_{1,2} \cdot p_{CS_{1,1(LARGE)}}$$

$$POF = \frac{1}{\text{paths}} \sum_{i=1}^{paths} POF_i$$

Importance Sampling

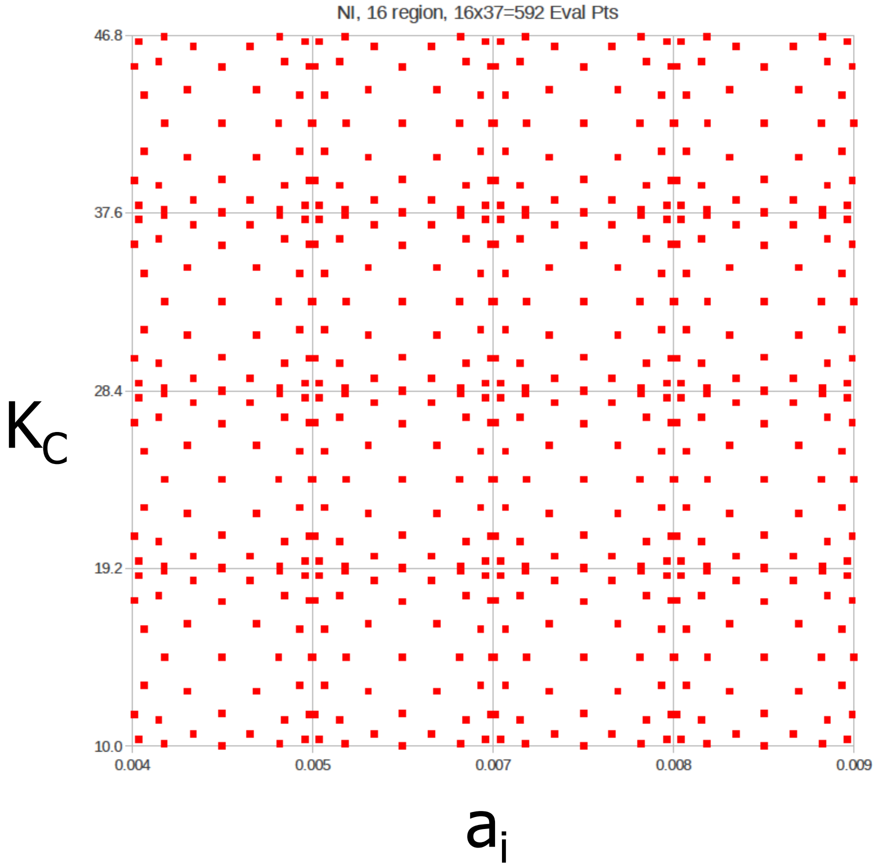
Only sample “important” region



- Important region changes with time
- Changes with inspections
- Must work in high dimensions

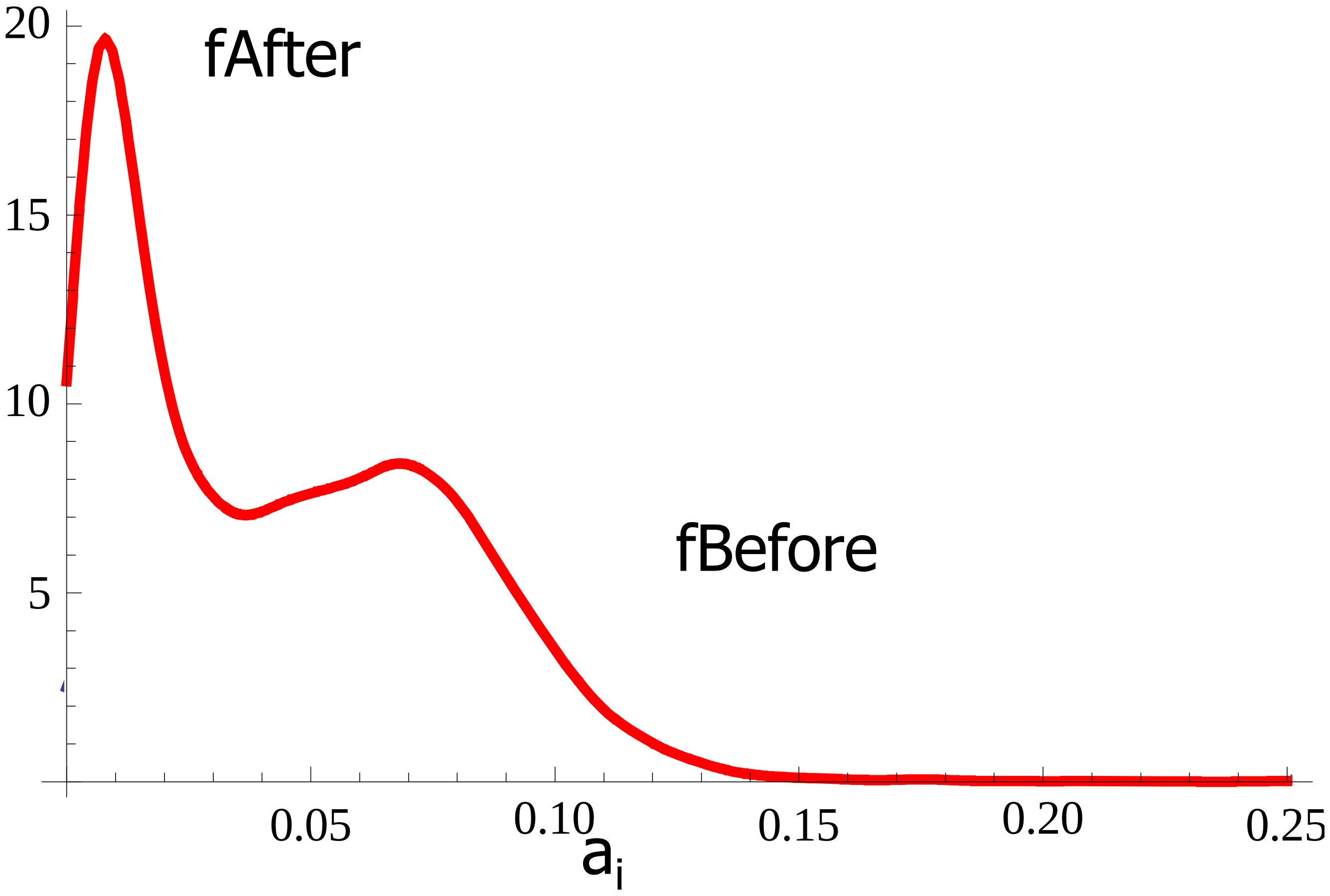
Numerical Integration

Evaluate the integrand at quadrature points. Adaptively adjust the concentration of points to provide a better approximation.



Numerical Integration

Challenges: accurately representing multi-modal PDFs in random variable space.

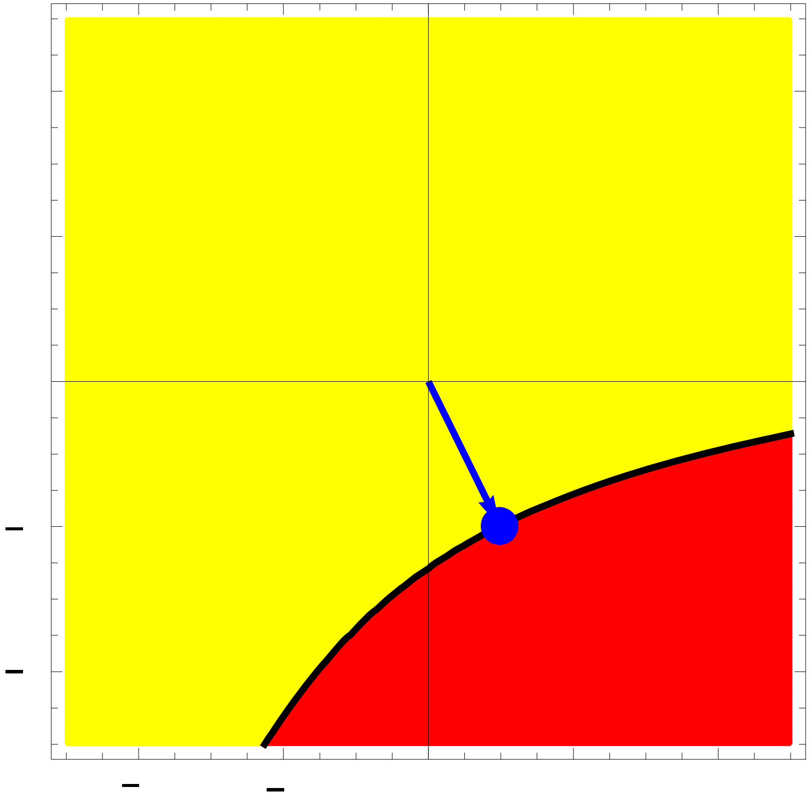


FORM

Very good and efficient for small probabilities

Sensitivities computed as a byproduct

Multi-mode crack size distribution may cause inaccuracies

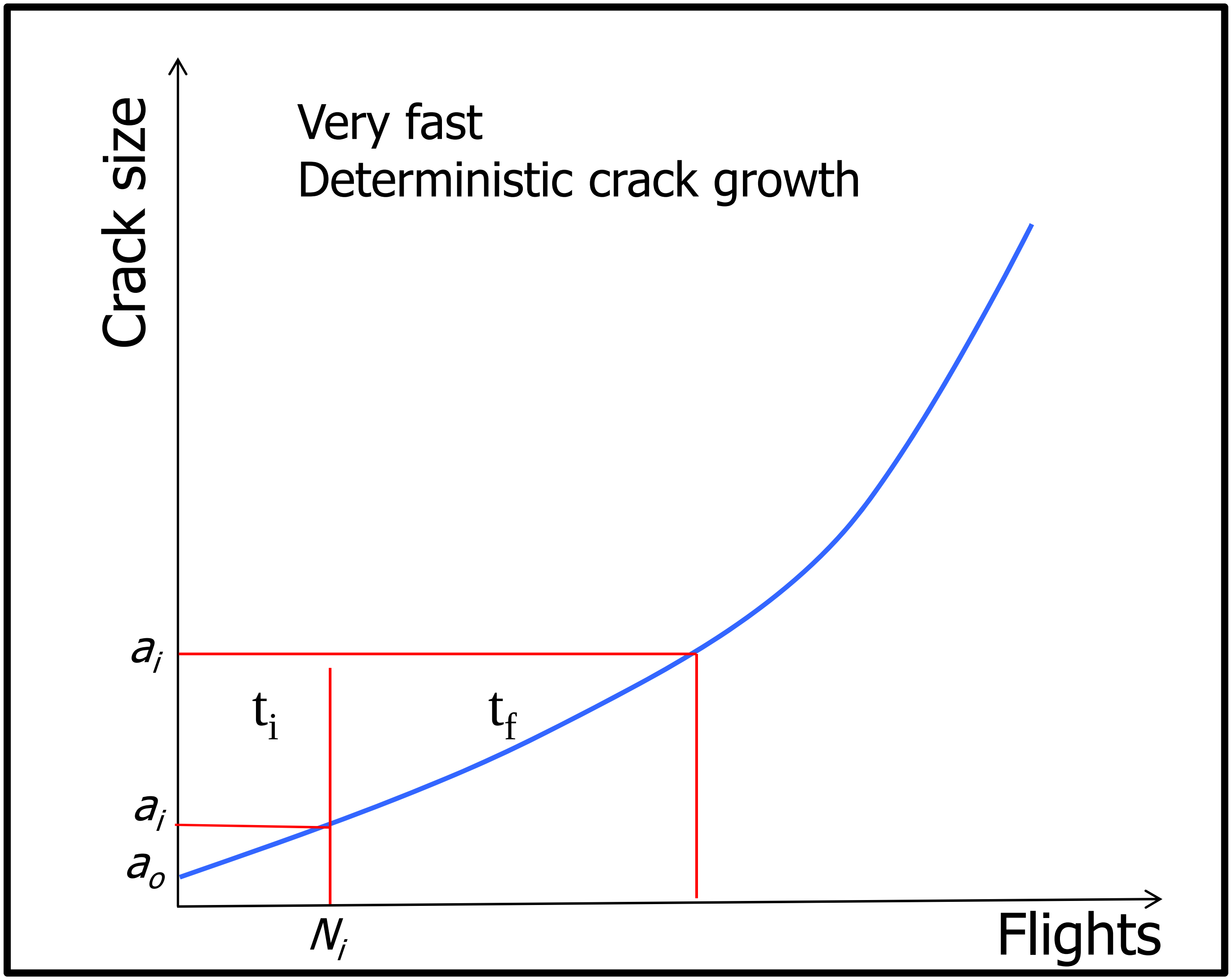


Why only 3 RVs?

- These are the only three random variables that matter (Right answer – if true)
    - Variations in other variables are insignificant
- Using only 3 RVs let’s one compute fast fracture mechanics (Wrong answer)
- Using only 3 RVs let’s one use a fast probabilistic algorithm, e.g., numerical integration (Wrong answer)

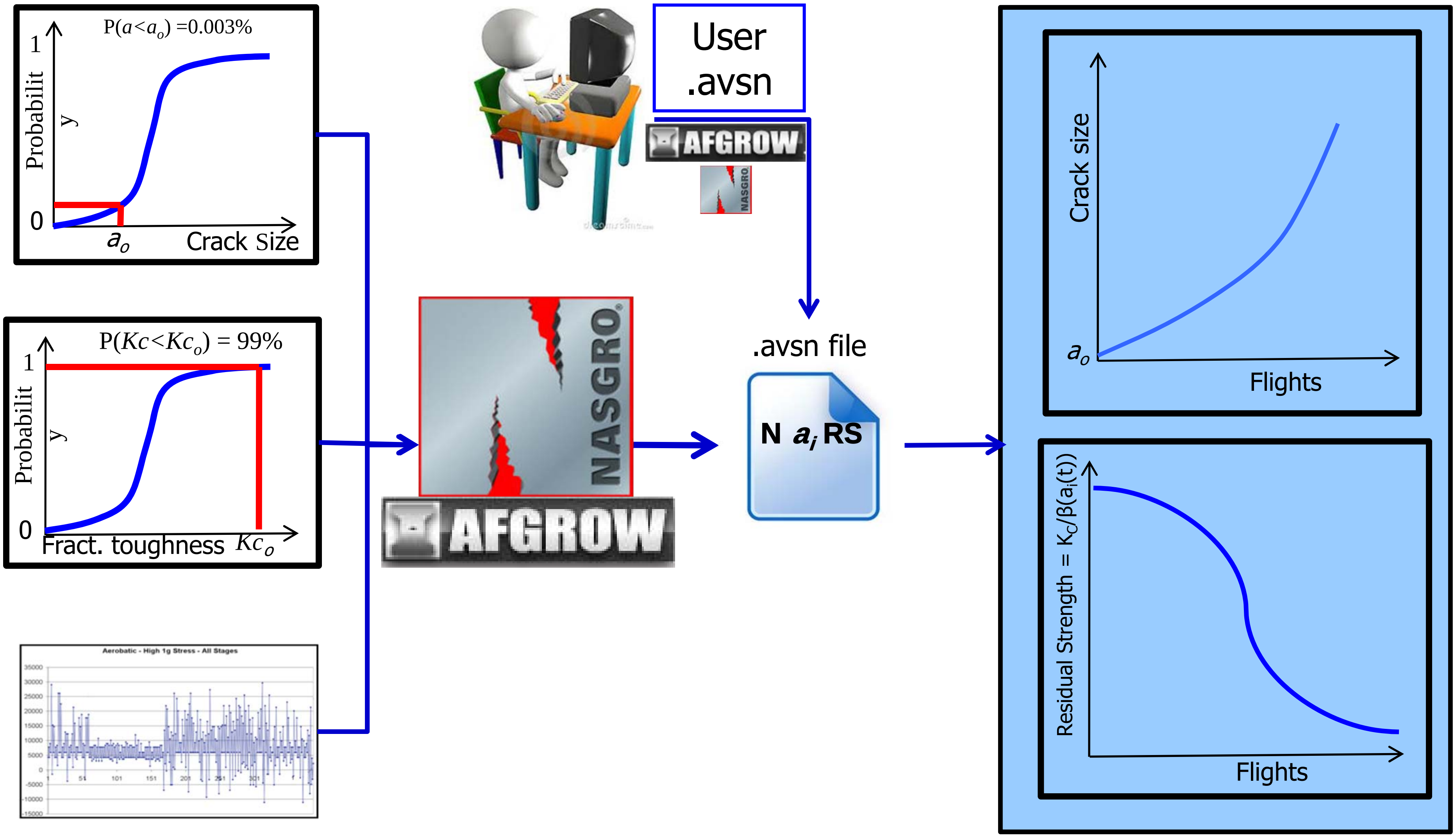
- Develop algorithms that allow a higher number of random variables. Make as efficient as possible.

Crack Growth Generation



Only a single fracture mechanics analysis required

Crack Growth Interface



Analysis Methods

- ✓Only a_i and K_c Random
- ✓Representative Spectrum
- ✓Inspection/Repair

Master Curve
Monte Carlo/
Numerical Integration

- ✓Hz, SFPOF, CTPOF

1X – Computational Cost

- ✓Multiple Rnd variables
- ✓Representative Spectrum
- ✓Inspection/Repair

Adaptive Kriging/Nasgro
Monte Carlo/
Numerical Integration

- ✓Hz, SFPOF, CTPOF

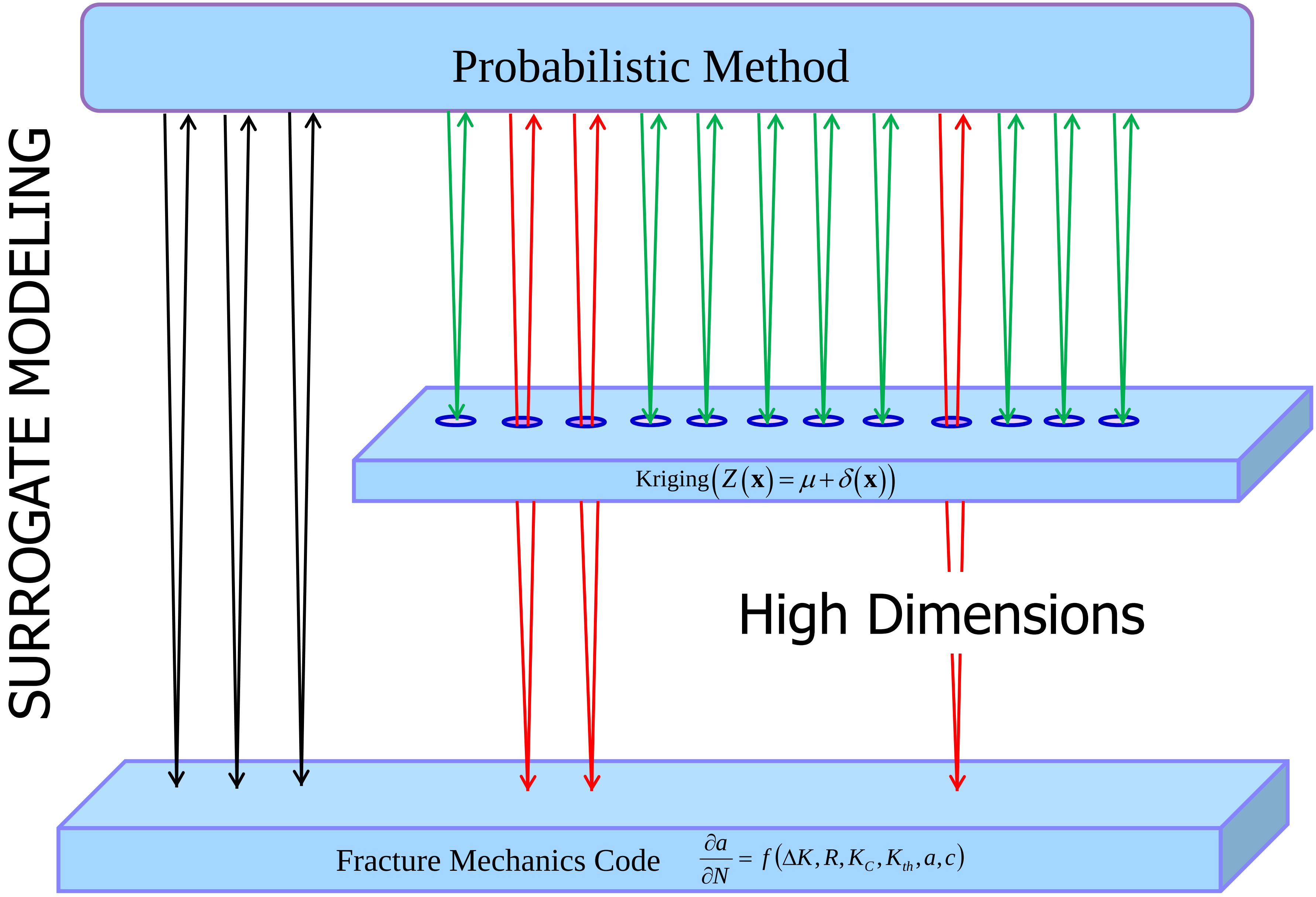
100X - Computational Cost

- ✓Multiple Rnd variables
- ✓Multiple Spectrum
- ✓Inspection/Repair

Full Nasgro/Afgrow
Monte Carlo/
Numerical Integration

- ✓Hz, SFPOF, CTPOF

1000X - Computational Cost



SURROGATE MODELING

Probabilistic Method

Kriging $\left(Z(\mathbf{x}) = \mu + \delta(\mathbf{x})\right)$

High Dimensions

Fracture Mechanics Code $\frac{\partial a}{\partial N} = f\left(\Delta K, R, K_C, K_{th}, a, c\right)$

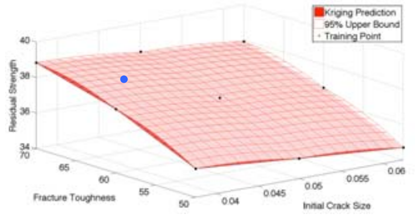
⬭ User Defined Error

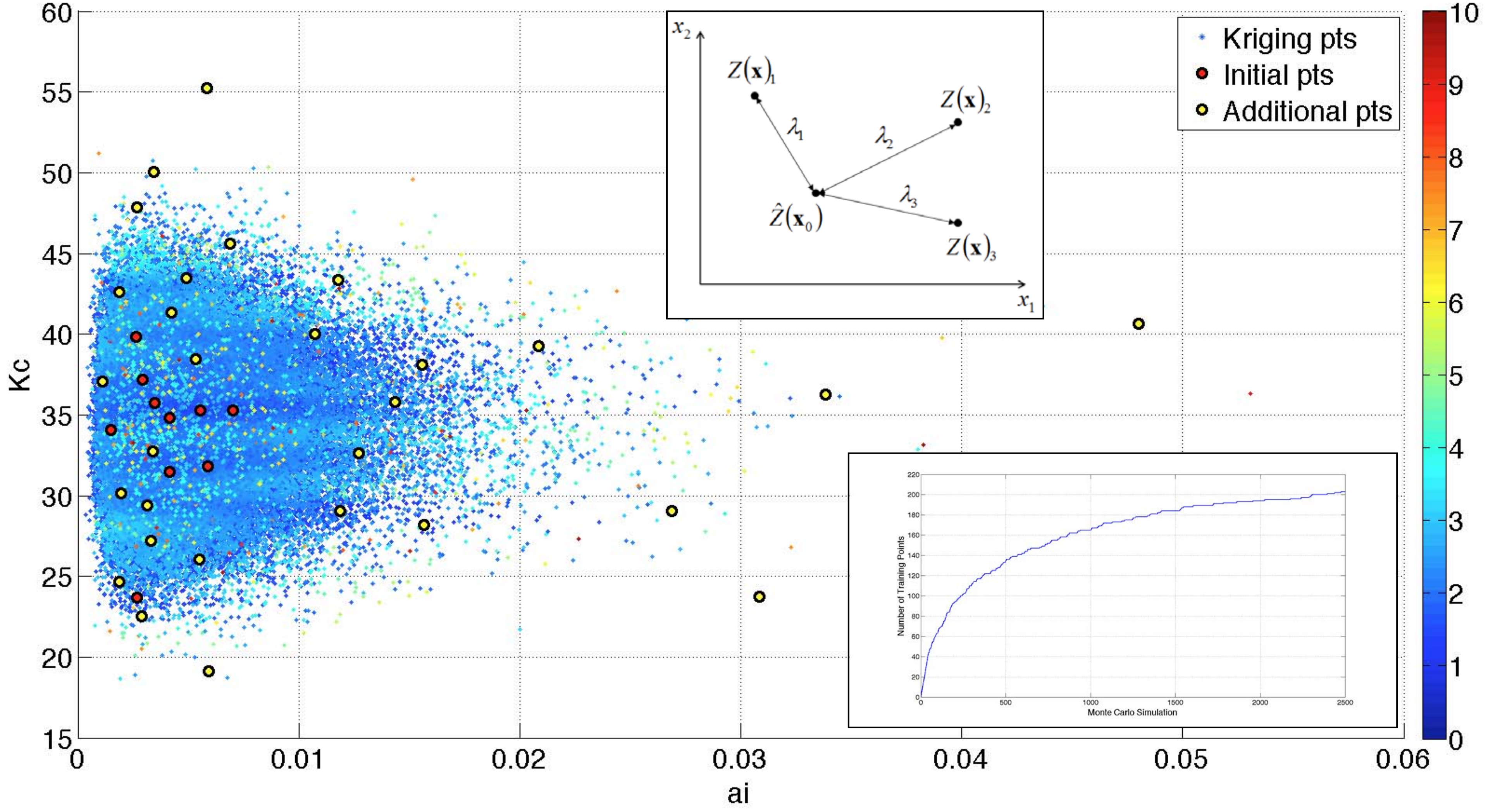
— Initial Training Points

— Additional Training Points (Kriging Error> User Error)

— Kriged Points (Kriging Error < User Error)

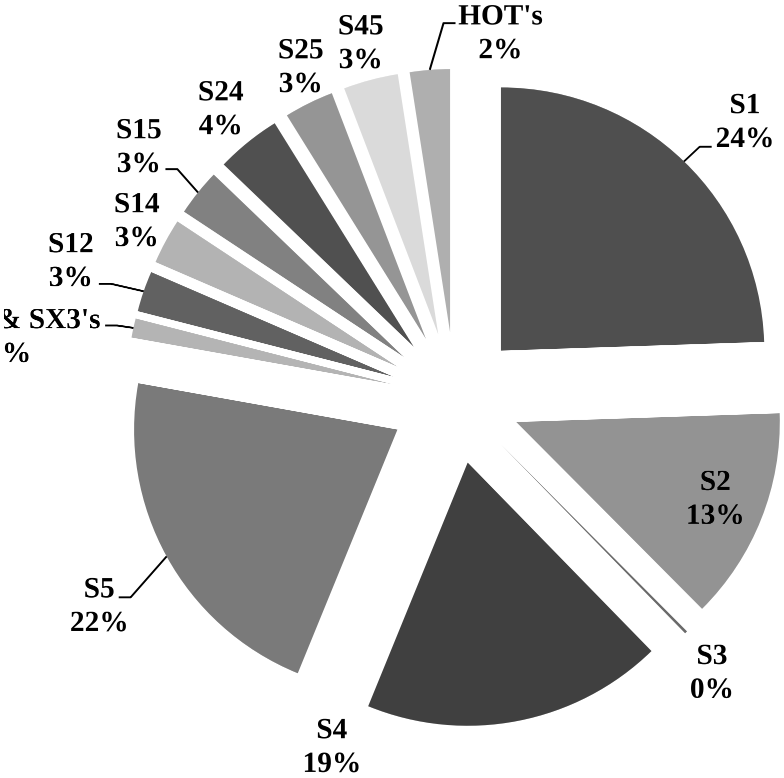
Kriging Schemetic





Sensitivity Analysis

Why no sensitivity analysis?
Too expensive?
Not well known>



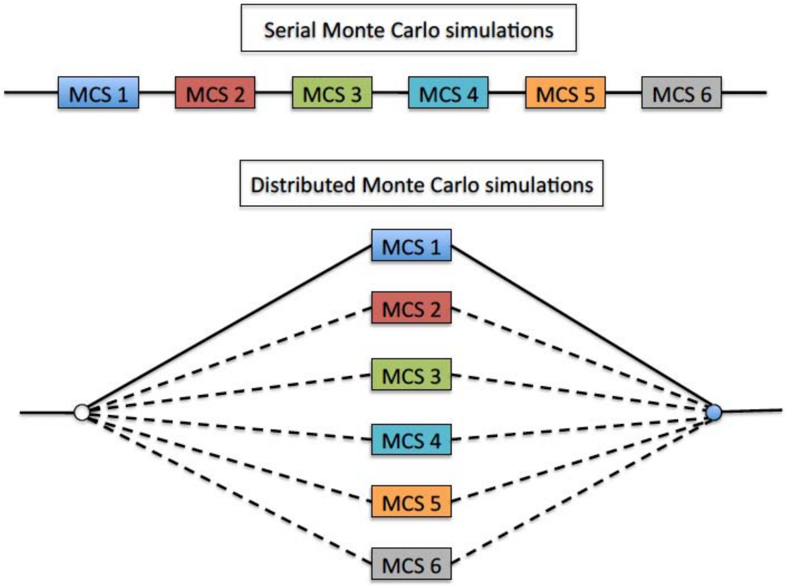
Global Sensitivity Analysis
Apportions the variance
Requires multiple analyses

Score Fn. Method
Partial derivative approach

$$\frac{\partial POF(t)}{\partial \mu_{ai}} = \int_{0}^{\infty} \int_{-\infty}^{\infty} \left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}} \right) \kappa_{\mu}(a) \right] f_{a_0}(a_0) f_{K_c}(K_c)\, da_0\, dK_c$$

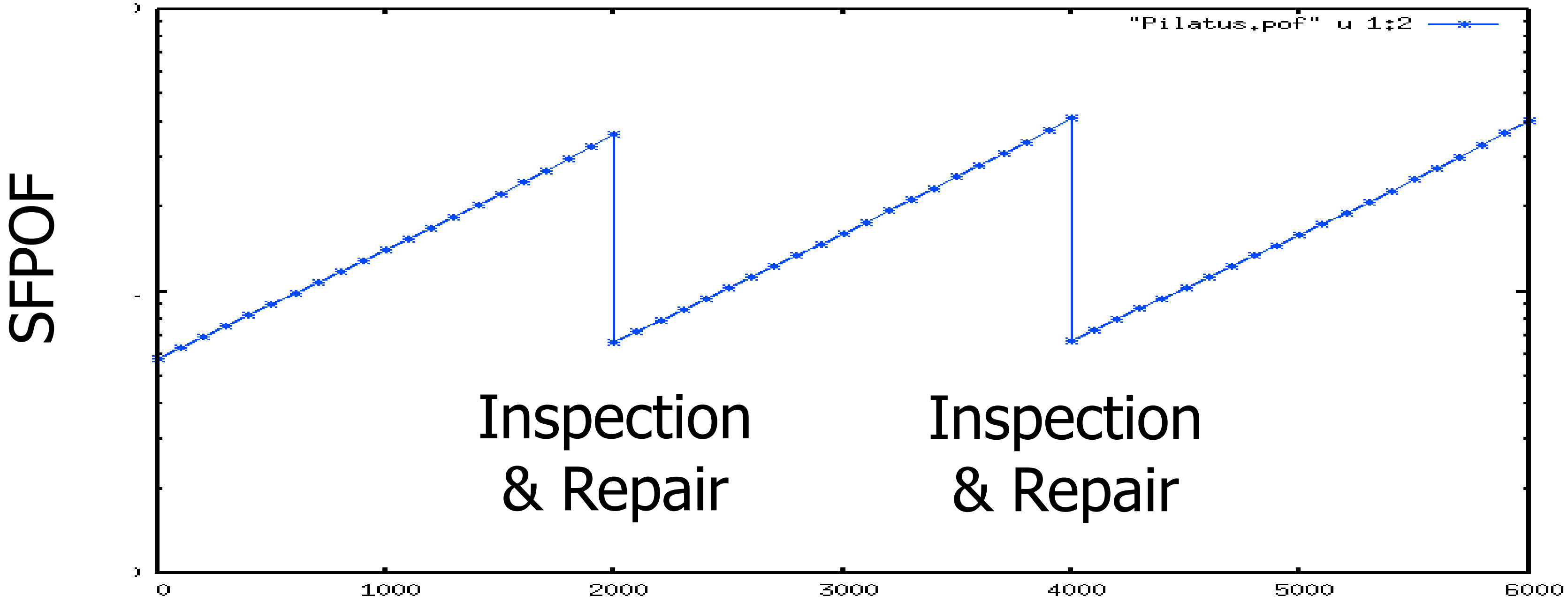
High Performance Computing

EVERYONE has multiple cores available (2-8).
Intel MIC chip (60 cores) potential game changer.
EVERYONE has a decent graphics card.
New standards in place and emerging
(OpenMP, MPI, cuda, OpenCL, OpenACC)



Summary

- Continue the push for an efficient, robust probabilistic algorithm to allow the user to to conduct a risk assessment of user-defined complexity.



Summary

- The POF integral associated with airframe risk assessment is deviously challenging due to small probabilities, inspection and repair
- A robust sampling-based approach has been implemented
- New methods are needed to address:
  - Larger no. of random variables
  - More flexibility
  - Improved efficiency
  - Sensitivity analysis
  - High performance computing

Acknowledgements

- Probabilistic Structural Risk Assessment and Risk Management for Small Airplanes, Sep 2007- Dec 2010, Federal Aviation Administration, Grant 07-G-011
- Probabilistic Damage Tolerance-Based Maintenance Planning for Small Airplanes, Sep. 2009-Aug. 2012, Federal Aviation Administration, Grant 09-G-016
- Probabilistic Fatigue Management Program for General Aviation, Sep. 2012-Aug. 2016, Federal Aviation Administration, Grant 12-G-012

SHM Application

- Can this methodology be applied to SHM, i.e., frequent, recurring inspections?
- Yes, but **ONLY** if the correlation between inspections is considered.