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Sensitivity Analysis for General Aviation Risk Assessment

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General Aviation structural risk assessment is of vital importance to ensure airworthiness and safety. However, it is even more important to understand the role of all the different variables and their importance or sensitivity over the airplane life in order to reduce the risk. Based on this idea, different sensitivity methods (scatter plots, parallel box plots, and global sensitivity) were applied to the software SMART (Small Aircraft Risk Technology). SMART is probabilistic fatigue linear damage software developed by the University of Texas at San Antonio (UTSA) to conduct risk assessment in general aviation (GA) airplanes. SMART computes the airplane structural life considering variables such as: maneuver and gust load limit factors, ground stress, one-g-stress, flight length-velocity matrix, flight length-weight matrix, exceedance curves, Miner's damage coefficients, and stress life curves, considering randomness in some of the variables. The results from the sensitivity methods indicate that the variables one-g stress, gust and maneuver loads, PSN curve and damage coefficient play an important role in the airplane life and more caution should be focused on these variables.

Nomenclature

DC = Damage coefficient
E [•] = Expected value
LT = Load transfer
OH = Open hole
PSN = Probabilistic Stress-Life curve
S_i = First order index
S_{ij} = Second order index
S_{ijk} = First order index
S_T = Total effect index
V (•) = Variance
X = Vector of random variables
Y = Response of the model

I. Introduction

MOST general aviation (GA) aircraft are designed for safe-life based upon a crack initiation type failure mechanism, e.g., Miner's rule; and risk assessment provides an important tool in the design and maintenance of these airplanes. Due to the complexity and variability of evaluating the risk in an airplane, a computational approach is required.

SMART [9] is software developed by UTSA to compute probabilistic risk assessment in small airplanes through linear damage based on Miner's Rule. SMART computes the hours and flights to failure of an airplane, based on real flight conditions. It takes into account probabilistic information on loading (gust and maneuver loads, sink rate, flight length-velocity matrix and flight length-weight matrix) and material behavior (Miner's Coefficient and Probabilistic stress Life curves).

A physical phenomenon like fatigue failure can be represented by a mathematical model which has inputs with variability that affect the response of the system. To analyze the influence of these variables on the response, a sensitivity analysis can be applied. Sensitivity analysis is useful to gain more knowledge about the behavior of a

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problem and helps in the decision-making process [4], [5], [6]. It can be done in any stage of the design process in order to determine the important factors related to the model response. There are a number of methods in the literature to apply sensitivity analysis, some of these methods include: scatter plots, parallel box plots, and analysis of variance (first order and total indices), to mention a few.

In order to have a better understanding of the impact of the variables affecting the life of an airplane, a sensitivity analysis was applied to a realistic general aviation example problem. To solve this problem the risk assessment software was used due to its flexibility and its computational low-cost, which allows efficient simulation of many different conditions for the airplane.

The results obtained were presented and analyzed in order to gain a better understanding of the behavior of the airplanes' life with respect to the variables and the conditions affecting their life, and at the same time comparing the sensitivity analysis methods.

II. SMART-LD

In many applications, fatigue life evaluation of structural components is conducted using a deterministic approach, and GA is not an exception. However, due to the number of uncertainties present and their impact on structural integrity a probabilistic approach is needed [7], [10]. In fatigue life assessments, both the material properties and the load characteristics are essential random variables and may exhibit significant variability.

The methodology in SMART [9] encompasses the required elements necessary to conduct a structural integrity evaluation. SMART considers real-world airplane-to-airplane and flight-to-flight variations such that a realistic risk assessment can be made of an aircraft structural detail.

To perform a risk evaluation, a methodology that follows the guidelines used for safe-life evaluation in FAA reports AFS-120-73 [1] and AC-23-13A [2] was incorporated in a computer code. The methodology calculates the flights/hours-to-failure or the safe-life (time to crack initiation) for GA and this methodology is explained step by step as follows:

- Variables such as airplane usage, load limit factors, ground stress, one-g stress, airplane velocity, and flight length are input by the user.
- According to the airplane usage (see Table 1), e.g., single-engine unpressurized instructional, pressurized usage, twin-engine general usage, etc. the respective data (exceedance curves, sink rate data, etc.) are loaded from internal libraries.

Table 1. Usage Options

Usage Options
Single engine unpressurized instructional usage
Single engine unpressurized personal usage
Single engine unpressurized executive usage
Twin engine unpressurized basic instructional usage
Twin engine unpressurized general usage
Pressurized usage
Agricultural special usage
User defined

- Realizations of the random data such as: sink rate velocity, airplane velocity, flight duration, etc. needed for Monte Carlo sampling are generated.
- For each Monte Carlo sample, the code generates a characteristic stress spectrum that includes all the flight stages: gust, maneuver, taxi, ground-air-ground, and landing and rebound. The methodology process through this point is shown in Fig. 1.

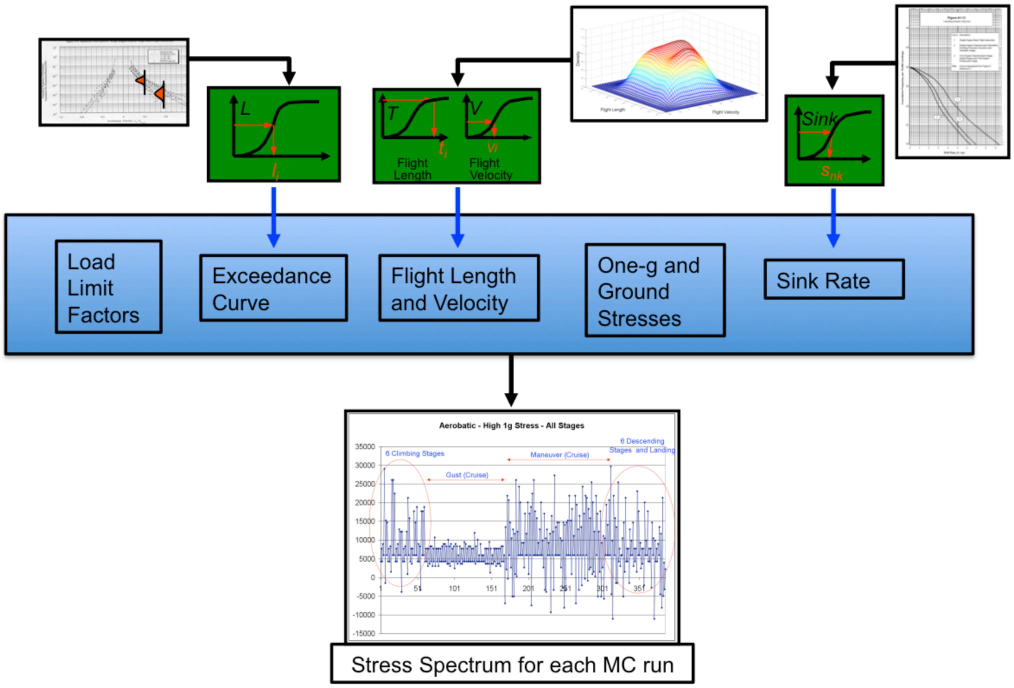

Figure 1. Schematic of risk assessment methodology for the spectrum generation

- Damage is accumulated for each Monte Carlo sample using Miner's rule until Miner's critical value is reached and flights-to-failure is recorded as shown in Fig. 2.

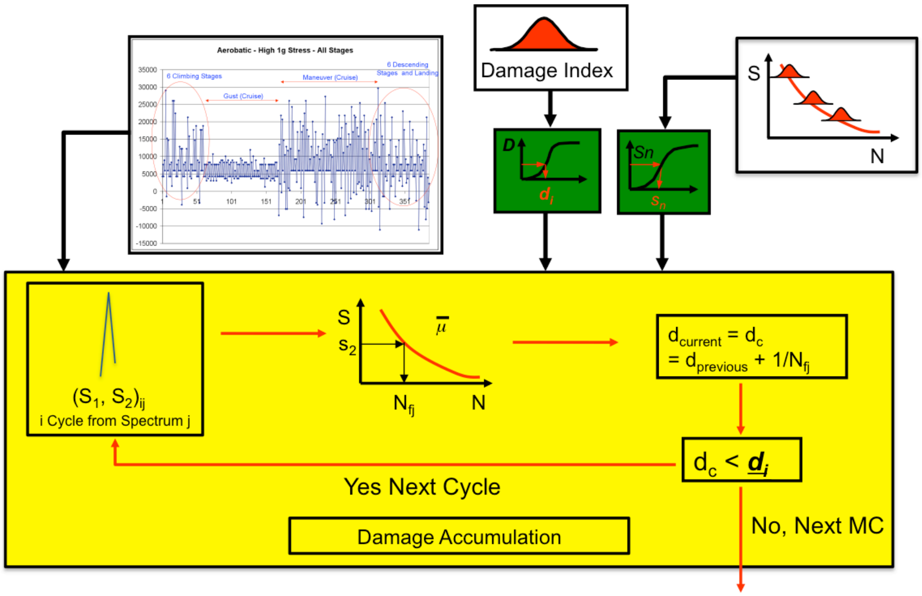

Figure 2. Schematic of risk assessment methodology for the damage accumulation

- When the Monte Carlo sampling is finished, the random variables and flights/hours-to-failure are stored for post-processing processed to determine the probability distribution of flights/hours-to-failure (probabilities, mean, standard deviation, confidence intervals, hazard function).

Given the significant airplane-to-airplane and flight-to-flight variations, an essential ingredient of the SMART methodology was to investigate, develop and model probability distribution functions (pdfs) of the critical input data as flight duration, aircraft speed, sink rate velocity, the damage index coefficient, and Probabilistic S-N curves.

III. Sensitivity Methods

A sensitivity analysis was performed in SMART to identify the importance of the variables over the airplane life. The methods used for the sensitivity analysis were: scatter plots, parallel box plots, and variance-based methods (global sensitivity). The results obtained through these methods were compared, in order to have an idea of the accuracy of the methods.

A. Scatter Plots and Parallel Box Plots

Scatter plots are two-dimensional plots of the sample points versus the corresponding response points [3]. This constitutes an effective graphical tool to understand the influence of the variables over the life and to assess qualitative relationships, according to the shape of the scatter plot. Depending in the complexity of the problem, the number of samples required can increase. If a random variable is important, the points will follow a defined pattern; if a random variable is not important there is not any definite pattern in the points.

Parallel box plots are a graphical approach to visualize higher-dimensional data. It shows if the distribution has a long tail, its central value, and variability. It is an effective approach to search for patterns in multivariate data.

B. Variance based method

Global sensitivity analysis (GSA) [11], [12] is a technique used to identify the importance of a variable based on how much its variance contributes to the total variance of a system response.

A variable X_i is fixed at a particular value and the system variance is computed for $X_{\sim i}$ (all factors but X_i). The difference between the conditional variance for X_i and the original system variance indicates the importance of X_i; if the system variance reduces significantly, the variable is important. To avoid the dependence when X_i is fixed, a large number of conditional variances for different fixed values are computed, and an expected value (average) of these variances is used.

In a generic model as:

$$Y = f(X_1, X_2, \ldots, X_k) \tag{1}$$

Each factor X_i can be fixed to a particular value x^*_i; then the variance of Y (response) can be computed as:

$$V_{X\sim i}\left(Y / X_i = x_i^*\right) \tag{2}$$

The conditional variance computed by Eq. (2) is different from the original variance of Y; this difference is a measure of the relative importance of X_i. To avoid the dependence on x^*_i, the average of the measure of all possible points of x^*_i is used:

$$E_{X_i}\left(V_{X_{\sim i}}\left(Y / X_i = x_i^*\right)\right) + V_{X_i}\left(E_{X_{\sim i}}\left(Y / X_i = x_i^*\right)\right) = V(Y) \tag{3}$$

The second term in the right side of the equation is called the first-order effect of X_i on Y, and the first-order sensitivity index is defined as:

$$S_i = \frac{V_{X_i}\left(E_{X_{\sim i}}\left(Y / X_{\sim i}\right)\right)}{V(Y)} \tag{4}$$

The value of S_i is always a number between 0 and 1 and indicates the influence of each variable by itself, in the total variance of the model. The larger the value of S_i, the more important the variable is.

To measure all the effects of a variable (including interactions with others) the total index is used. It is computed as:

$$S_{T_i} = 1 - \frac{V_{X_{\sim i}}\left(E_{X_i}\left(Y / X_{\sim i}\right)\right)}{V(Y)} \tag{5}$$

If the total index is close to zero, the variable is considered non-influential and can be modeled as deterministic.

The interaction between variables (interaction terms) can be obtained with a similar procedure as the one used for the first order term. In this case, the interaction variables of interest are fixed and the expected value is computed.

IV. Example Problem Description

To analyze the sensitivity of the response with respect to each of the variables used by SMART (see Table 2), the life of single-engine basic instruction usage airplane is computed; for this analysis a total of 20,000 Monte Carlo samples were used. The characteristics of the airplane, variables, and parameters are shown in Table 3 and Fig. 3 and Fig. 4. Scatter plot analysis and the parallel box plot analysis were developed based on these samples.

Table 2. Variables used by SMART

Variable	Type
Gust/Maneuver Load Exceedances	Probabilistic (Lognormal) [10]
Aircraft Velocity and Flight Duration	Probabilistic (Joint PDF with correlated variables)
Sink Rate	Probabilistic
Damage index	Probabilistic (Normal or Weibull Distribution) [9]
Maneuver Load Limit Factors	Deterministic
Gust Load Limit Factors	Deterministic
Ground Stress	Probabilistic (Joint PDF with correlated variables)
One-g-stress	Probabilistic (Joint PDF with correlated variables)
P-S-N	Probabilistic (Determined from regression modeling of constant amplitude tests) [9]

Table 3. Conditions used for the sensitivity analysis

Variable						Value						
Usage						Single-Engine Unpressurized Usage Basic flight Instruction						
Design LLF Maneuver						3.8-1.5						
Design LLF Gust						3.4-1.2						
Ground Stress (psi)						-3500						
One-g- stress						8500						
Flight Length and Velocity Matrix (Figure 3)												
		Average Speed during flight, %VNO or %VMO										
Flt. Time (h)	Percentage of flights.	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.980	1.00
0.50	0.02	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.55	0.04	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.60	0.06	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.65	0.08	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.70	0.10	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.75	0.12	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.80	0.16	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.85	0.12	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.90	0.10	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.95	0.08	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
1.00	0.06	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
1.05	0.04	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
1.10	0.02	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
Flight Length and Weight Matrix (Figure 4)												
		Average Speed during flight, %VNO or %VMO										
Flt. Time (h)	Percentage of flights.	0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.980	1.00
0.55	0.02	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.50	0.04	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.60	0.06	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02

Variable						Value						
0.65	0.08	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.70	0.10	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.75	0.12	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.80	0.16	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.85	0.12	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.90	0.10	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
0.95	0.08	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
1.00	0.06	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
1.05	0.04	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
1.10	0.02	0.02	0.05	0.08	0.10	0.12	0.26	0.12	0.10	0.08	0.05	0.02
Average Velocity (Vno/Vmo, knots)						195						
Miner's Rule Damage Factor						Weibull : OH(α=3.37,β=0.38,γ=0.51) LT(α=1.13,β=2.35,γ=1.24)						
MCSAMP						20,000						
SN curve						PSN_ASTM						
Analysis Type						Damage						
SSF (Direct input)						3.1						

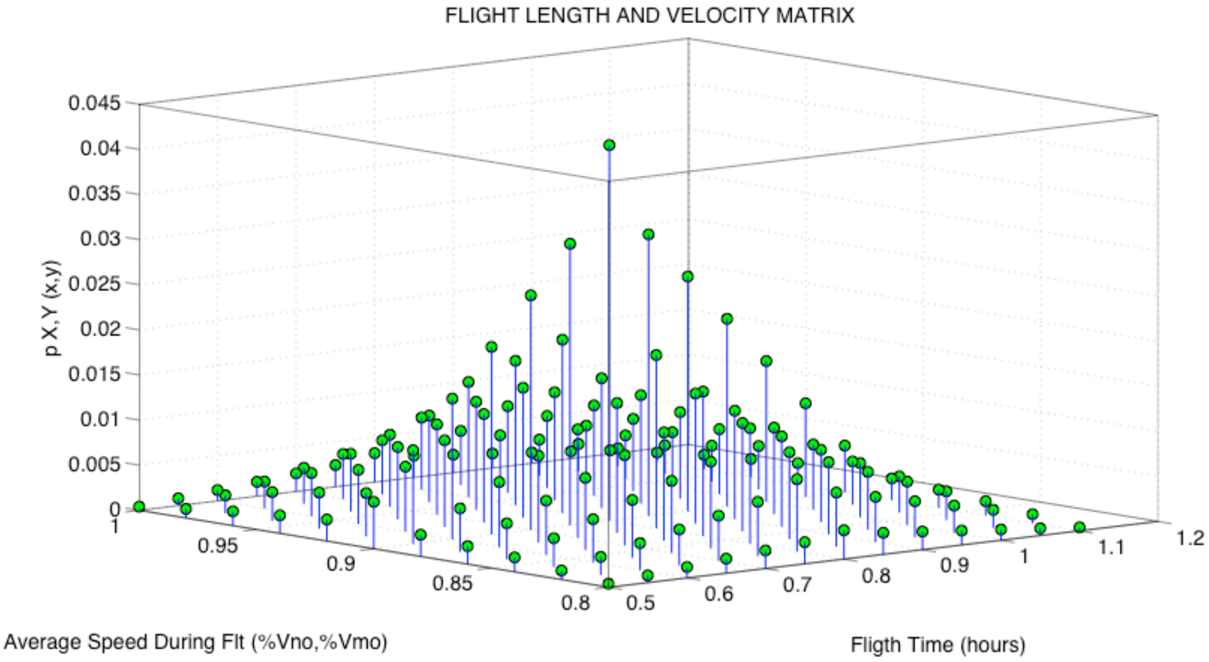

Figure 3. Flight length and velocity matrix

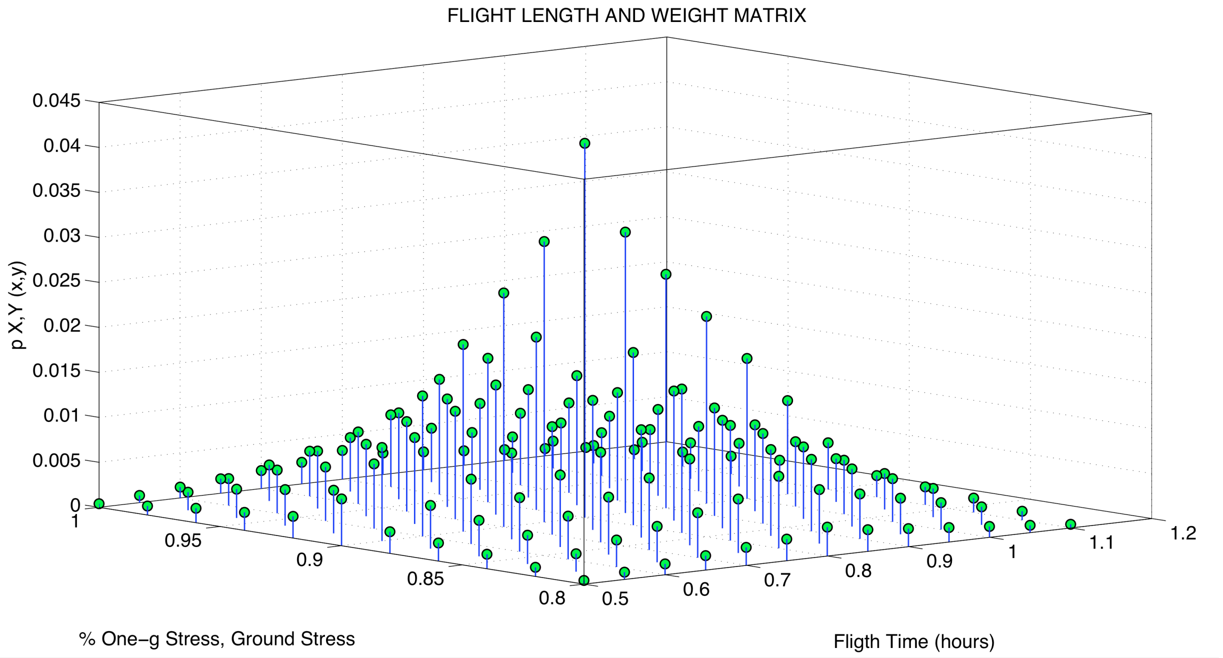

Figure 4. Flight length and weight matrix

To compute the indices from the global sensitivity analysis, a different approach should be used. To compute "the variance of an expected value" according to Eq. (4), a nested Monte Carlo method is used. The number of samples used was 2000 by 2000 samples.

To get a better understanding about the behavior of the variables, the sensitivity analysis (scatter plots, box plots, and GSA) was applied to four different cases, varying the stress-life curves and the value of the damage coefficient. All these cases were computed using an ASTM PSN curve (see Fig. 5 and Fig. 6). The cases were:

- Open hole, Miner's damage coefficient equals to one.
- Load transfer (50% Hi-lok), Miner's damage coefficient equals to one.
- Open hole, Miner's damage coefficient random (Fig. 7)
- Load transfer (50% Hi-lok), Miner's damage coefficient random (Fig. 8)

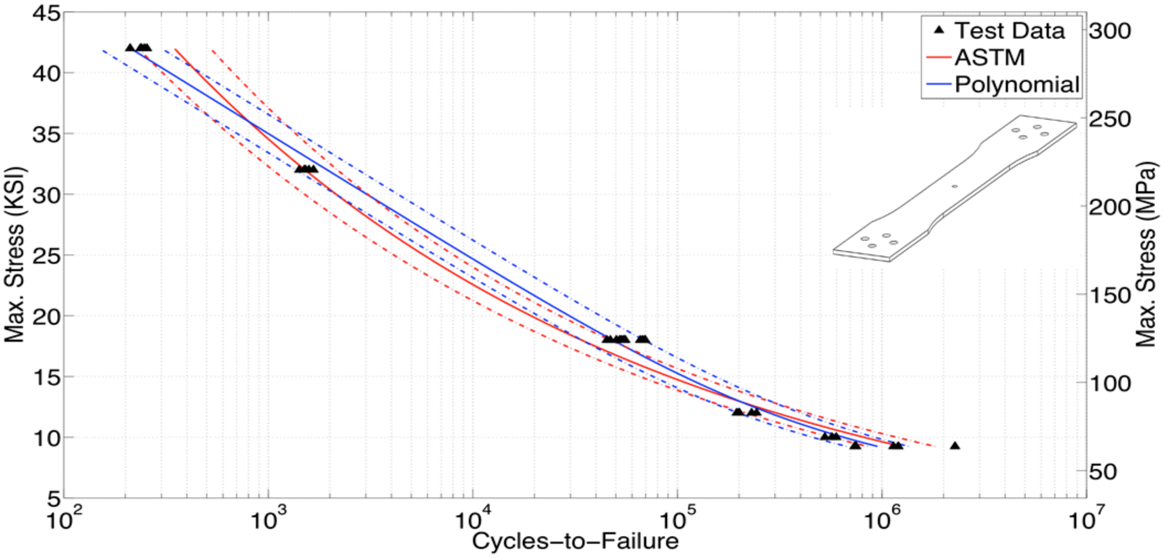

Figure 5. PSN curve open hole (1.5" wide, mean stress 3 Ksi)

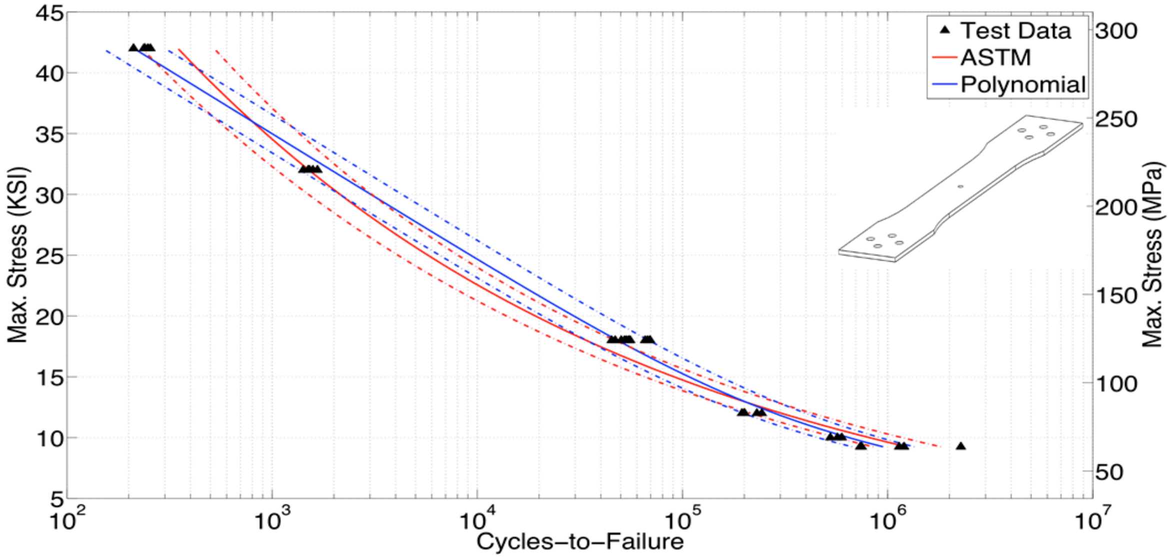

Figure 6. PSN curve open hole (1.5” wide, mean stress 6 Ksi)

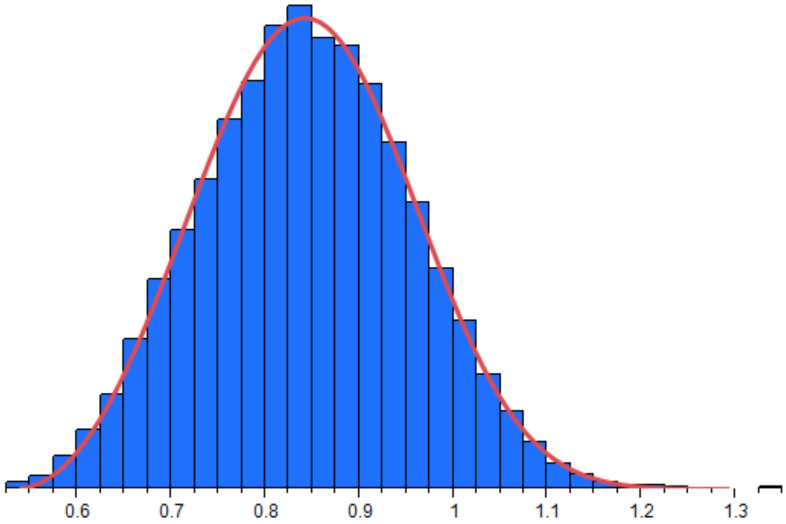

Figure 7. Distribution damage coefficient OH case

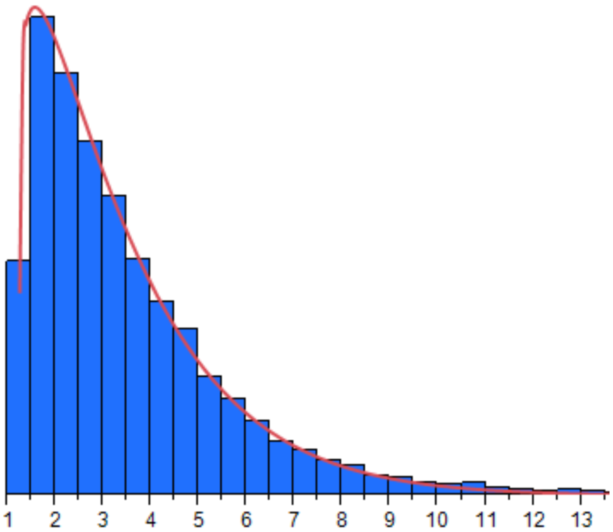

Figure 8. Distribution damage coefficient LT case

One of the drawbacks of global sensitivity analysis is the difficulty to handle correlated variables. However, this issue can be avoided by "grouping" the correlated variables. To be able to compute the indices for correlated variables, those variables must be fixed at the same time. Based on this, the groups and variables analyzed for GSA were:

- Group 1: Flight duration, flight velocity, one-g stress (Fig. 3 and Fig. 4).
- Group 2: Damage coefficient (DC) and Probabilistic Stress-Life curve (PSN)
- Sink rate
- Gust load
- Maneuver load

Ground stress is not included above because it is fully correlated to one-g stress; hence the sensitivity index is the same.

V. Results and Discussion

This section contains the results obtained from the sensitivity analysis applied to the four cases mentioned above. The sensitivity methods applied were scatter plots, parallel box plots and global sensitivity analysis.

A. Hours to Failure

Table 4 shows the values of mean and standard deviation for the four different cases described in the previous section. It can be observed that life is higher for the load transfer cases than for the open hole cases; the load transfer case presented higher failures as can be seen in Fig. 9. Also, it is observed that the variance for the load transfer case is higher, due to the long tail presented by the distribution of damage coefficient (Fig. 8). It should be noticed that the difference in means and variances between the open hole and the load transfer cases is more notorious for the case of random damage coefficient, indicating the high impact of this variable over the life. The mean life for the open hole case with random damage coefficient is less, compared to the case with damage coefficient equals to one.

Table 4. Parameters hours to failure for ASTM curve for OH and LT

	OH		LT	
	DC=1	DC random	DC=1	DC random
Mean hours to failure	14,199	11,967	21,132	73,137
Variance hours to failure	2.7 E+7	2.2 E+7	6.5 E+7	2.8 E+9

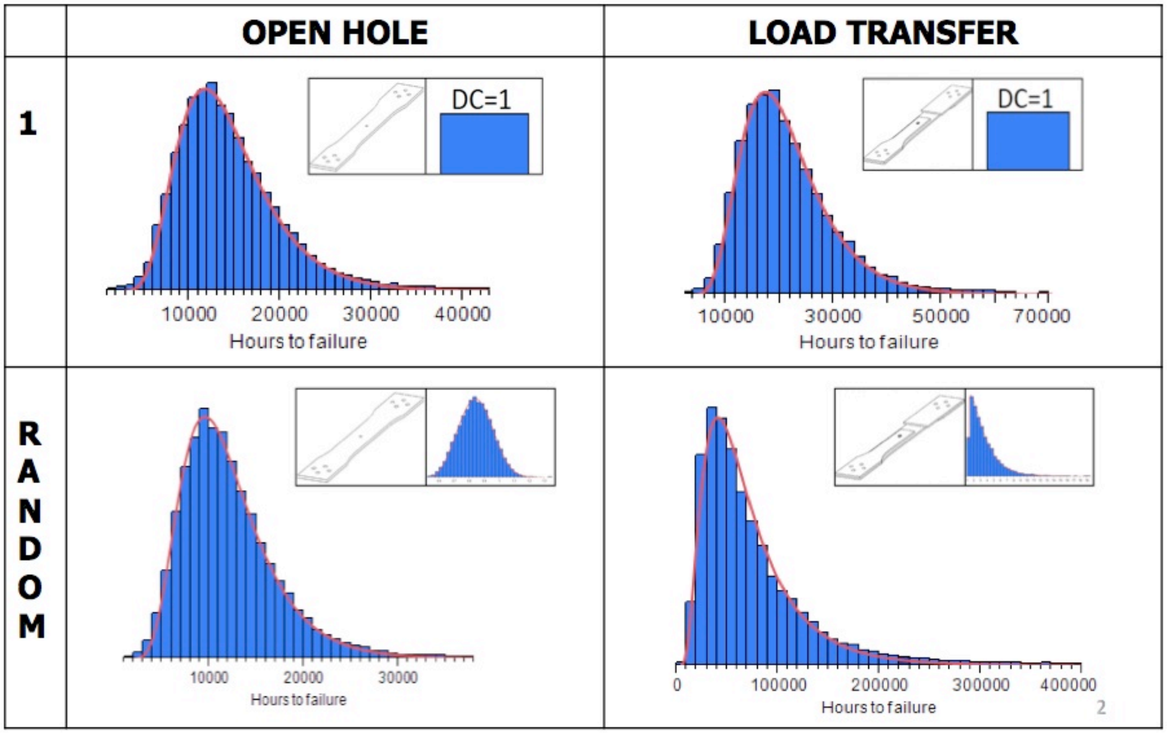

Figure 9. Hours to failure all cases

B. Scatter Plots

The scatter plots between the life and the different random variables for all the cases are presented in Fig. 10 to Fig. 13. In general, the scatter plots showed that flight duration, flight velocity, and sink rate have not importance over the life, because the shape of the points does not has a defined pattern. The rest of the variables seem to have an importance over the life, depending on the case as follow:

1. Open hole DC=1

One-g stress (also ground stress) was the variable with the most defined pattern from the scatter plots. This pattern seemed linear. Gust and maneuver loads also had some pattern indicating their influence over the life. PSN seemed to have a low influence over the life, because the two groups of points observed.

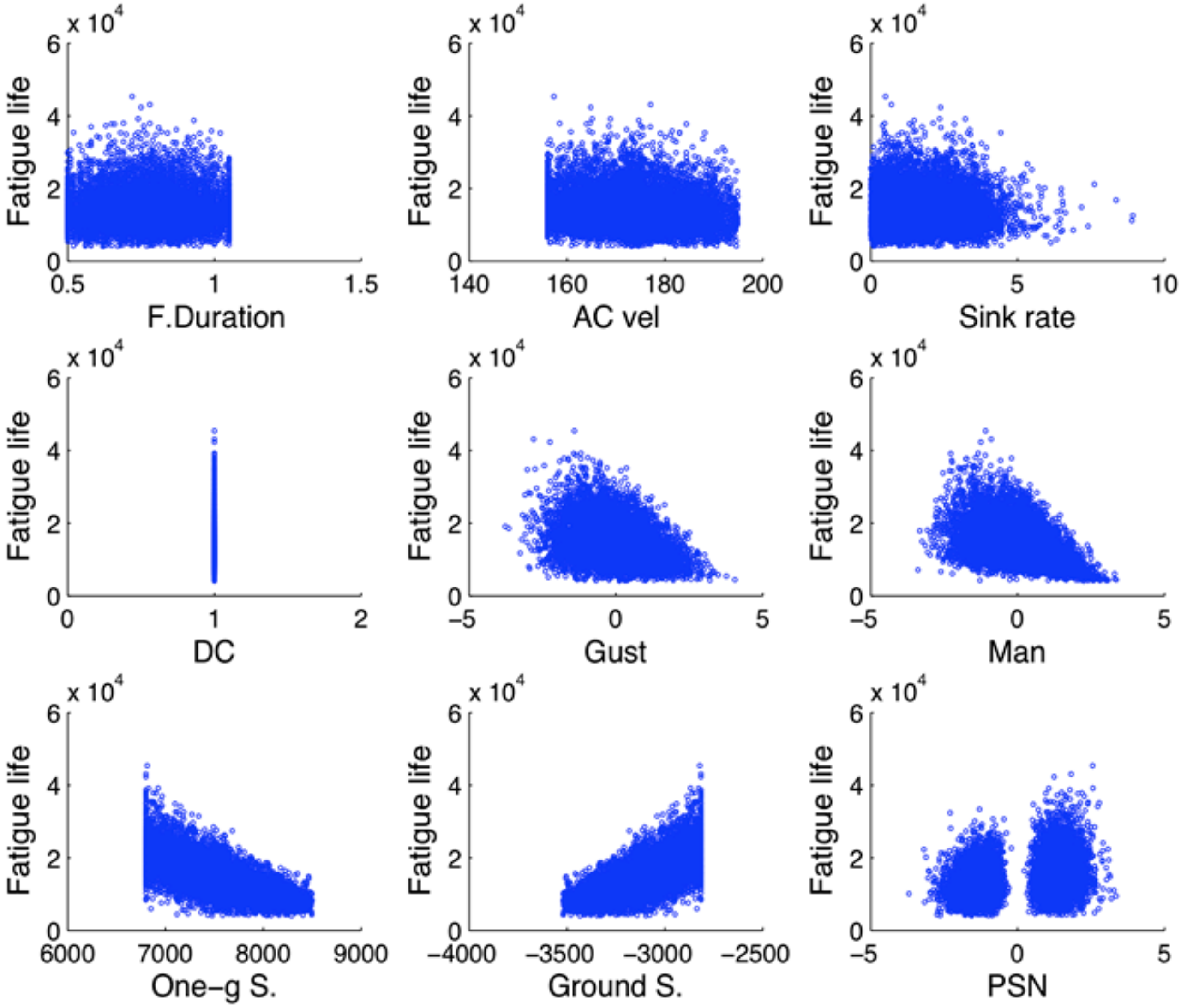

Figure 10 .Scatter plot matrix OH with damage coefficient equals to 1 (ASTM curve)

2. *Open hole DC random*

One-g stress, gust, and maneuver loads were the most important variables affecting the life, but this importance was less than in the previous case, because the points had more scatter. The shape of PSN was more elongated now, indicating that its importance increased. Damage coefficient also had some importance and the scatter of its points is similar to the points of gust, indicating that the importance of these two variables is comparable.

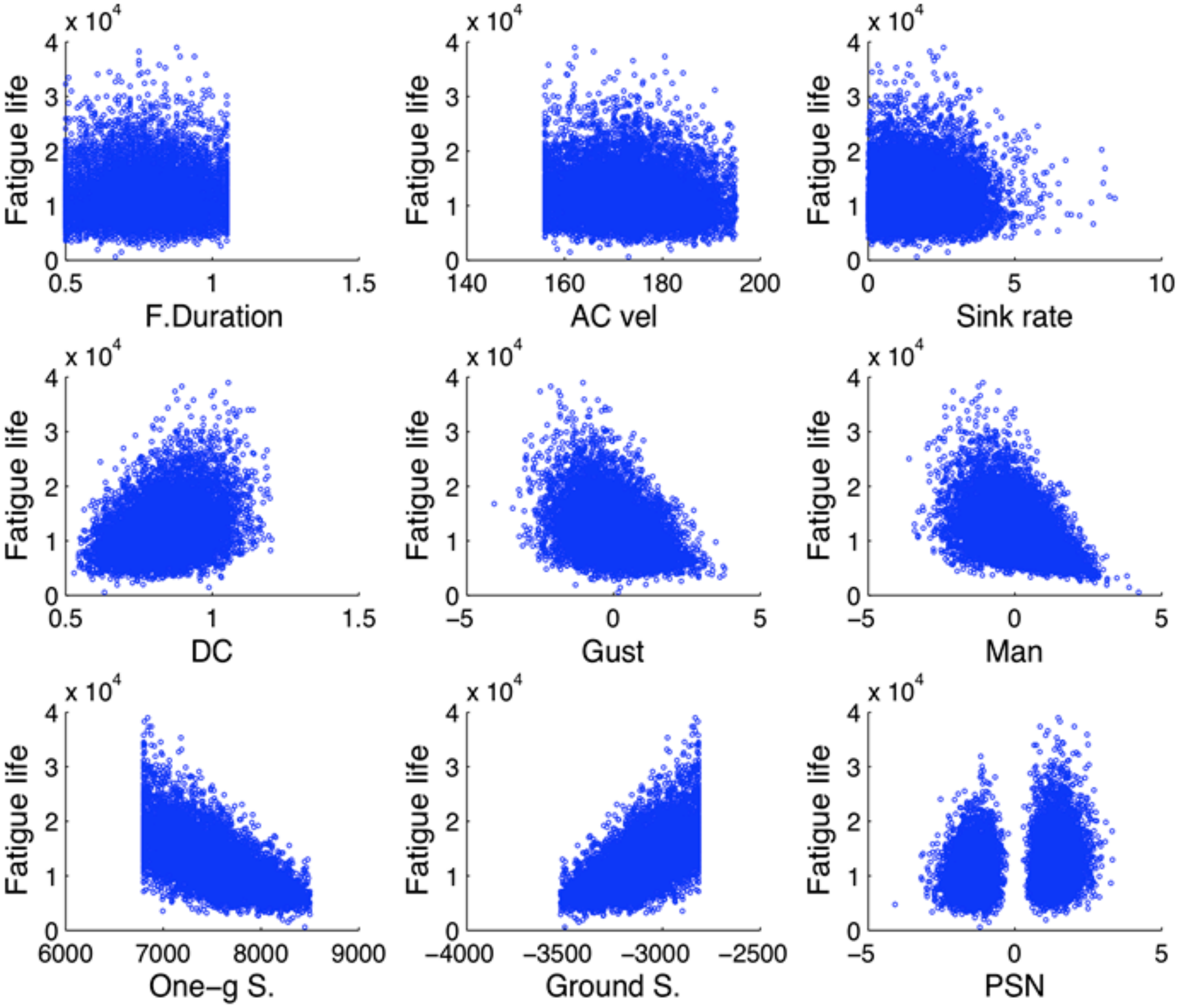

Figure 11. Scatter plot matrix OH with damage coefficient random (ASTM curve)

3. Load transfer DC=1

Although the points of scatter plots for one-g stress, maneuver, and gust loads presented some trend, the relationships between this factors and the life was not as strong as in the previous case. PSN seemed to have some influence over the life, even though it did not have a very well defined shape.

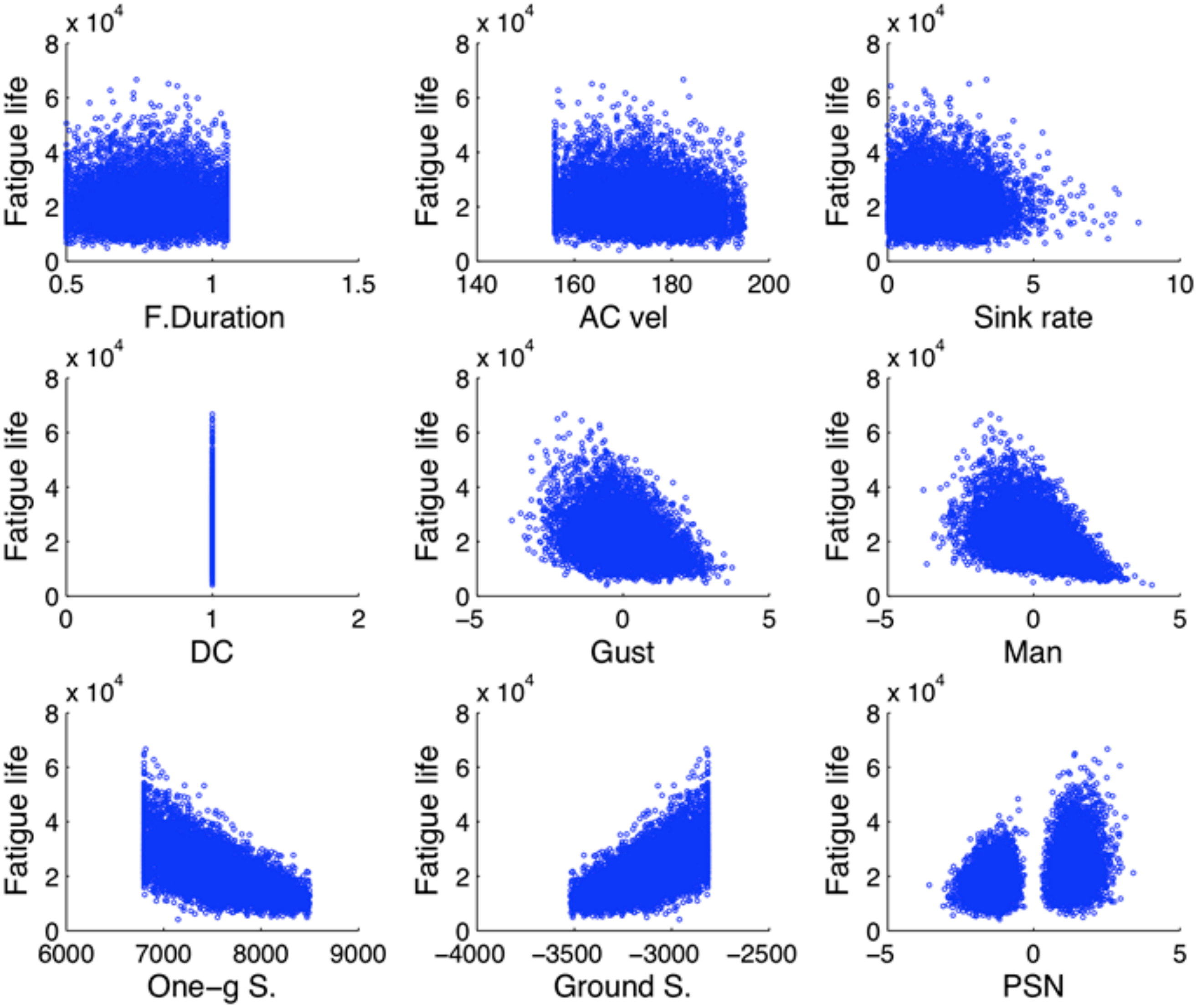

Figure 12. Scatter plot matrix LT with damage coefficient equals to 1 (ASTM curve)

4. Load transfer DC random

The importance of damage coefficient was notorious for this case. The scatter of the points was well defined. One-g, maneuver, and gust loads were still important variables, but their influence was reduced; the scatter of their points increased. PSN appeared to increase its influence because the scatter of the points looked more defined in this case.

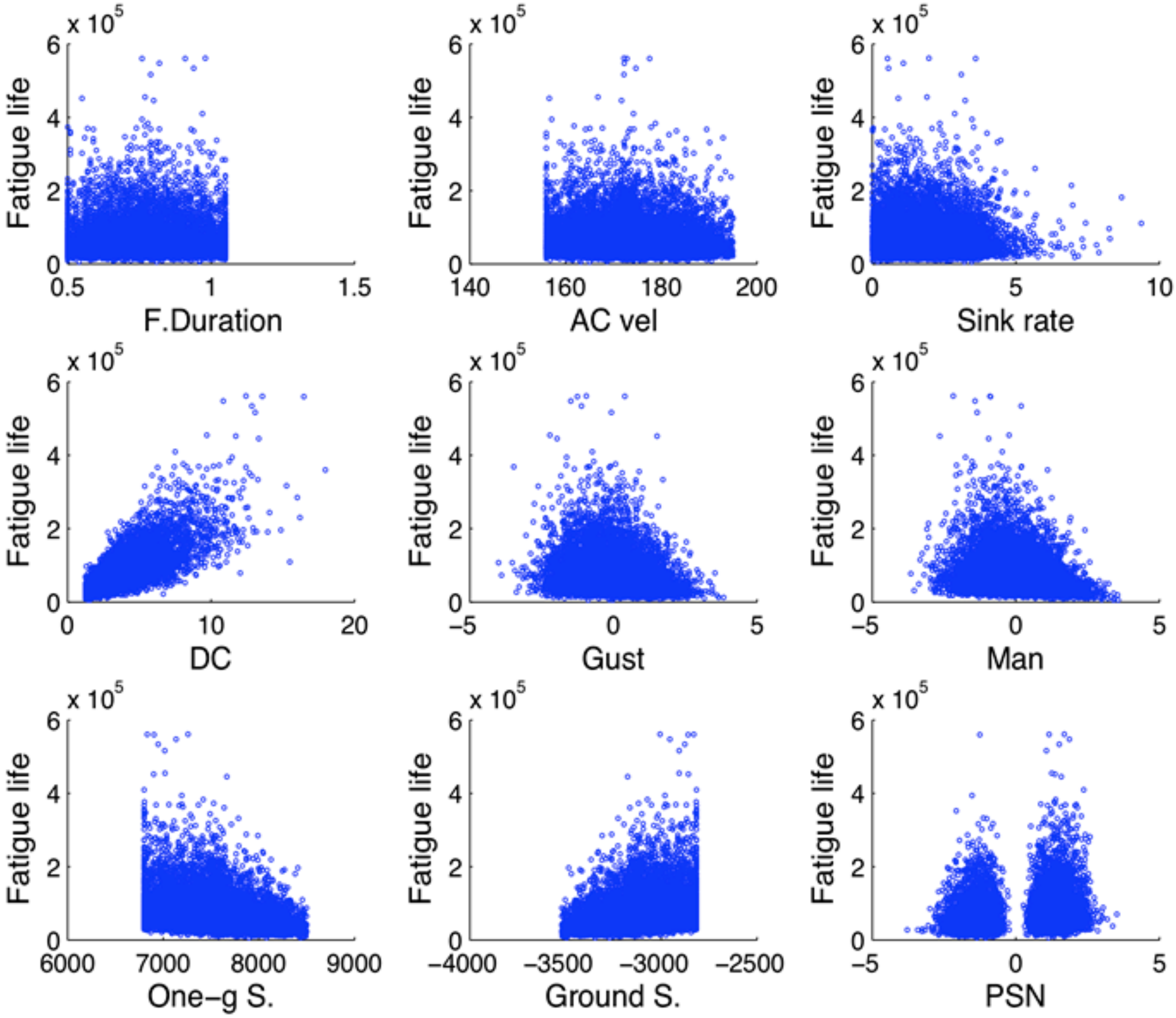

Figure 13. Scatter plot matrix LT with damage coefficient random (ASTM curve)

C. Parallel Box Plots

The box plots relating life with all the random variables are presented in Fig. 14 to Fig. 17. The selected areas in red are the samples with low failures. The percentage of samples selected is around 5% .The incidence of the variables varies depending on the cases, however, there are some facts common to all the cases, that cause low life failures. These facts were:

- High values of one-g stress, gust and maneuver loads, and flight velocity.
- Low values of ground stress, PSN curve, and Damage coefficient.

It is observed that the mean of the area selected displaced more in some cases than others; this can be interpreted as a measure of the influence of the variables over the life. Based on that, the variables having a considerable influence over the life were one-g stress, gust and maneuver loads, PSN curve and damage coefficient. For the open hole case, one-g stress, gust and maneuver factors affected the life the most, meanwhile for the load transfer case PSN and damage coefficient had higher effect over the life.

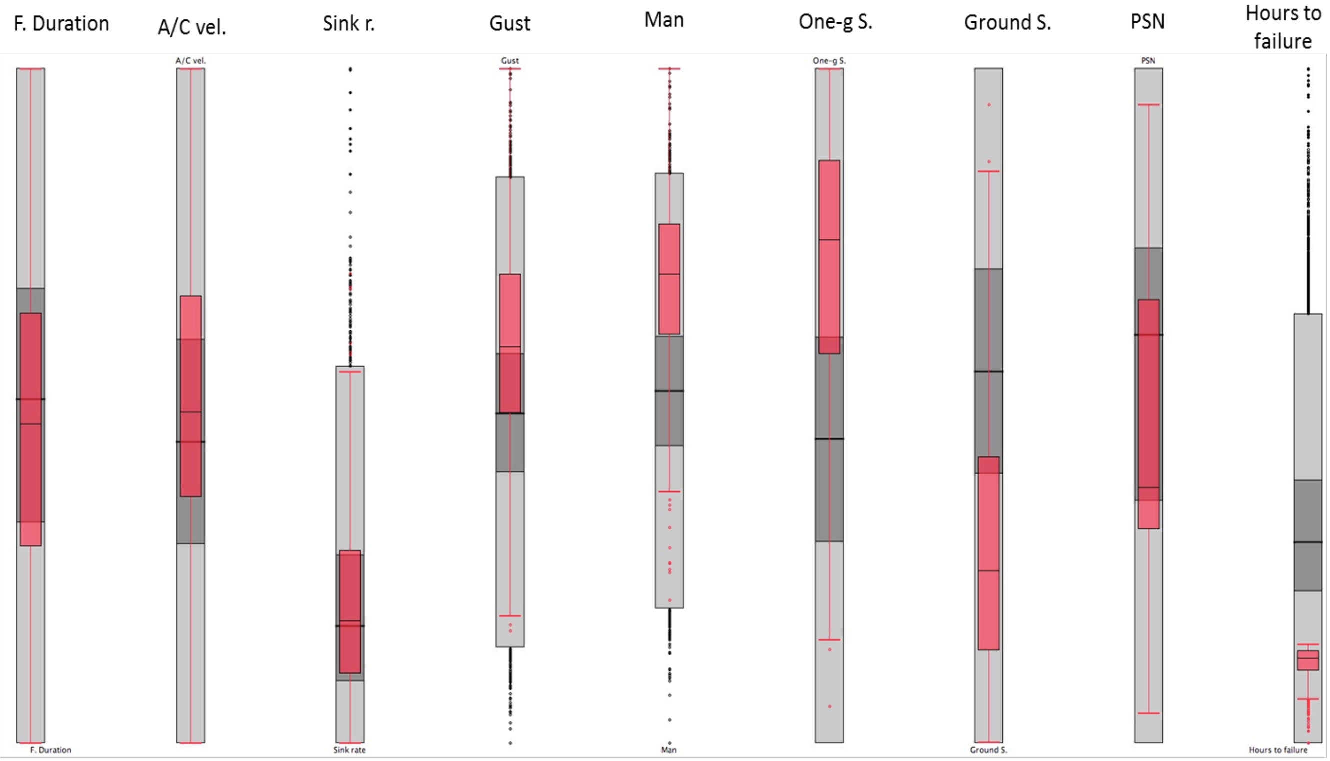

Figure 14. Parallel box plots OH with damage coefficient equals to 1 (ASTM curve)

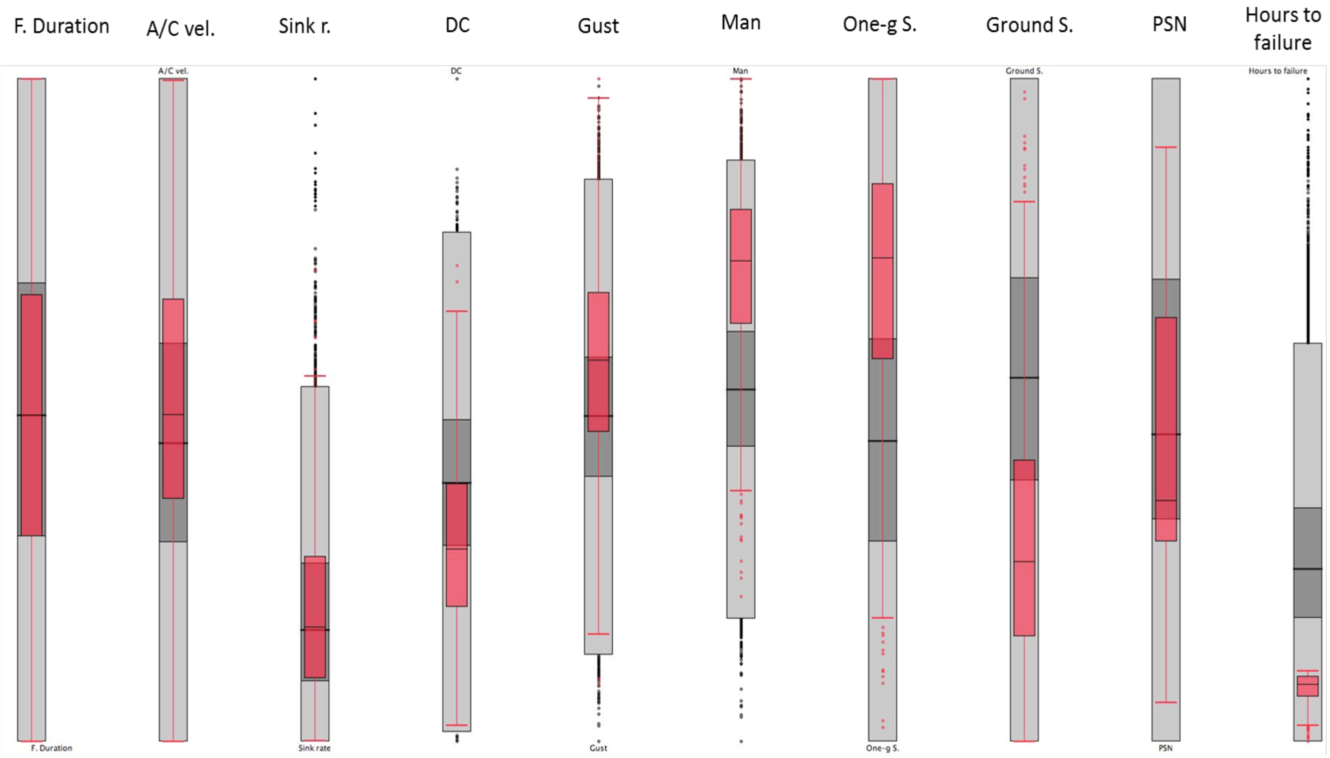

Figure 15. Parallel box plots OH with damage coefficient random (ASTM curve)

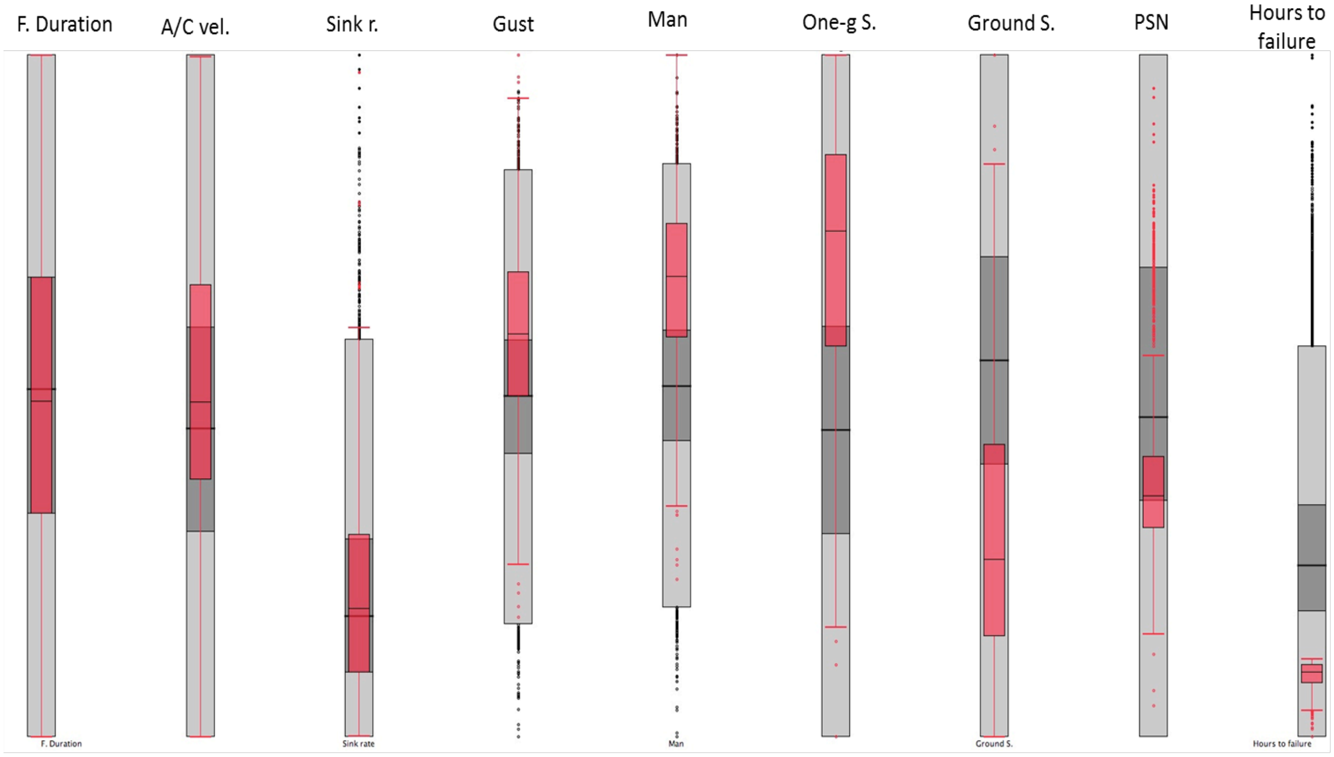

Figure 16. Parallel box plots LT with damage coefficient equals to 1 (ASTM curve)

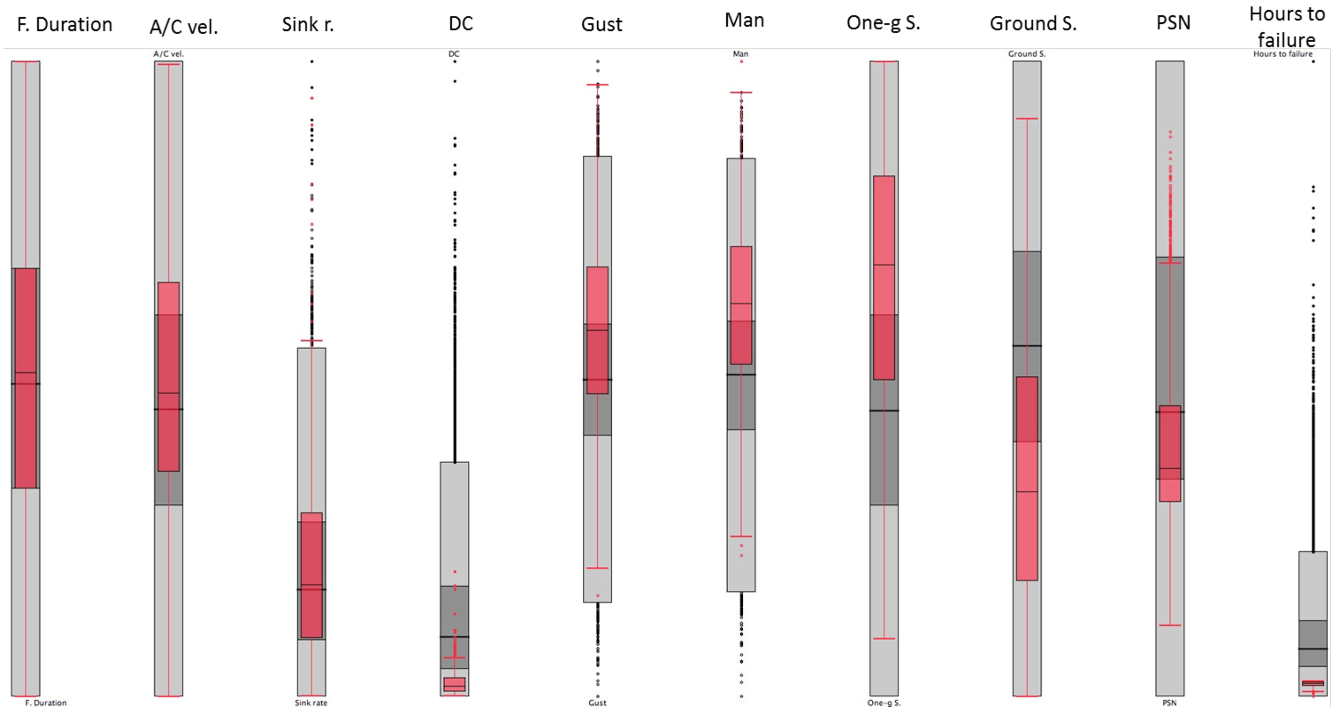

Figure 17. Parallel box plots LT with damage coefficient random (ASTM curve)

D. Global Sensitivity Analysis

To compute the indices corresponding to the global sensitivity, one variable was fixed to a specific value, and the rest of them were generated randomly (2000 samples) and the mean of the life was computed. Then, another specific value was generated and fixed for the same variable, and the mean of the life was computed again with the same number of samples. This process was repeated 2000 times, and the variance of the means of life computed previously (2000 in total), was computed and compared with the total variance of the system. This ratio is the first order index and represents the influence of the variable over the model.

Table 5 to Table 8 shows the normalized values for the first, second and third order indices for all the cases studied. These indices represents the variance provided by the variable itself (or in combinations with others), to the total variance of the response. These indices show the importance of the variable; the higher the index the more important the variable is. Fig. 18 to Fig. 21 shows pie charts of the indices, for all the cases. The indices are a number between zero and one, but the percentage values were used in order to represent the results better.

It is observed that the first order indices, for cases were damage coefficient was deterministic, (equals to one), represented around 80-90 % of the variance of the model, meanwhile for the cases where damage coefficient was random, the first order indices represented above 90% of the total variance. This is due to the variance coming from the damage coefficient.

From the global sensitivity analysis results, it is observed that for the open hole cases (with damage coefficient random and fixed), one–g stress, gust, and maneuver loads were the variables with the higher influence over the variance of the model, by themselves or through interaction terms. This fact indicates the importance of these variables for this case. When damage coefficient was a random variable, it affected the variance considerably, which indicates that is also an important variable. PSN and sink rate did not have any influence over the variance of the system by themselves, only through interactions terms.

For the load transfer cases, one –g stress, gust, and maneuver loads were the variables with the higher influence over the variance of the model when the damage coefficient was fixed. Their influence was about the same, as the corresponding open hole case. However, when damage coefficient was random, this last one and PSN became the most important variable, due to influence over the variance. One–g stress, gust, and maneuver loads had some influence over the variance of the response, but not as high as in the previous cases. Below is a detailed description of the results obtained.

1. *Open hole DC=1*

The first order terms represented 80% of the total variance of the system, the second order terms 65% and the third order terms 14%, of the total variance of the model. The dominant variable was Group 1, conformed by one-g stress, flight velocity, and flight duration. This group contributed 45% of the total variance. Maneuver and gust loads (22% and 13% respectively) were also important variables. These three variables also had interaction terms between them, with some importance (Group1, gust and maneuver factors, 8%). PSN had some influence over the variance of the life, but through interactions terms (PSN and Maneuver factor, 3%). Sink rate had a similar situation (sink rate, gust and maneuver loads, 4%).

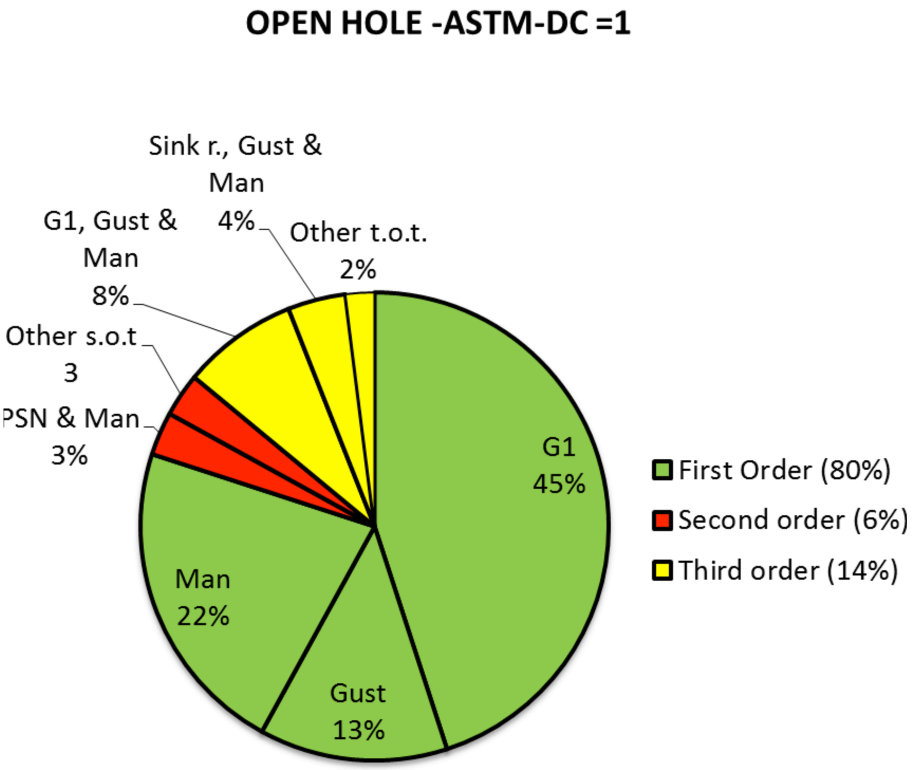

Figure 18. Pie chart OH damage coefficient equals to 1 (ASTM curve)

Table 5. Sensitivity indices obtained for OH, damage coefficient equals to 1 (ASTM curve)

Variable	First order indices, S_i
Flight duration, Flight velocity, and One-g stress	0.45
Sink rate	0.00
PSN and Damage coefficient	0.00
Gust load	0.13
Maneuver load	0.22
Total Si	**0.80**
	Interaction terms , S_{ij}
PSN and maneuver load	0.03
Other second order terms	0.03
	Interaction terms , S_{ijk}
Group 1, gust and maneuver loads	0.08
Sink rate, gust, and maneuver loads	0.04
Other third order terms	0.02
Total	**1.00**

2. *Open hole DC random*

The first and second order terms signified a 93% and 7% of the total variance of the system, respectively. Group 1, gust and maneuver loads conserved the same percentage of variance (45%, 12% and 23%) of the previous case. Damage coefficient represented 13% of the variance. The interaction terms (second order in this case) are formed by Group 1, gust and maneuver factors.

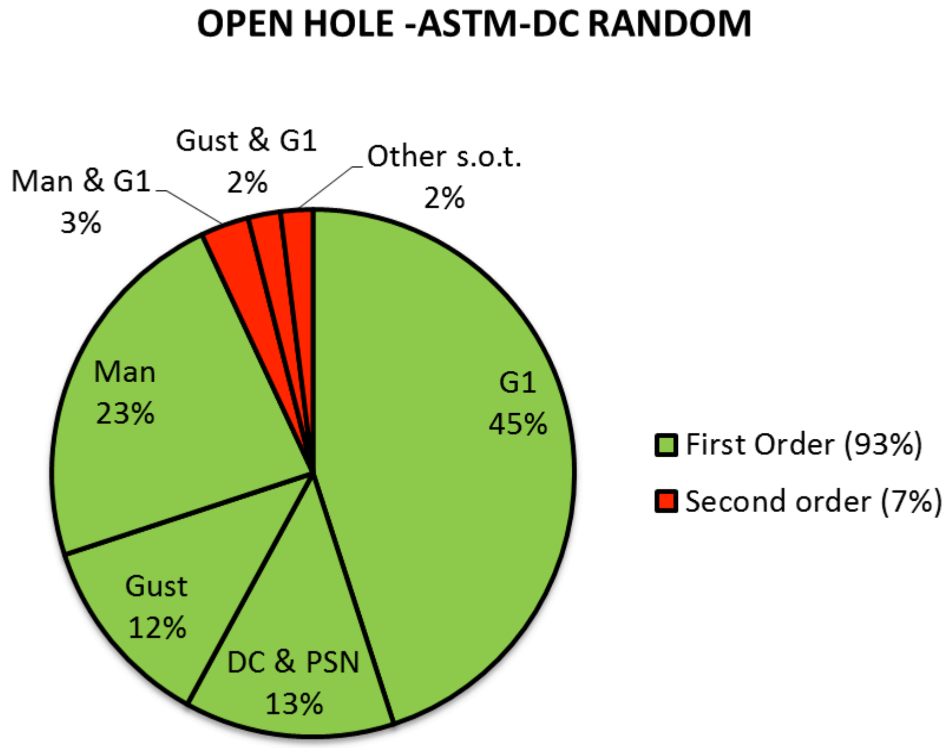

Figure 19. Pie chart OH damage coefficient random (ASTM curve)

Table 6. Sensitivity indices obtained for OH, damage coefficient random (ASTM curve)

Variable	First order indices, S_i
Flight duration, Flight velocity, and One-g stress	0.45
Sink rate	0.00
PSN and Damage coefficient	0.13
Gust load	0.12
Maneuver load	0.23
Total *Si*	**0.93**
	Interaction terms , S_{ij}
G 1and maneuver load	0.03
G1 and gust load	0.02
Other second order terms	0.02
Total	**1.00**

3. *Load transfer DC=1*

The first order terms represented 89%, the second order terms 9% and the third order terms 2% of the total variance of the system. Group 1, gust and maneuver loads were the most important variables (49%, 16%, 24%); theirs indices increased compared with the corresponding open hole case. These terms also affected the life through interaction terms. PSN had a low influence over the variance through an interaction term (PSN and Group 1, 3%).

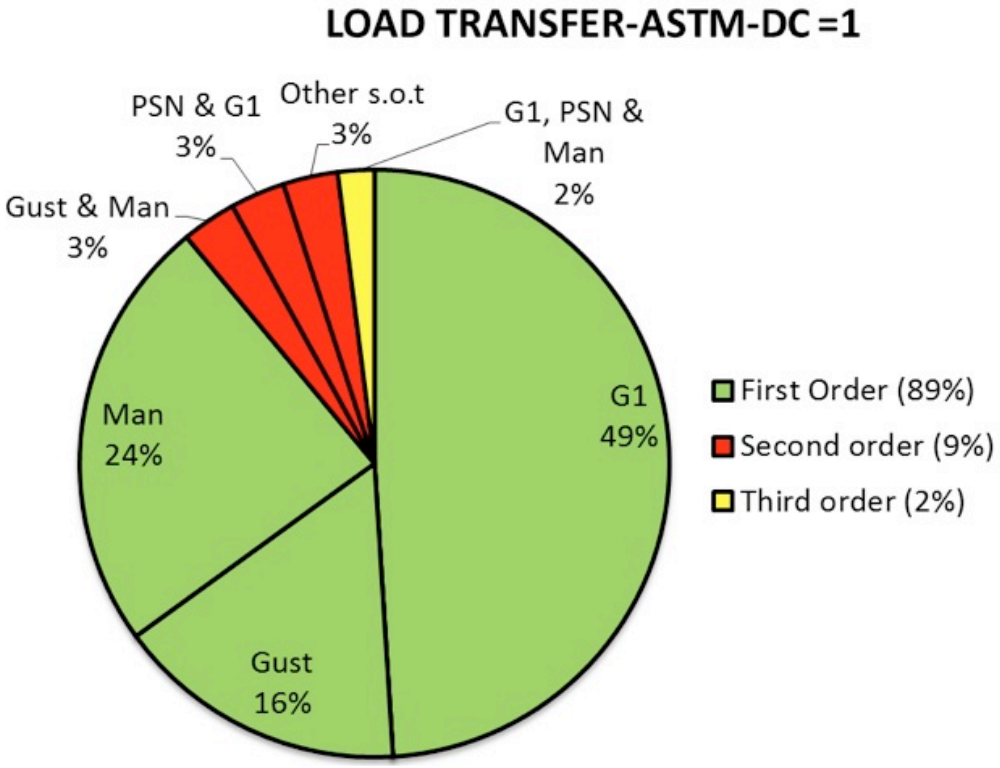

Figure 20. Pie chart LT damage coefficient equals to 1 (ASTM curve)

Table 7. Sensitivity indices obtained for LT, damage coefficient equals to 1 (ASTM curve)

Variable	First order indices, S_i
Flight duration, Flight velocity, and One-g stress	0.49
Sink rate	0.00
PSN and Damage coefficient	0.00
Gust load	0.16
Maneuver load	0.24
Total *Si*	**0.89**
	Interaction terms , S_{ij}
Gust and maneuver load	0.03
PSN and G1	0.03
Other second order terms	0.03
	Interaction terms , S_{ijk}
Group 1, PSN and maneuver loads	0.02
Total	**1.00**

4. *Load transfer DC random*

In the load transfer case, 97 % of the variance was represented by the firs order terms, where Group2, conformed by damage coefficient and PSN, was the most important variable (75%). Group1, gust and maneuver loads were important variables, but with less influence than in previous cases (12%, 4%, 6%). The third order terms represented 3% of the variance, being the most relevant term, the one formed by maneuver and Group 1 (2%).

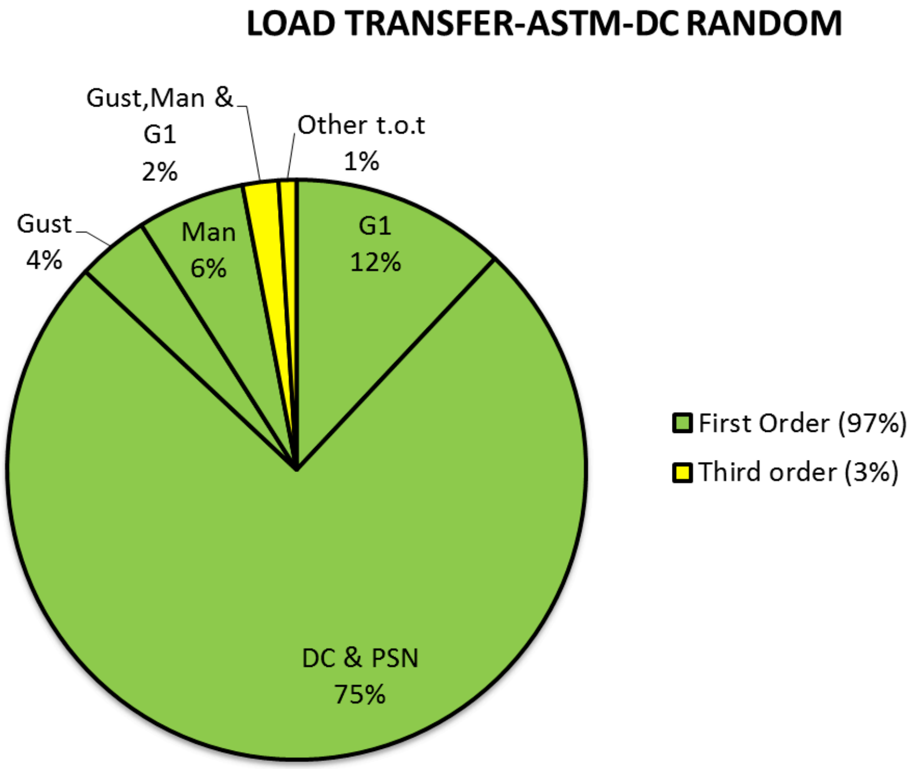

Figure 21 Pie chart LT damage coefficient equals random (ASTM curve)

Table 8. Sensitivity indices obtained for LT, damage coefficient random (ASTM curve)

Variable	First order indices, S_i
Flight duration, Flight velocity, and One-g stress	0.12
Sink rate	0.00
PSN and Damage coefficient	0.75
Gust load	0.04
Maneuver load	0.06
Total *Si*	**0.97**
	Interaction terms , S_{ijk}
Group 1, gust and maneuver loads	0.02
Other third order terms	0.01
Total	**1.00**

To obtain a better understanding of the model, the same four cases described above were repeated using a PSN polynomial curve. The comparison between the ASTM and the polynomial curves are presented Fig. 22. It can be observed that PSN and damage coefficient are the most important variables for the polynomial case, for both open hole and load transfer. Group 1, gust and maneuver loads still had influence over the life, but this importance decreased substantially.

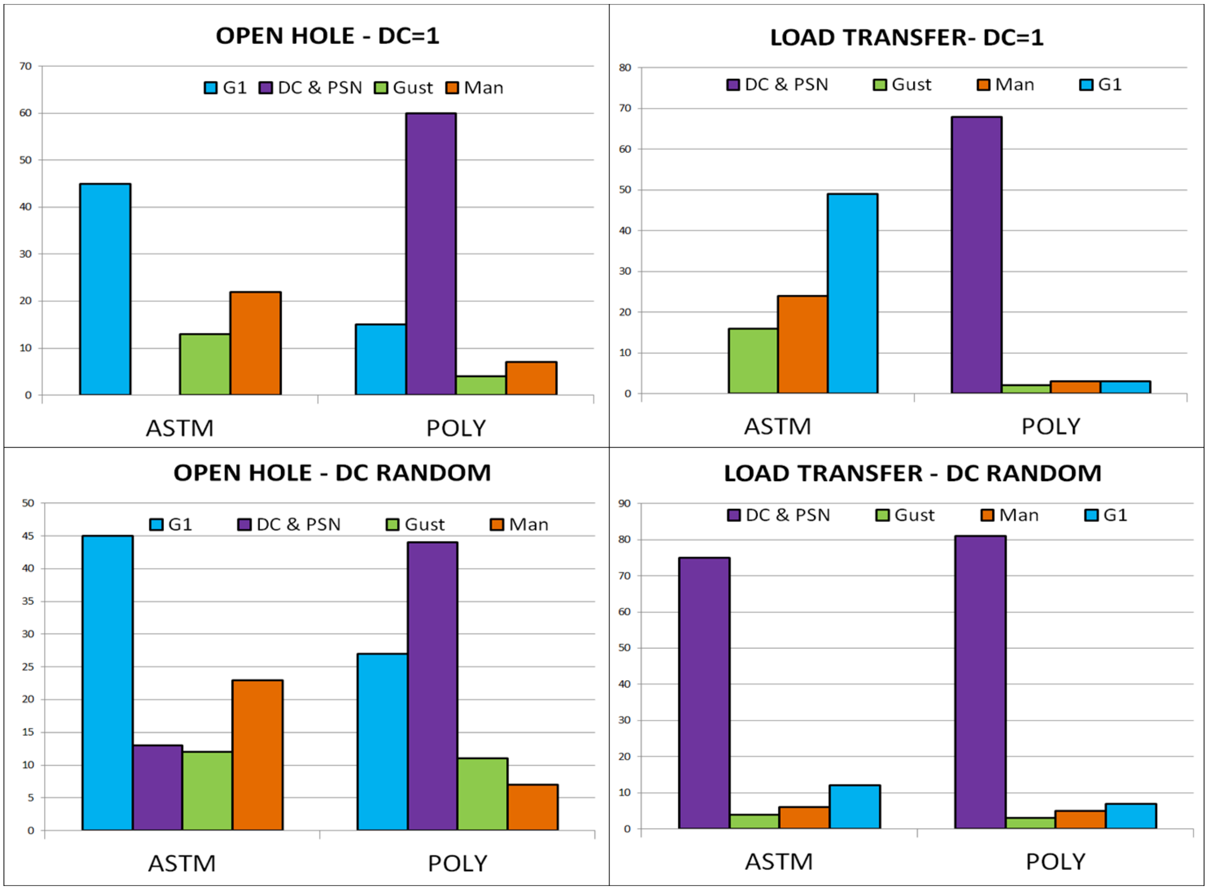

Figure 22. Comparison between ASTM and polynomial curves. *a) OH damage coefficient equals to 1, b) OH damage coefficient random, c) LT damage coefficient equals to 1, d) LT damage coefficient random*

VI. Summary Remarks and Conclusions

A sensitivity analysis was used to analyze the influence of the variables over the life in a risk assessment problem for general aviation using the software SMART. The methods used for the sensitivity analysis were scatter plots, box plots, and global sensitivity.

The results obtained from the three sensitivity methods were consistent, indicating that the variables that most affect the life are one-g stress, ground stress, gust and maneuver loads, PSN, and damage coefficient. Sink rate does not directly affect the life; it affects the life through some interaction terms, but this interaction is low compared to the other variables.

For the open hole case the one-g stress, gust loads, and maneuver loads are the most important variables. PSN and damage coefficient had more importance for the load transfer case, especially for the polynomial case.

The relationship between life in airplanes and the input variables is complex, but a sensitivity analysis provided very valuable insights in understanding the behavior of the model.

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VIII. References

[1] FAA Report AFS120-73-2, “Fatigue Evaluation of Wing and Associated Structure on Small Airplanes.” 1973.

[2] FAA Report AC23-13A, “Fatigue Fail-Safe, and Damage Tolerance Evaluation of Metallic Structure for Normal, Utility, Acrobatic, and Commuter Category Airplanes.” 2005.

[3] Golden, P., Millwater, H. and Yang, X.,“Probabilistic fretting fatigue life prediction of Ti-6Al-4V”. International Journal of Fatigue, 2010. 32(12): p. 1937-1947.

[4] Millwater, H., “Universal properties of kernel function for probabilistic sensitivity analysis”, Probabilistic Engineering Mechanics, 2009. 24(1): p. 89-99.

[5] Millwater, H., and Wieland, D. ,”Probabilistic sensitivity-based ranking of damage tolerance analysis elements”, Journal of aircraft, 2010. 47(1):p. 161-171.

[6] Millwater, H., Bates,A., and Vazquez, E.,” Probabilistic sensitivity methods for correlated normal variables”, International Journal of reliability and Safety, 2011. 5 (1): p. 1-20

[7] Nuss, M., Shiao, M., “Probabilistic Approach to fleet Risk Managements of Small Airplanes.” Aging Aircraft Conference, Atlanta GA, March 2006.

[8] Ocampo, J. and Millwater, H. SMART User manual. 2010, version 1.0

[9] Ocampo,J., Millwater,H., Singh, G., Smith, H., Abali, F., Nuss, M., Reyer, M., and Shiao,M., “Development of a Probabilistic Linear Damage Methodology for Small Aircraft”, Journal of aircraft, 2011. 48(6):p. 2090-2106.

[10] Reyer W. M., “Probability Basis of Safe-Life Evaluation in Small Airplanes.” Aging Aircraft Conference, 2006. Atlanta, March 2006.

[11] Saltelli,A., Campolongo,T., Gatelli, J., and Tarantola, M. , Global sensitivty anallysis. The primer. 1st Edition ,2008.Wiley

[12] Saltelli,A., Campolongo,T., Gatelli, J., Tarantola, S ,and Ratto, M. , Sensitivty anallysis in practice.. 1st Edition ,2005.Wiley