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Probabilistic Risk Assessment for Small Airplanes

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This paper describes the development of a probabilistic methodology that can perform risk assessment of small airplanes. The objective was to develop a comprehensive probabilistic methodology to allow Federal Aviation Administration (FAA) engineers to conduct a risk assessment of general aviation (GA) structural issues in support of policy decisions. Requisite-supporting technology and data issues, in particular, probability distributions of relevant inputs, were investigated and developed so that a realistic risk assessment could be obtained. Example problems are presented to demonstrate the methodology that includes the calculation of flights (or hours)-to-failure and the probability-of-failure for a specified number of flying hours. Representative sensitivity studies were also conducted to determine significant variables.

Nomenclature

FAA	=	Federal Aviation Administration
GA	=	General Aviation
PDF	=	Probability Density Function
S-N	=	Stress-Life
MPI	=	Messaging Passing Interface

I. Introduction and Overview

IN 1991, Congress mandated that the FAA establish an Aging Aircraft Program. The focus of this program was age-related structural problems with airplanes used in public transportation. At the time, Congress excluded the GA fleet from the mandate. However, the FAA determined that as the GA fleet continues to age, there is a concern about ensuring the continued airworthiness of the diverse GA fleet. To guide future efforts in addressing the effects of aging on GA airplanes, the Small Airplane Directorate developed an FAA Aging GA Roadmap that serves as a guide to proactively manage the overall airworthiness of aging GA airplanes. One of the four major focus areas of the Roadmap is data-driven risk assessment and risk management. As a result, a research and development program

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was undertaken to develop the required methodology, computer software, and supporting data to conduct structural risk assessments.

II. Methodology

In many applications, fatigue life evaluation of structural components is conducted using a deterministic approach, and GA is not an exception. However, due to the number of uncertainties present and their impact on structural integrity a probabilistic approach is needed[1,2]. In fatigue life assessments, both the material properties and the load characteristics are essential random variables and may exhibit significant variability.

The methodology in this work encompasses the required elements necessary to conduct a structural integrity evaluation, and moreover, considers real-world, airplane-to-airplane and flight-to-flight variations such that a realistic risk assessment of an aircraft structural detail can be performed. Table 1 shows a summary of the variables used to conduct the risk assessment.

Variable	Type
Gust/Maneuver Load exceedances	Probabilistic: (lognormal distributions at different acceleration fractions levels)
Aircraft Velocity and Flight Duration	Probabilistic: (Joint pdf with correlated variables)
Sink Rate	Probabilistic
Damage Index	Probabilistic: (normal or Weibull distribution)
Maneuver Load Limit Factors	Deterministic
Gust Load Limit Factors	Deterministic
Ground Stress	Deterministic
One-g Stress	Deterministic

Table 1. Code Variable Classification.

To perform a risk evaluation, two different methodologies that follow the FAA guidelines used for safe-life evaluation (AFS-120-73[3] and Advisory Circular (AC) 23-13A[4]) were incorporated in a computer code. The first methodology calculates the flights/hours-to-failure or the safe-life (time to crack initiation) for GA and this methodology is explained step by step as follows:

- Variables such as airplane usage, load limit factors, ground stress, one-g stress, airplane velocity, and flight length are input by the user.
- According to the airplane usage, e.g., single-engine unpressurized Instructional, pressurized usage, twin-engine general usage, etc. the respective data (exceedance curves, sink rate data, etc.) are loaded from internal libraries.
- Realizations of the random variables such as: sink rate velocity, airplane velocity, flight duration, etc. needed for Monte Carlo sampling are generated. A weighted mix of usages is allowed.
- For each Monte Carlo sample, the code generates a characteristic stress spectrum that includes the flight stages: gust, maneuver, taxi, ground-air-ground, and landing and rebound. The methodology process through this point is shown in Figure 1.
- Damage is accumulated for each Monte Carlo sample using Miner's rule until Miner's critical value is reached and flights/hours-to-failure is recorded as shown in Figure 2.
- When the Monte Carlo sampling is finished, the random variables and flights/hours-to-failure are post-processing to determine the distribution of flights/hours-to-failure (mean, standard deviation, confidence intervals) and to identify the significant random variables.

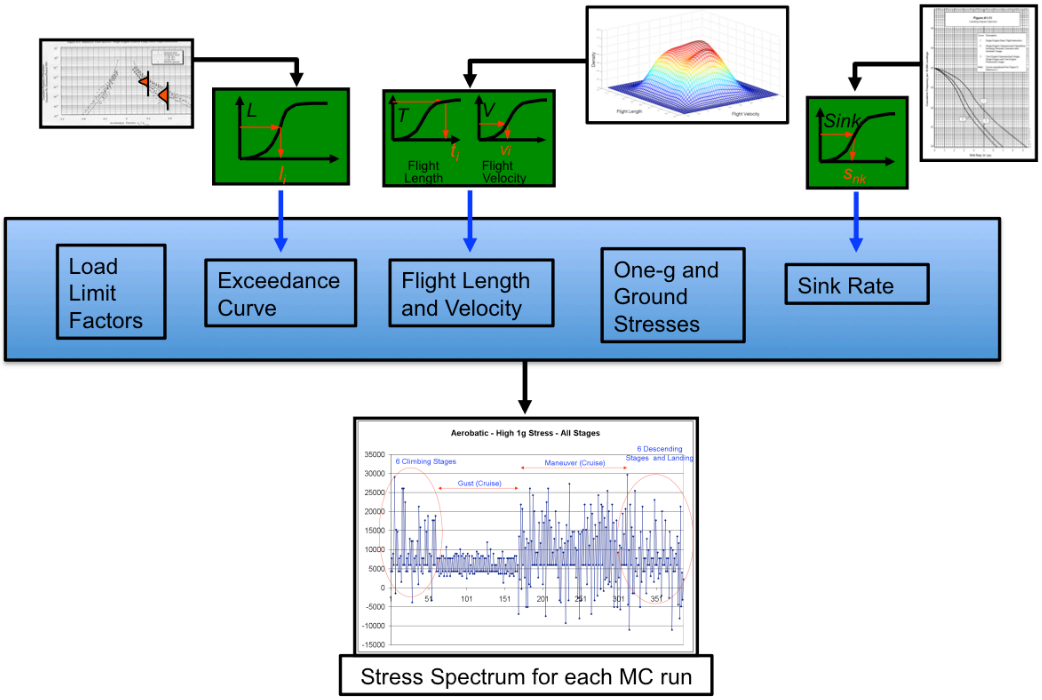

Figure 1. Schematic of Risk Assessment Methodology for the Spectrum Generation.

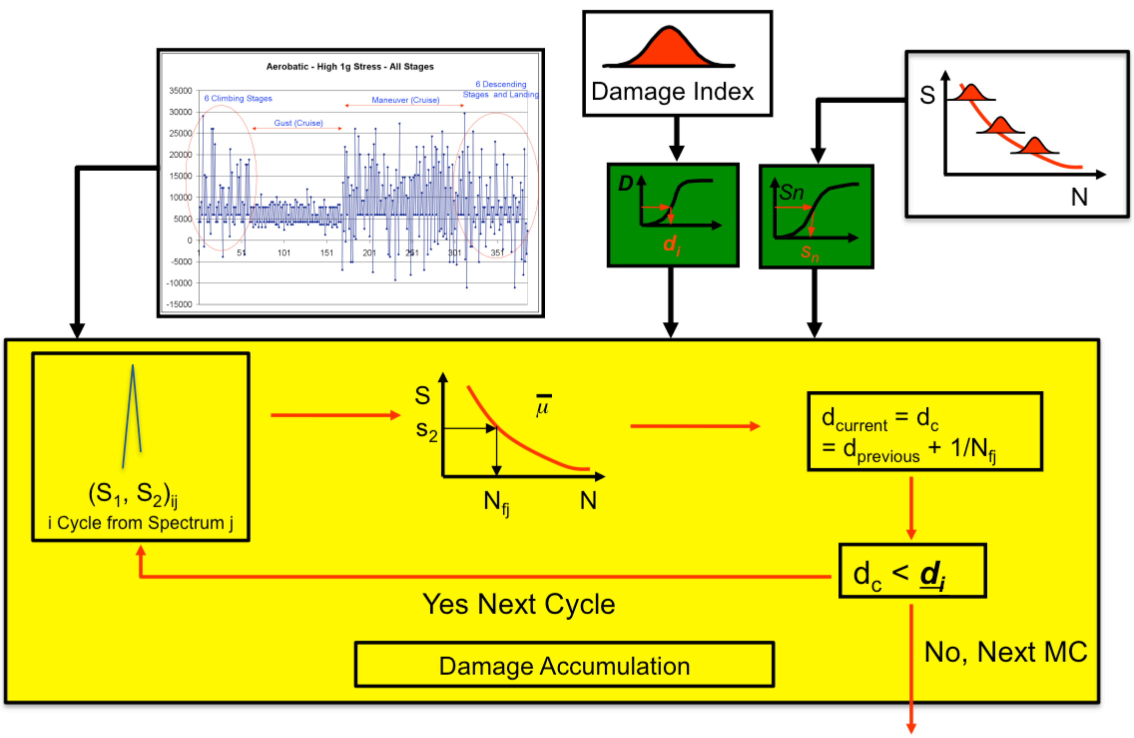

Figure 2. Schematic of Risk Assessment Methodology for the Damage Accumulation.

The second methodology calculates the accumulated damage condition and the probability-of-failure given any number of hours flown by the airplane. The methodology is explained as follows:

- The same steps showed for the first methodology to generate the stress spectrum are used in this methodology – See Figure 1.

- Damage is accumulated for each Monte Carlo sample until the flight hours specified by the user is reached. The accumulated damage is recorded as shown in Figure 3.
- If the damage recorded is larger than the random Miner's damage coefficient generated for that sample, a failure is counted.
- When the Monte Carlo sampling is finished, the random variables, the accumulated damage, and the failures are post-processed in order to obtain the probability-of-failure and the significant random variables.

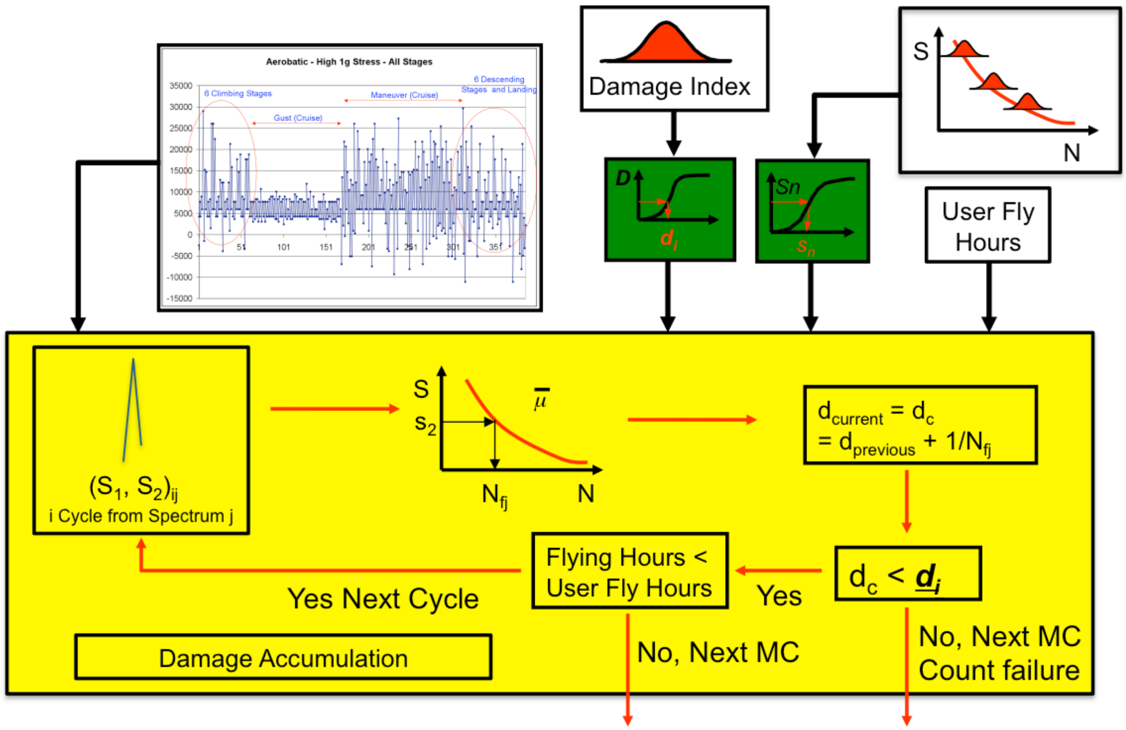

Figure 3. Schematic of Risk Assessment Methodology for Damage Accumulation.

Given the significant airplane-to-airplane and flight-to-flight variations, an essential ingredient of the methodology was to investigate, develop, and model probability distribution functions (PDFs) of the critical input data such as flight duration, aircraft speed, sink rate velocity, the damage index coefficient, etc. Probabilistic S-N curves are being developed using data developed under an experimental program conducted by Wichita State University under a separate program.[6, 7]

Sensitivity analysis is an essential ingredient of a risk assessment. The Monte Carlo sampling results are post-processed to predict the risk of failure and the associated sensitivities. The sensitivities indicate the relative importance of the inputs on the life estimation. Various sensitivity methods are available such as scatter plots, segmented PDFs, regression, and others.[8]

The Monte Carlo sampling risk assessment methodology is well suited to parallel implementation. Therefore, the Monte Carlo samples were distributed to multiple processors using the OpenMP and MPI parallel methods.[9, 10] Significant speed-ups were obtained for both methods; 6.64 using 8 processors with OpenMP and 87.5 using 96 processors with MPI.

III. Numerical Examples

Two numerical examples are presented to demonstrate the methodology. For the first example, the airplane was assumed to have flown in a mixed usage, first Instructional usage and then Personal usage in equal periods of time: 50 percent in Instructional usage and 50 percent in Personal usage. For the second example, the airplane was assumed to have flown for 10,800 hours on Instructional usage and has been flying for 2,000 hours on Personal usage thereafter.

Probability distributions for the inputs have been investigated and developed for both usages. Table 2 and Figure 4 show correlated flight length and flight velocity data for Instructional usage. Table 3 and Figure 5 show flight length and flight velocity data for Personal usage. Exceedance curves and sink rate data have been developed from the FAA report AC-23-13A.[4]

		Average Speed During Flight, % Design Velocity						
Flight time (Hours)	% of Flights	1.00	0.95	0.90	0.85	0.80	0.75	0.70
0.25	0	0	0	0	0	0	0	0
0.50	0.05	0	0	0.05	0.25	0.6	0.1	0
0.75	0.15	0	0	0.25	0.4	0.3	0.05	0
1.00	0.35	0.05	0.15	0.45	0.3	0.05	0	0
1.25	0.1	0.05	0.15	0.45	0.3	0.05	0	0
1.50	0.1	0.05	0.3	0.5	0.15	0	0	0
1.75	0.2	0.05	0.3	0.5	0.15	0	0	0
2.00	0.05	0.15	0.55	0.2	0.1	0	0	0

Table 2. Flight Length and Airspeed Data for Instructional Usage.

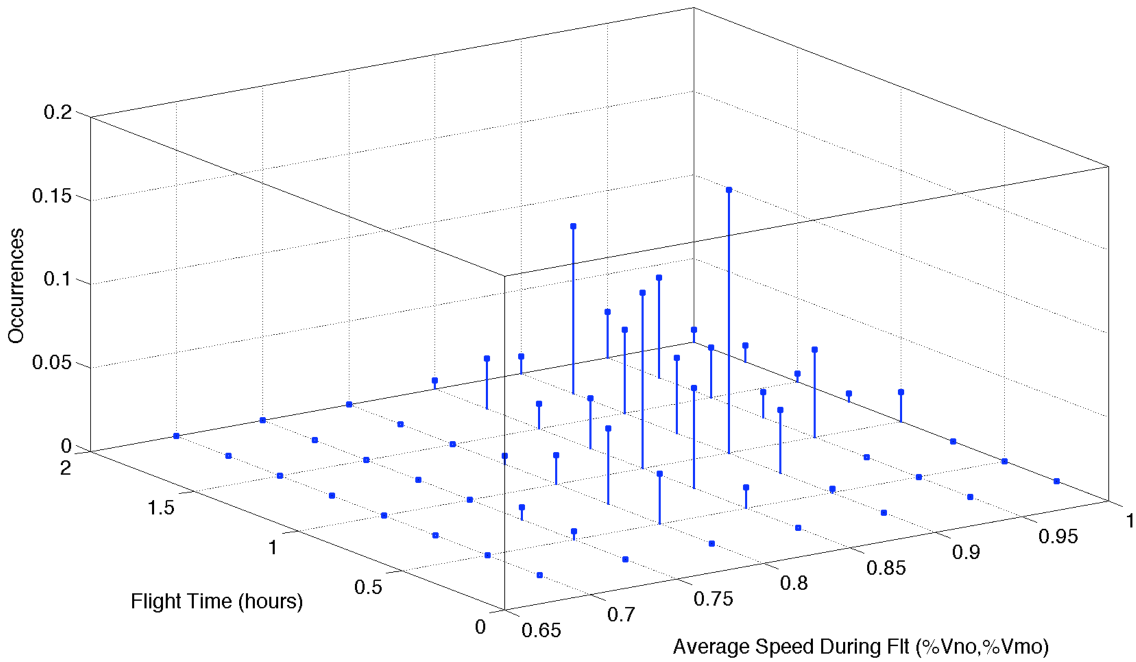

Figure 4. Flight Lengths and Flight Velocity Joint PDF for Instructional Usage.

		Average Speed During Flight, % Design Velocity					
Flight time (Hours)	% of Flights	1.00	0.95	0.90	0.85	0.80	0.75
1.00	0.35	0.05	0.15	0.45	0.3	0.05	0
1.25	0.1	0.05	0.15	0.45	0.3	0.05	0
1.50	0.1	0.05	0.3	0.5	0.15	0	0
1.75	0.2	0.05	0.3	0.5	0.15	0	0
2.00	0.05	0.15	0.55	0.2	0.1	0	0

Table 3. Flight Length and Airspeed Data for Personal Usage.

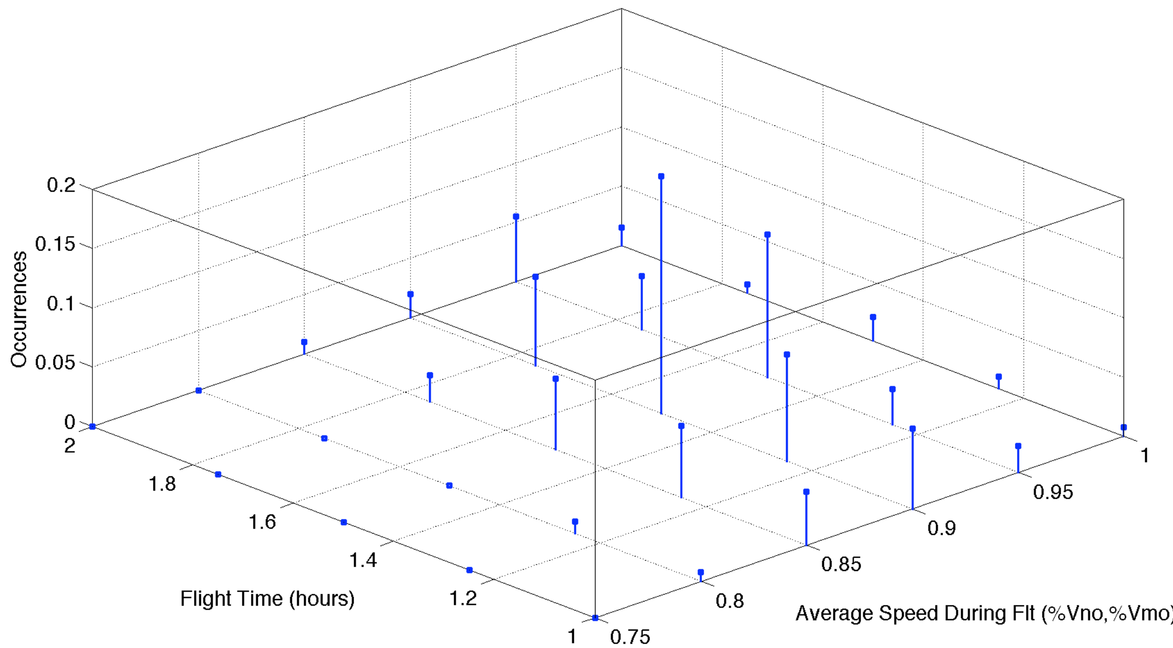

Figure 5. Flight Lengths and Flight Velocity Joint PDF for Personal Usage.

A. Example One

A safe-life and sensitivity analysis was conducted to determine the flights/hours-to-failure taking into account both usages. The data used for this analysis is contained in Table 4.

Variable	Characteristics	
Gust/Maneuver Load exceedances	Probabilistic exceedances curves for Instructional and Personal usage were taken from reference 3.	
Sink Rate	Sink rate values were taken from reference 5.	
Maneuver Load Limit Factors	Instructional Usage	+2.80 -2.50
	Personal Usage	+2.40 -2.20
Gust Load Limit Factors	Instructional Usage	+2.15 -2.15
	Personal Usage	+2.30 -2.30
One g stress	Instructional Usage	+7410
	Personal Usage	+7900
Ground Stress	Instructional Usage	-4520
	Personal Usage	-4800
Aircraft Velocity	Instructional Usage	160
	Personal Usage	170
Damage Index	Normal distribution with mean 1.0 and standard deviation 0.2	

Table 4. Analysis Data.

This analysis was performed using a weighted usage analysis, i.e., it was assumed that the airplane was used 50 percent of the time in Instructional usage and 50 percent of the time in Personal usage. A total of 20,000 samples were run and Table 5 shows the statistical results obtained from the analysis. Probability plotting versus different PDFs are shown in Figure 6. The flights-to-failure distribution clearly follows a lognormal distribution. The PDFs of flights-to-failure and hours-to-failure are shown in Figure 7 and Figure 8, respectively.

The sensitivity analysis using correlation coefficients is shown in Table 6 for both usages (Instructional and Personal), in Table 7 only for Personal usage, and in Table 8 only for Instructional usage. Higher correlation coefficients indicate more importance. The results show all variables except sink rate are significant. For Personal usage, the gust is more significant relative to maneuver, whereas for Instructional usage the reverse is true.

95% Confidence Interval	Flights-to-Failure Mean	95% Confidence Interval
17,310	17,431	17,551
95% Confidence Interval	**Hours-to-Failure Mean**	**95% Confidence Interval**
21,214	21,321	21,427
95% Confidence Interval	**Flights-to-Failure Standard Deviation**	**95% Confidence Interval**
8,634	8,704	8,777
95% Confidence Interval	**Hours-to-Failure Standard Deviation**	**95% Confidence Interval**
7609	7,672	7,736

Table 5. Safe-life Analysis Results.

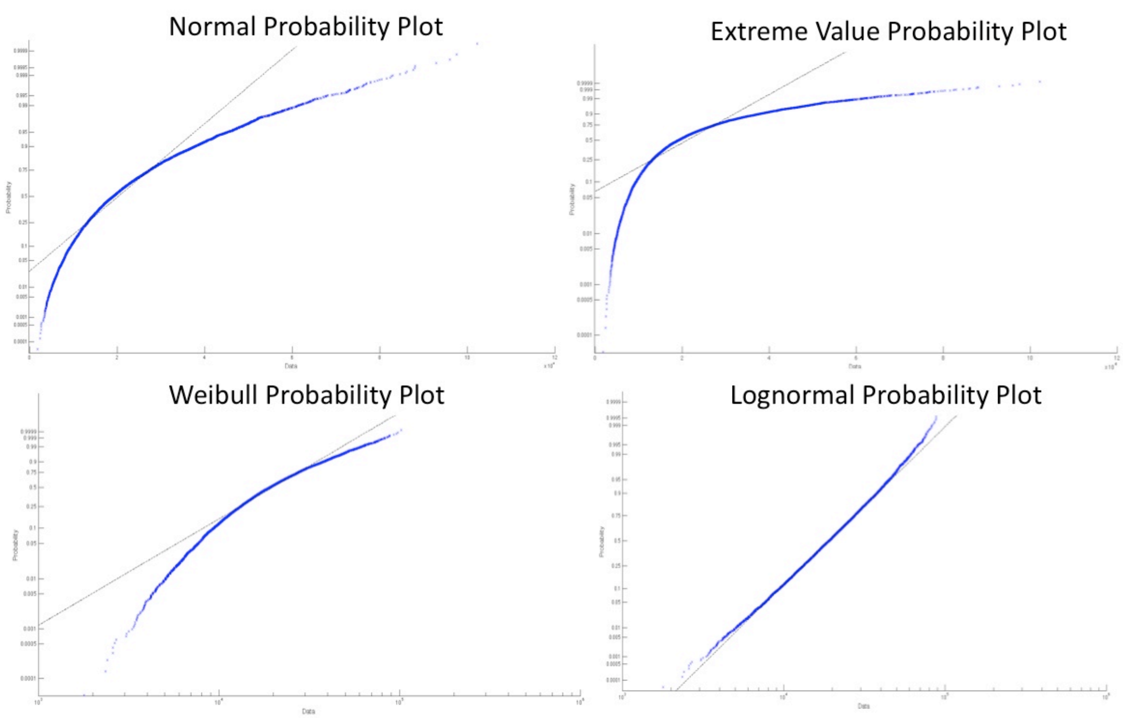

Figure 6. Flights-to-Failure Probability Plot Testing.

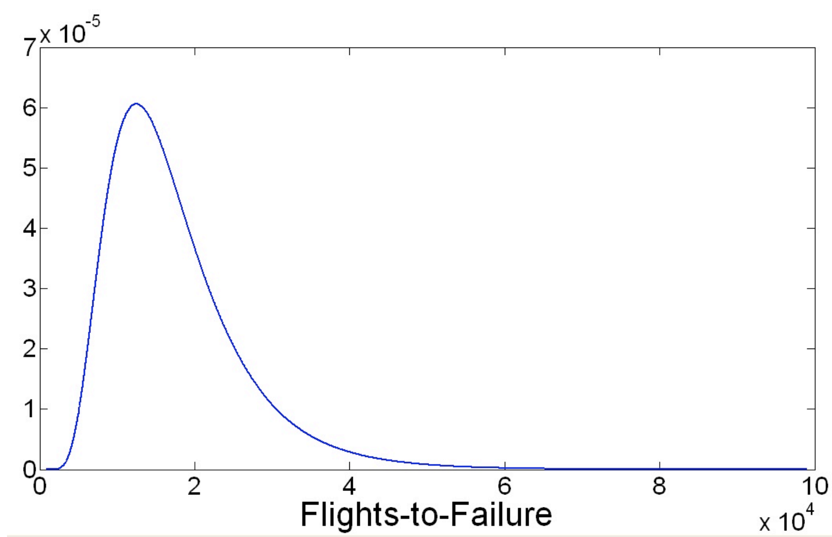

Figure 7. Flights-to-Failure PDF.

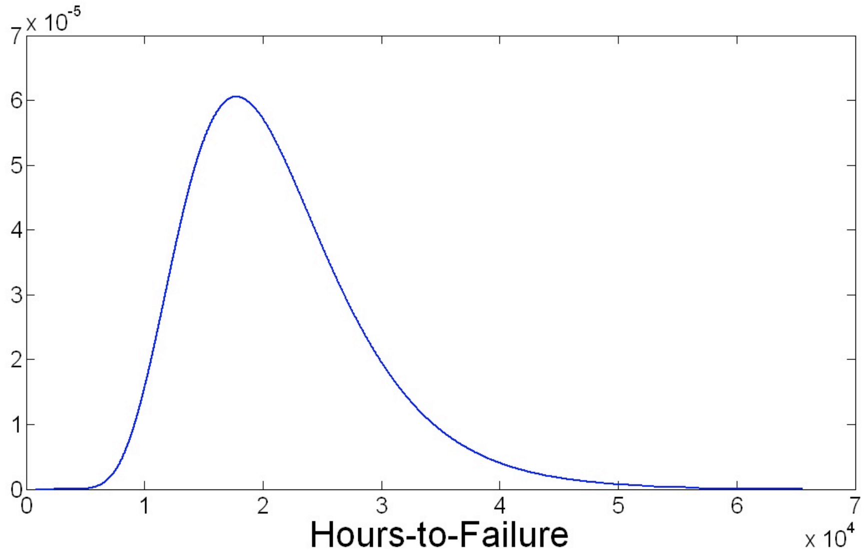

Figure 8. Hours-to-Failure PDF.

	Flights-to-failure	Flight Length	Flight Speed	Sink Rate	Miner's Coefficient	Gust Factor	Maneuver Factor
Flights-to-failure	1	-0.5988	-0.4356	-0.0288	0.4068	0.3389	0.3936

Table 6. Correlation Analysis Personal and Instructional Usage.

	Flights-to-failure	Flight Length	Flight Speed	Sink Rate	Miner's Coefficient	Gust Factor	Maneuver Factor
Flights-to-failure	1	-0.4716	-0.2262	-0.0403	0.5423	0.5851	0.2401

Table 7. Correlation Analysis Personal Usage.

	Flights-to-failure	Flight Length	Flight Speed	Sink Rate	Miner's Coefficient	Gust Factor	Maneuver Factor
Flights-to-failure	1	-0.5959	-0.4149	-0.0335	0.3882	0.2489	0.5279

Table 8. Correlation Analysis Instructional Usage.

Qualitative sensitivity analysis in the form of scatter plots is presented in Figure 9 where the red dots represent Instructional usage and blue dots represent Personal usage. The Y axis is the flights-to-failure. From the qualitative analysis it is clear that the red dots are concentrated more heavily towards early flights-to-failure than the blue dots. Hence one can conclude that Instructional usage is more severe than Personal usage. The quantitative analysis supports this conclusion because the correlation coefficient between the variables and flights to-failure is larger for Instructional usage than Personal usage. The one exception is the gust factor because the one-g stress for Personal usage was assumed to be larger than for Instructional usage (it was assumed larger because Personal usage could involve the carriage of more than two passengers and more fuel) making this variable more severe in Personal usage (compare Table 7 and Table 8).

Another important analysis can be done between flight stages (gust, maneuver, taxi, landing and rebound, and ground-air-ground). Figure 10 presents an estimated joint PDF between different stages obtained from the Monte Carlo sampling. This type of analysis helps to identify interactions between the different stages. Correlation coefficients between the damage from the different flight stages are presented in Table 9.

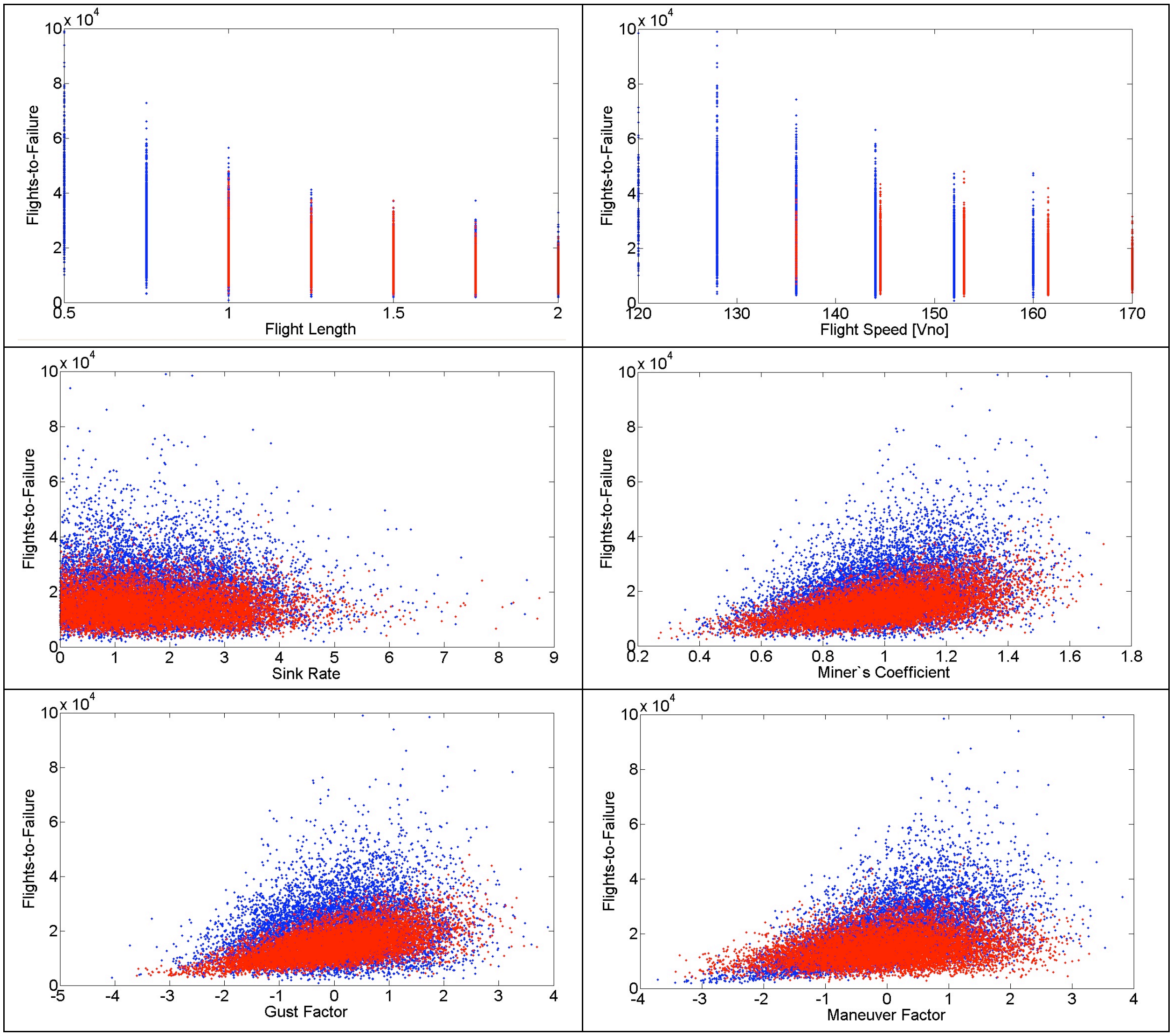

Figure 9. Correlation Analysis Instructional (Red) Personal (Blue).

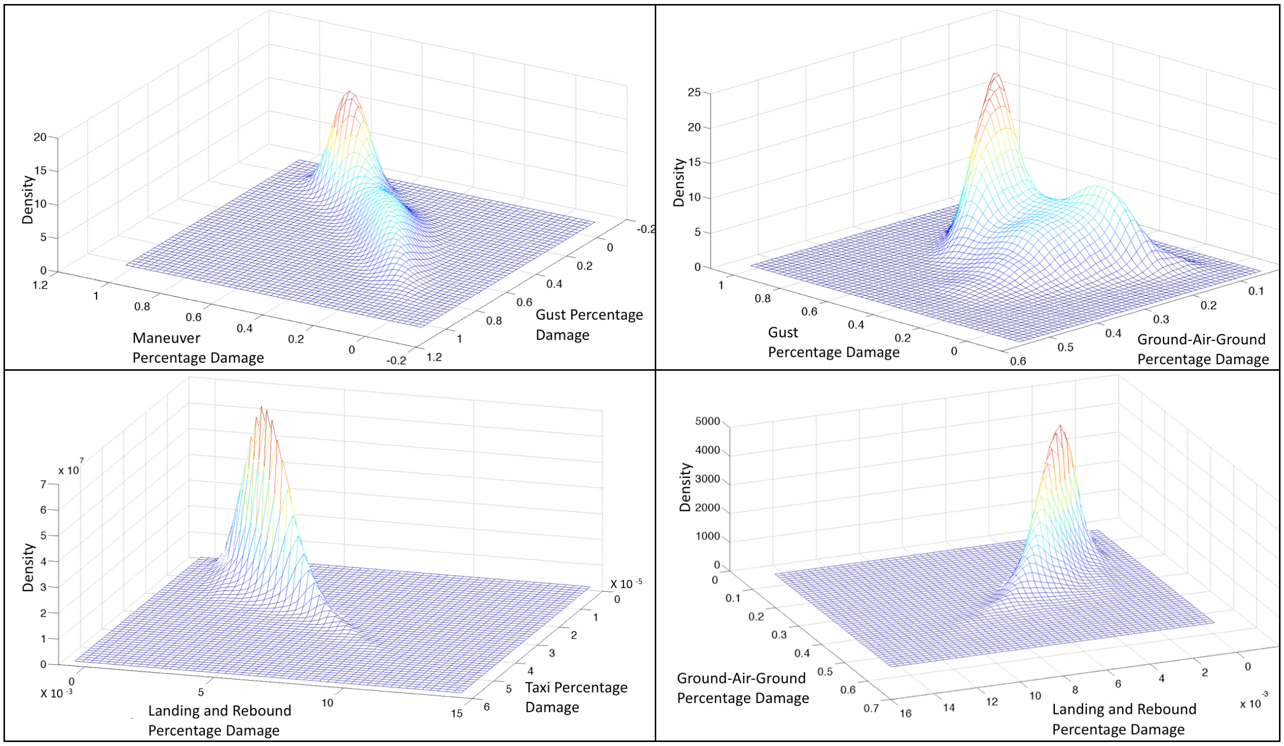

Figure 10. PDF Stages Analysis.

	Gust	Maneuver	Taxi	Landing and Rebound	Ground-air-Ground
Gust	1	-0.4599	-0.7063	-0.7321	-0.6827
Maneuver	-0.4599	1	-0.7961	-0.8473	-0.7799
Taxi	-0.7063	-0.7961	1	0.9553	0.9468
Landing and Rebound	-0.7321	-0.8473	0.9553	1	0.9508
Ground-air-Ground	-0.6827	-0.7799	0.9468	0.9508	1

Table 9. Correlation Stage Percentage Damage.

B. Example Two

Using the methodology where the user specifies a number of flying hours for each usage, the second analysis was conducted to calculate the probability-of-failure if the aircraft is flown for another 1,000 and 2,000 additional hours under the Personal usage, in addition to the 10,800 Instructional and 2,000 in Personal usage already flown. The main advantage of this analysis is to have estimate the risk (probability-of-failure) of an airplane flying any number of additional hours from its current condition. The data used for this analysis is the same as for example 1, see Table 4.

The distribution of accumulated damage and Miner's damage coefficient are presented in Figure 11. Results from the analysis showed an increment in the mean damage of about 7.8 percent from flying an additional 1,000 hours and about 15.6 percent from flying additional 2,000 hours. The damage increment is linear because the flight conditions did not change between 1,000 and 2,000 hours. Table 10 shows the change in the probability-of-failure for each of the three runs, showing an increment of the probability-of-failure when the number of flying hours increases, as expected.

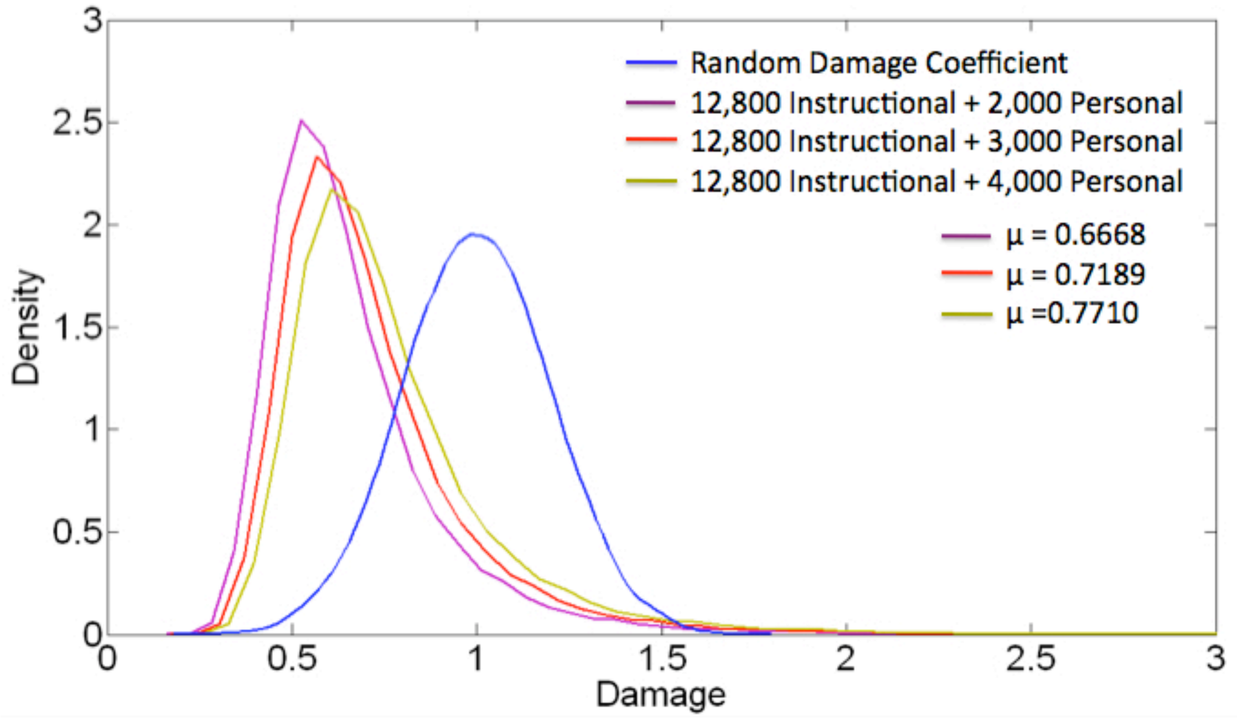

Figure 11. Mean Damage Results.

Current Condition 10,800 hours in Instructional Usage and 2,000 Hours in Personal Usage
0.1209
10,800 hours in Instructional Usage, 2,000 Hours in Personal Usage and an additional 1,000 in Personal Usage
0.1576
10,800 hours in Instructional Usage, 2,000 Hours in Personal Usage and an additional 2,000 in Personal Usage
0.1995

Table 10. Probability-of-Failure Results.

IV. Conclusion

Probabilistic fatigue evaluation of General Aviation aircraft is vital in order to provide important insight into the severity or criticality of a potential structural issue. For this reason, a probabilistic risk assessment methodology and computer software was developed such that FAA engineers can perform a risk assessment of a structural issue. Due to significant airplane-to-airplane and flight-to-flight variations, probability density functions of the critical variables were investigated and developed.

The methodology and software were demonstrated on a structural risk assessment example and the conclusions are presented as follows:

Sink rate does not play an important role on fatigue evaluation; therefore, further investigation of this variable would not be necessary. However, variables such as flight duration, Miner's coefficient, gust and maneuver loading are important to the problem. Correlation factors and scatter plots supported the importance of the different variables in the problem.

Flight distance and flight duration ranked high in the sensitivity analysis. Long flights and high velocities permit more occurrences of maneuver and gust loading, leading to early failures. Flight length is a big factor on the number of ground-air-ground cycles; since shorter flights increase the number of ground-air-ground cycles and consequently decrease the safe-life of the airplane.

From Figure 10 and Table 9, it is clear that high correlation occurs between gust and maneuver stages in flight percentage damage. This is because these flight stages share some of the inputs such as load limit factor and one g stress. The same relationship occurs between taxi, and landing and rebound that share the ground stress. Ground-air-ground shows high correlation with maneuver; the high correlation is because the maximum value for the ground-air-ground stage is always extracted from gust or maneuver. The flight stage (gust or maneuver) that presents the highest correlation with respect to ground-air-ground indicates that the maximum value was extracted from that flight stage. This correlation helps to show as well which stage between gust and maneuver contains the maximum peak stress. The high correlation with landing and rebound is because the minimum value for the ground-air-ground stage is always extracted from landing and rebound.

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