Probabilistic Damage Tolerance for Aircraft Fleets Using an Adaptive Crack Growth Fracture Mechanics Surrogate Model

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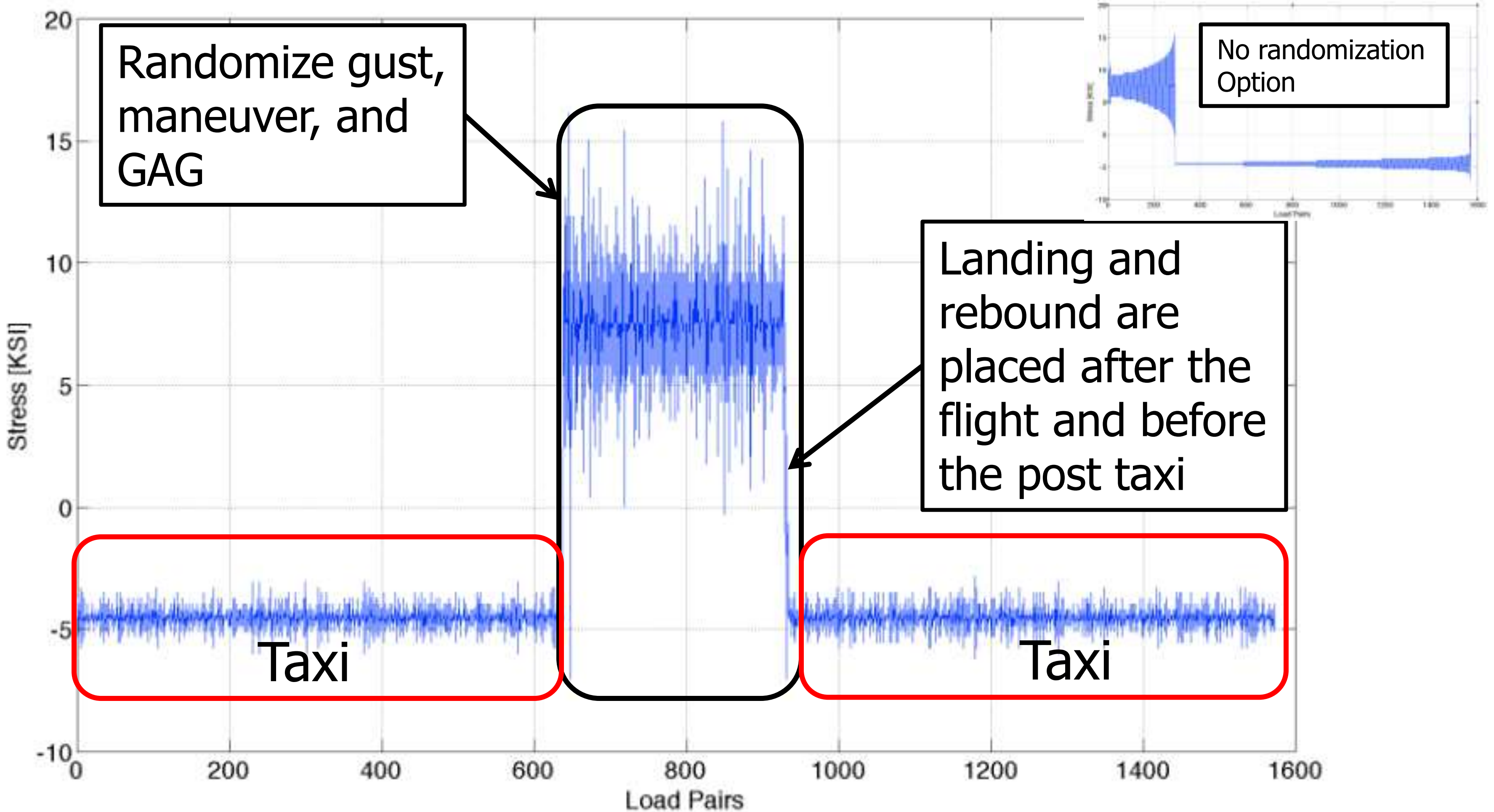
Motivation and Background

- Nowadays more aircraft fleets are using probabilistic damage tolerance analysis to ensure airworthiness.
- A comprehensive probabilistic damage tolerance method requires a combination of deterministic crack growth, inspection methods, probabilistic methods, and random variable modeling to provide probability-of-failure calculations.
- Crack growth evaluations are computational expensive, Kriging metamodels gives a solution to reduce the computational burden.

Methodology

Probabilistic Damage Tolerance for Small Airplanes



Randomize gust, maneuver, and GAG

No randomization Option

Landing and rebound are placed after the flight and before the post taxi

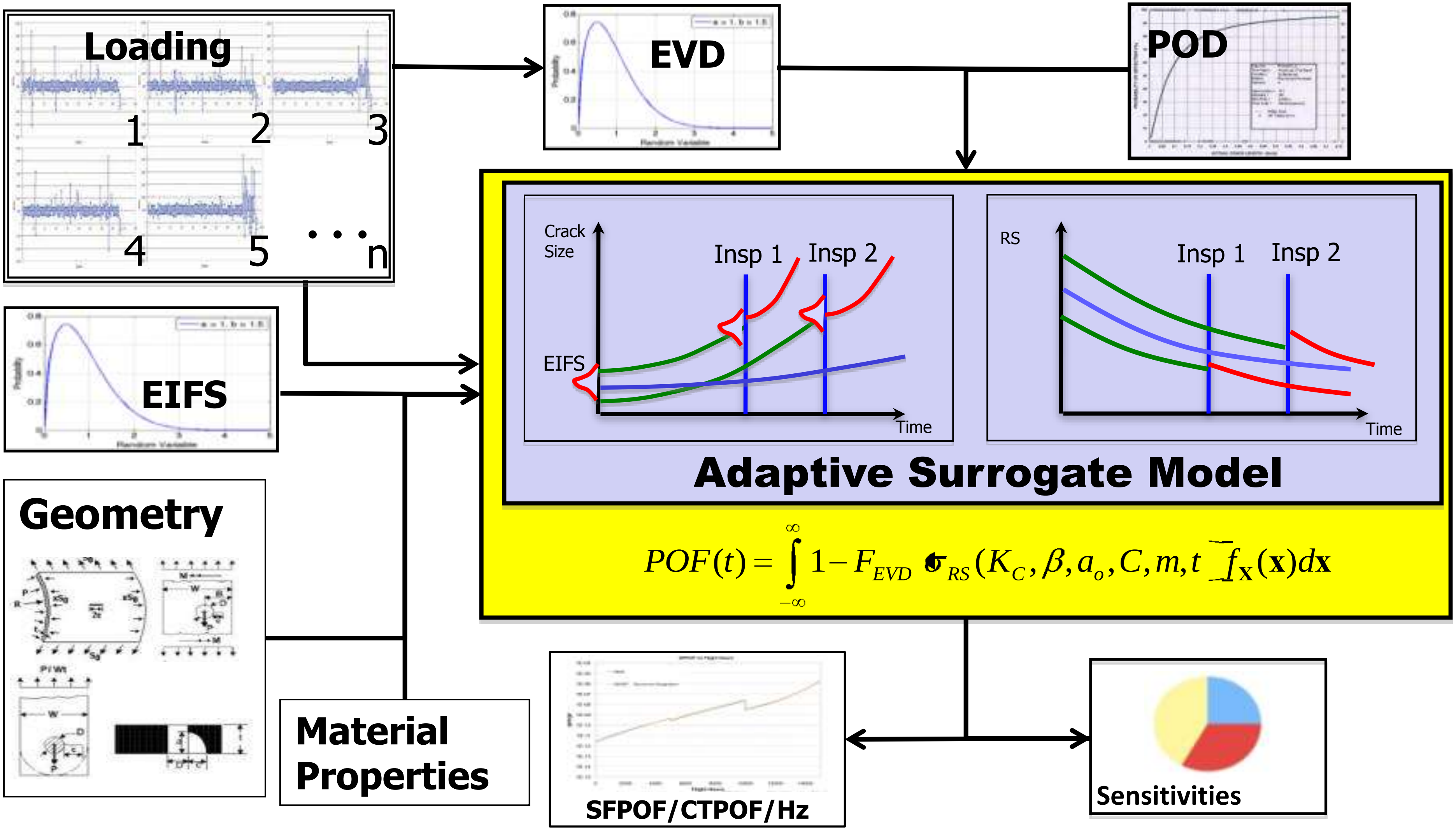
Taxi

Taxi

Randomize taxi loads and split half before the flight and half after the flight, Taxi load can be excluded from the analysis.

Methodology

Probabilistic Damage Tolerance for Small Airplanes



Loading

EVD

POD

EIFS

Geometry

Material Properties

Adaptive Surrogate Model

$$POF(t)=\int_{-\infty}^{\infty}\left[1-F_{EVD}\left(\sigma_{RS}(K_C,\beta,a_o,C,m,t)\right)\right]f_{\mathbf{X}}(\mathbf{x})\,d\mathbf{x}$$

SFPOF/CTPOF/Hz

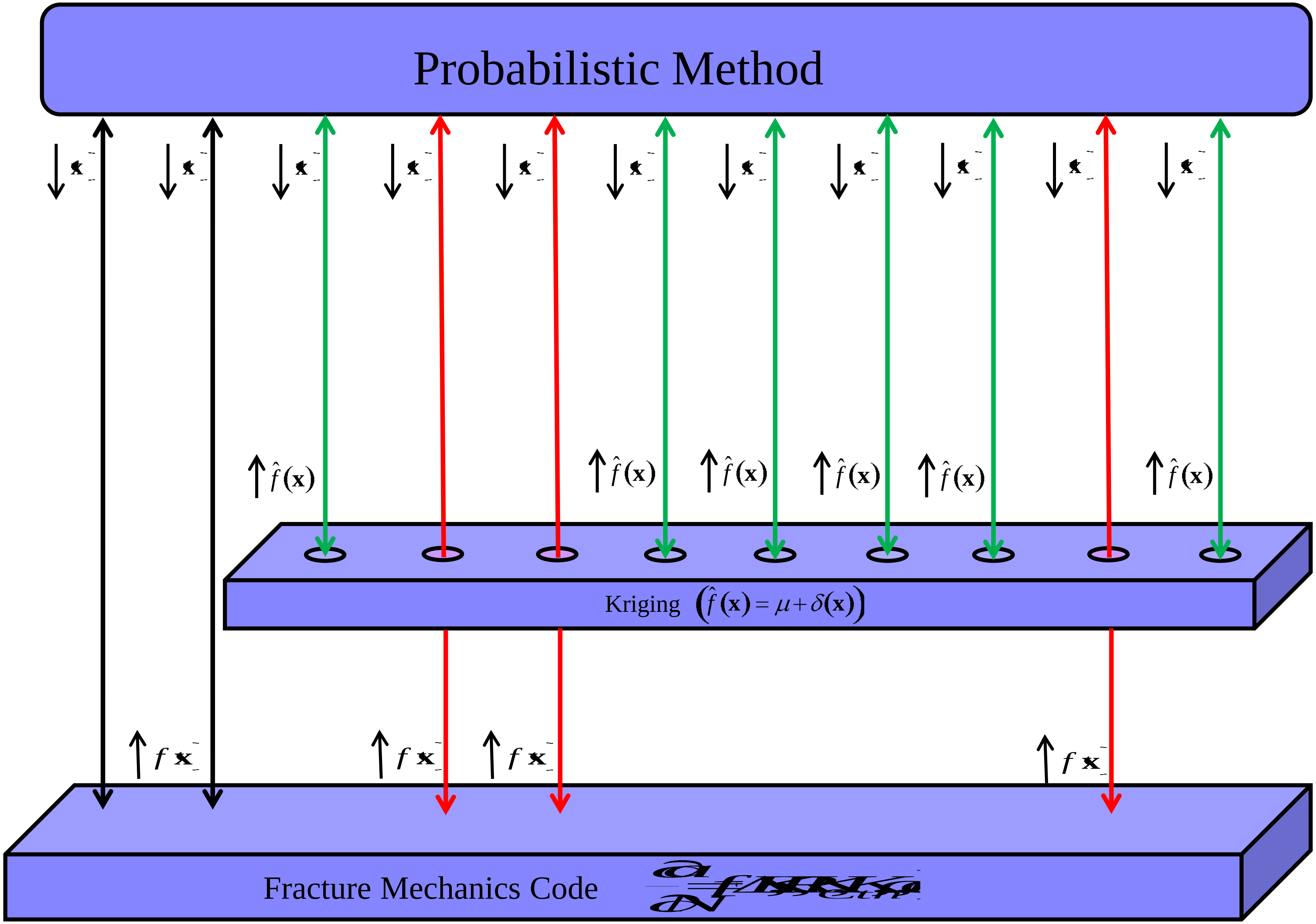
Sensitivities

Adaptive Residual Strength and Crack Growth Surrogate Model

$$POF(t)=\int_{-\infty}^{\infty}\left[1-F_{EVD}\left(\sigma_{RS}(K_C,\beta,a_o,C,m,t)\right)\right]f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}$$

- An adaptive Kriging surrogate model is used to reduce physics-based crack growth function calls, e.g., AFGROW, FASTRAN, UniGrow, NASGRO, etc.
  - Applicable to both:
    - POF calculations (residual strength predictions) and inspections (crack growth predictions)
  - Adaptive (self correcting):
    - additional crack growth function calls added as needed per user-defined error threshold.

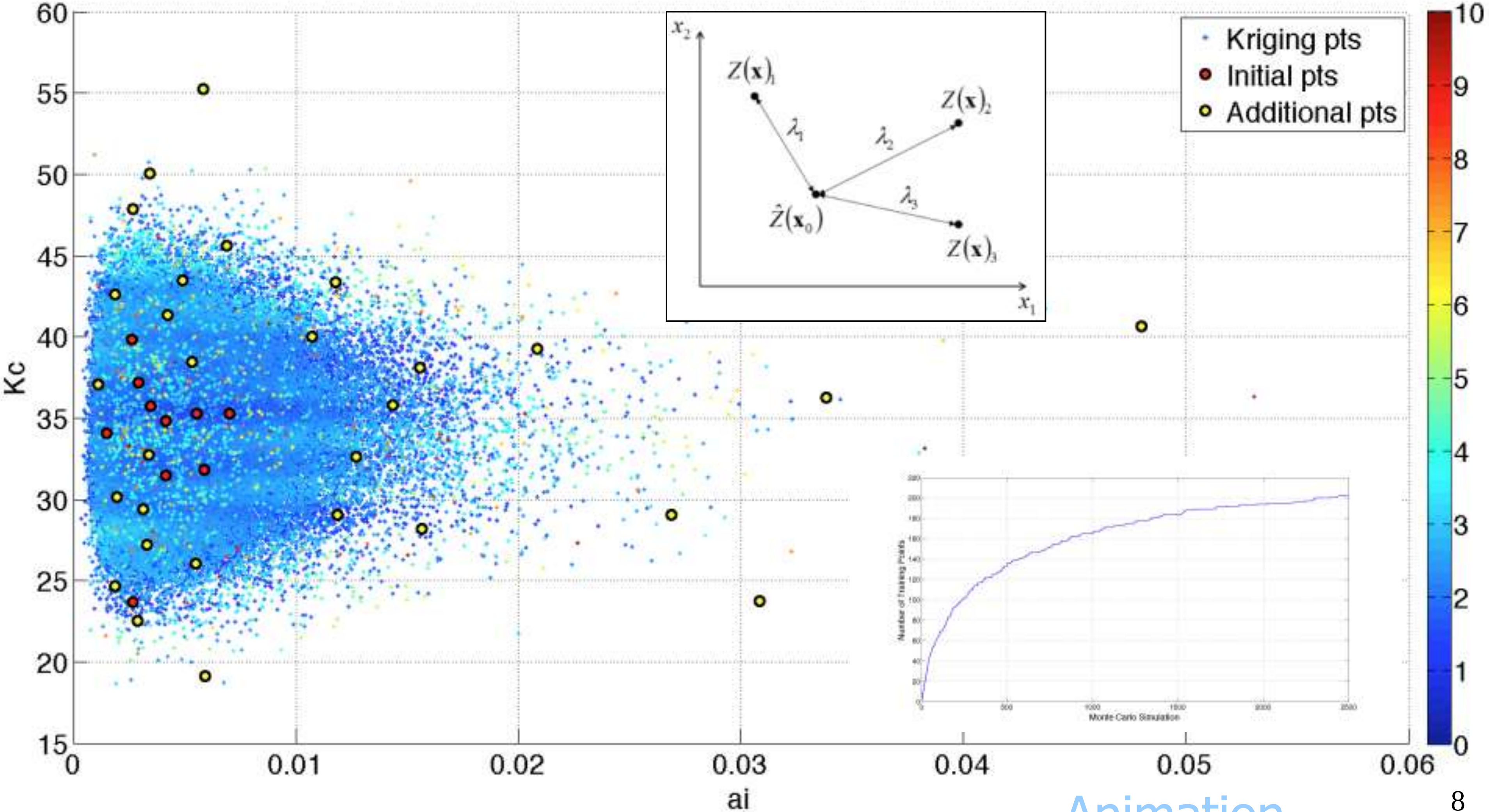
Surrogate Model Schematic



○ User Defined Error — Initial Training Points

— Additional Training Points (Kriging Error> User Error) — Kriged Points (Kriging Error < User Error)

Kriging Schemetic



Animation

Kriging Metamodeling

Mathematical Formulation

Replace the real model analysis with a surrogate model to reduce computational time. Kriging metamodel is an approximation of the Input/Output (I/O) function that is implied by the underlying simulation model.

Let $x_1, x_2, \ldots, x_n$ be "locations" where it is possible to observe data $z(x_1), z(x_2), \ldots, z(x_n)$. It is supposed the data are a realization of a stochastic process $Z(x)$

$$Z(x) = \mu + \delta(x)$$ (Ordinary Kriging)

Trend Function
(Mean of Process)

Gaussian Random Process
(Stochastic Component)

$$\hat{Z}(x_0) = \sum_{i=1}^{n} \lambda_i Z(x_i), \qquad \sigma_k{}^2 = \lambda_0 \gamma_0 = \sum_{i=1}^{n} \lambda_i \gamma(x_0 - x_i) + m$$

Kriging Adaptive Model

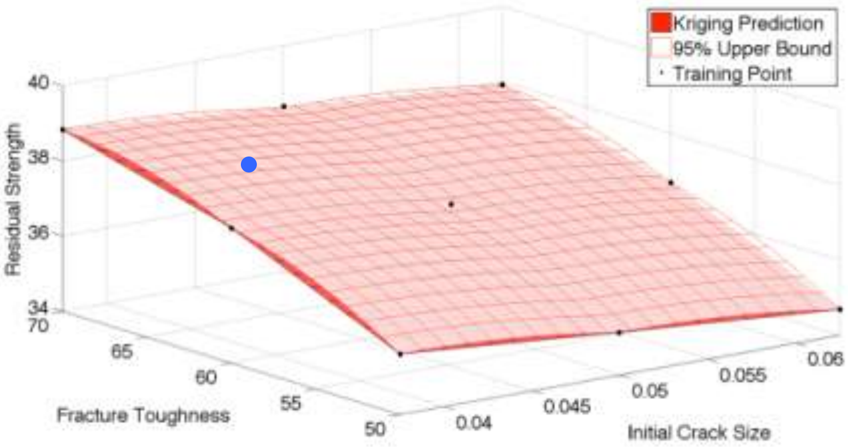
The error is calculated based on the Kriging variance and the assumption that $Z(x)$ is Gaussian

The 95% confidence bound from the prediction value can be computed as

$$A \equiv \left(A_{LB}, A_{UB}\right) \equiv \left(\hat{Z}(x_0) - 1.96\sigma_{\varepsilon}(x_0), \hat{Z}(x_0) + 1.96\sigma_{\varepsilon}(x_0)\right)$$

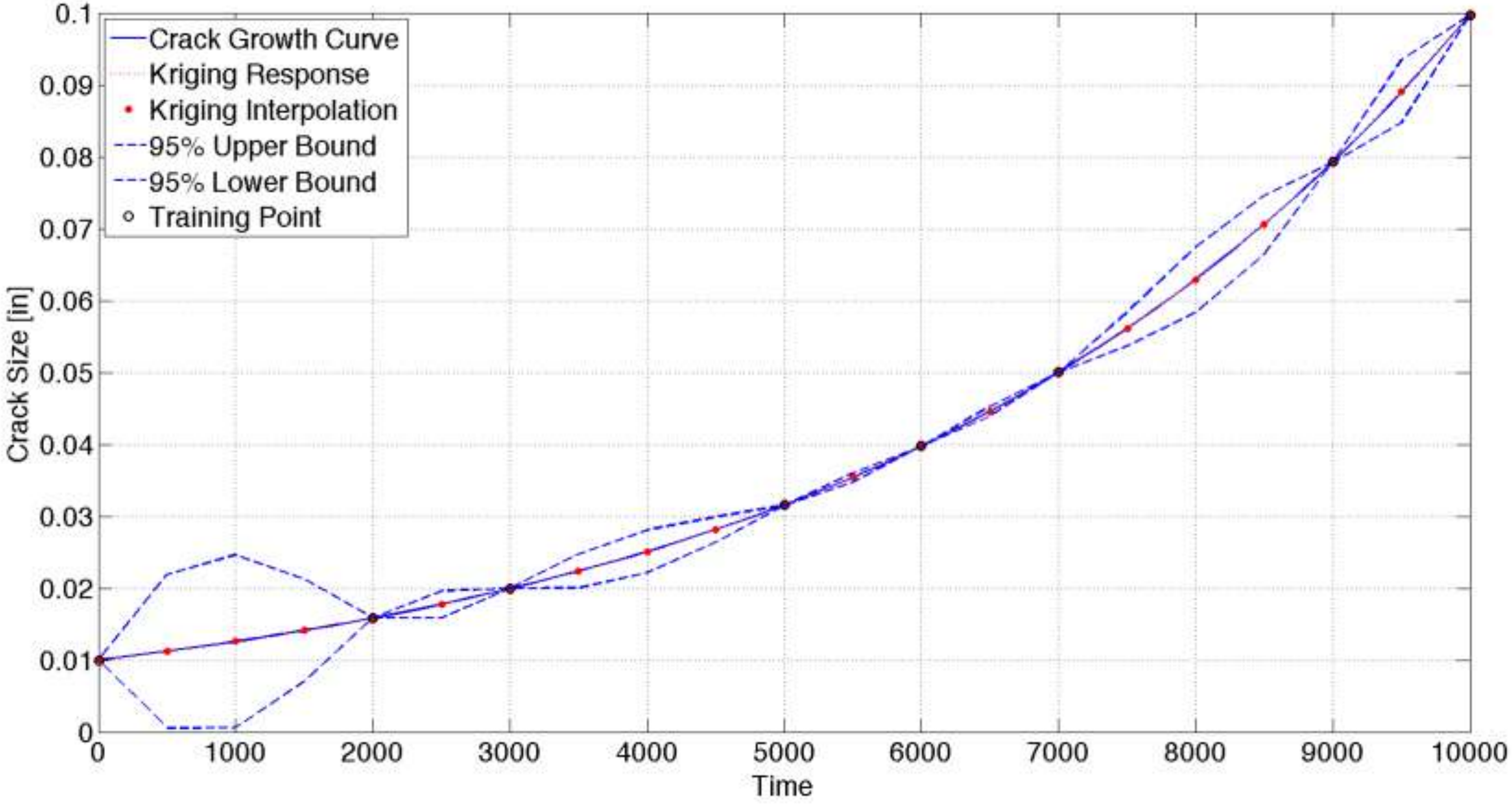
The error based on the 95% (99%) confidence bound can be computed as

$$error = \frac{abs\left(A_{LB} - \hat{Z}(x_0)\right)}{A_{LB}}$$



Kriging Error Prediction

Compute prediction variance and confidence bounds



Methodology

Kriging Metamodeling

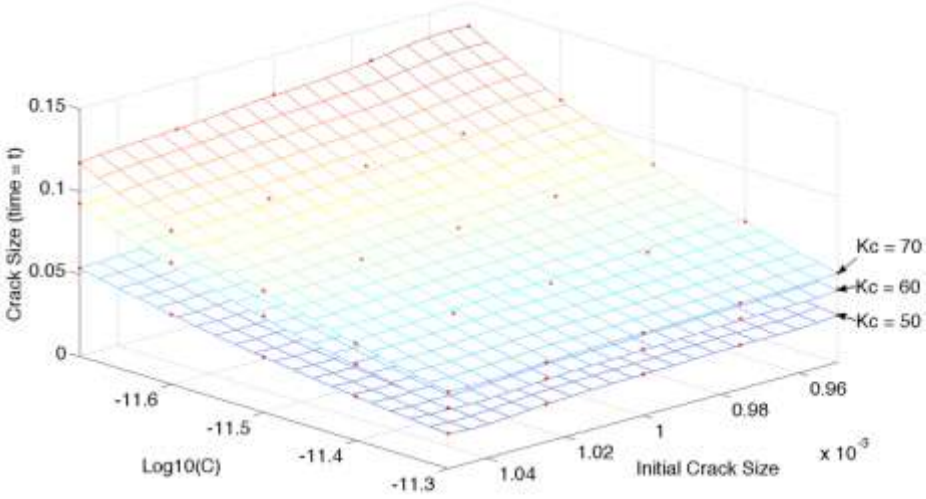
- Efficient Method to compute Crack Size (a) and Residual Strength (RS).
  - Few runs from the crack growth software used to train the Kriging surrogate Model (Gaussian process based on the correlations between the training data).

$$\frac{\partial a}{\partial N} = f(\Delta K, a, c)$$

$$\frac{\partial c}{\partial N} = f(\Delta K, a, c)$$

Kriging

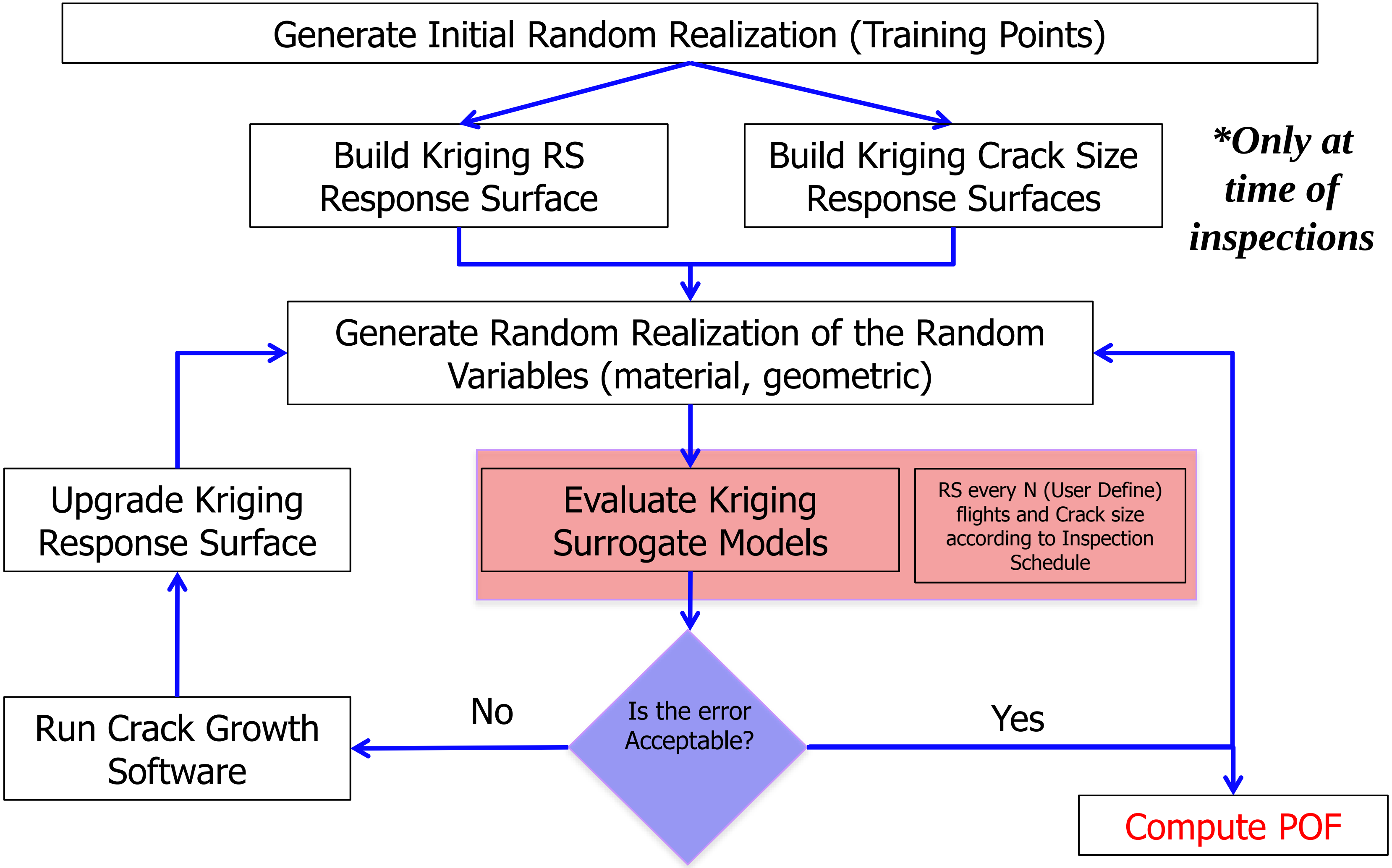
$$Z(\mathbf{x}) = \mu + \delta(\mathbf{x})$$



● Training point

  - After building the Kriging surface predict "a" and "RS".

Kriging Adaptive Model



Generate Initial Random Realization (Training Points)

Build Kriging RS Response Surface

Build Kriging Crack Size Response Surfaces

Only at time of inspections

Generate Random Realization of the Random Variables (material, geometric)

Evaluate Kriging Surrogate Models

RS every N (User Define) flights and Crack size according to Inspection Schedule

Is the error Acceptable?

No

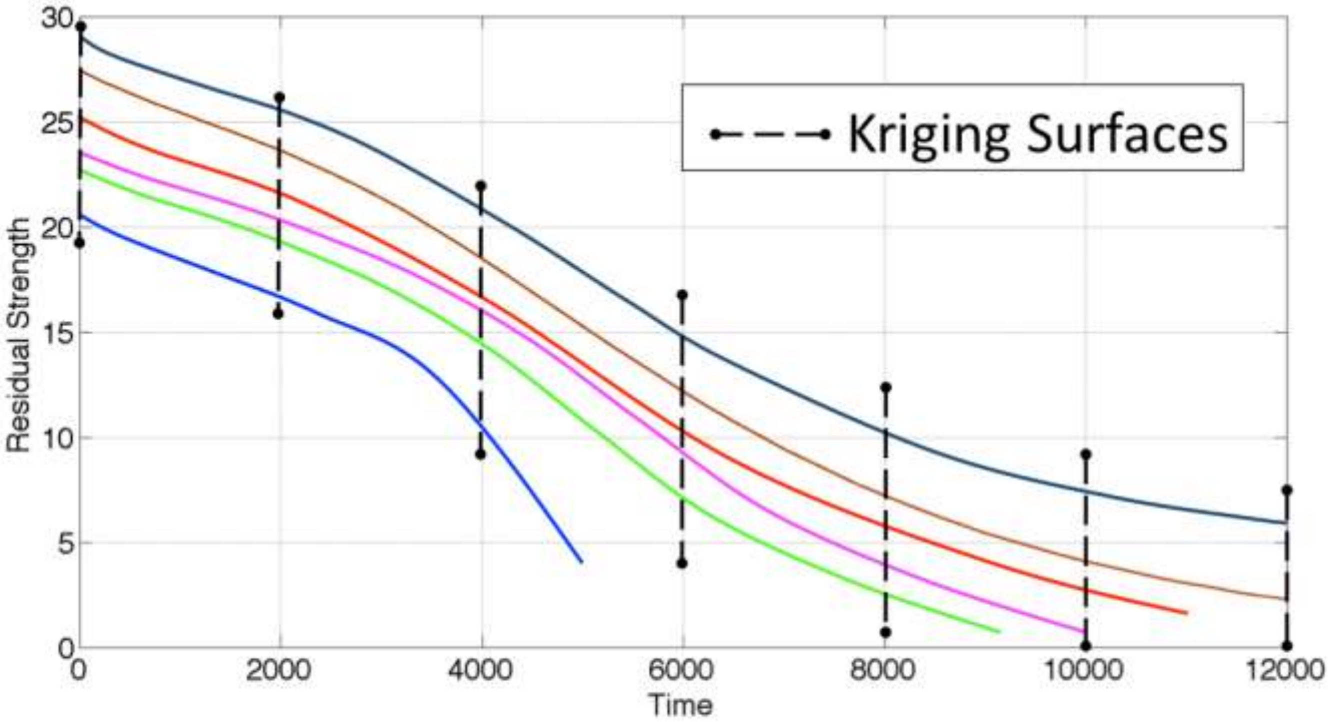
Run Crack Growth Software

Upgrade Kriging Response Surface

Yes

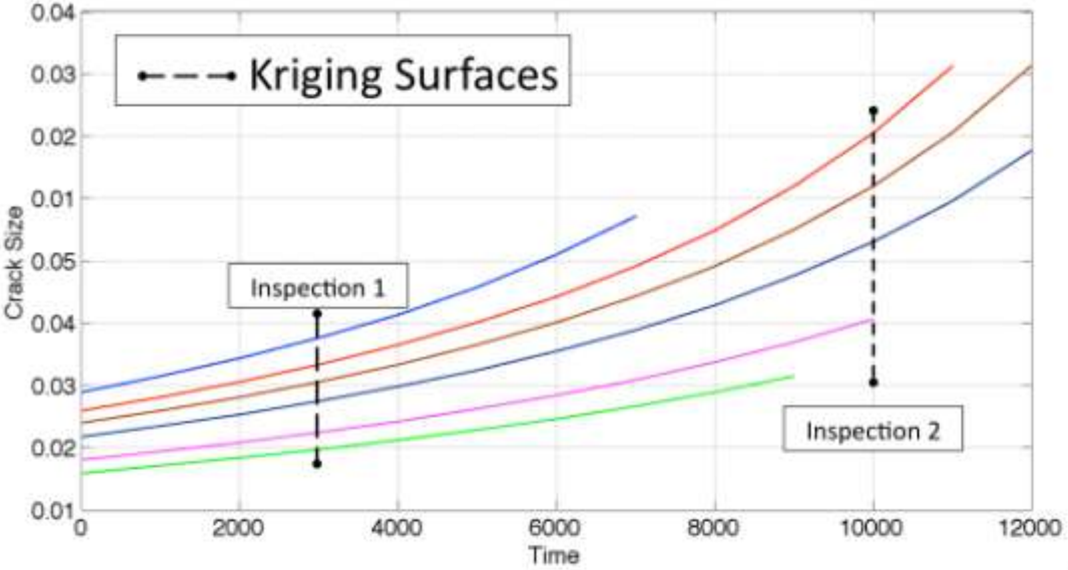
Compute POF

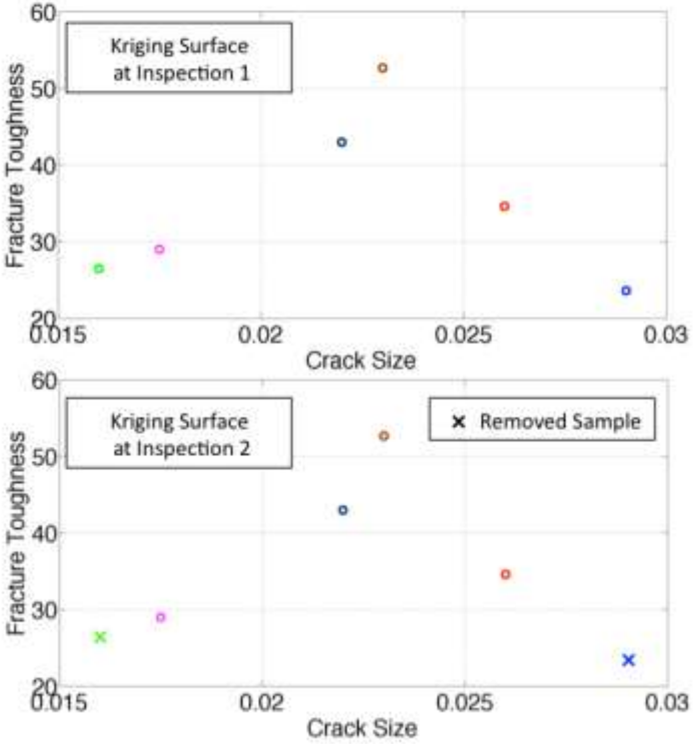
Time Dependent Surrogates



Residual strength Kriging surfaces are created anew at each time step requested by the user using non-failed realizations. Similarly for crack size estimates.

Crack Growth Surrogates

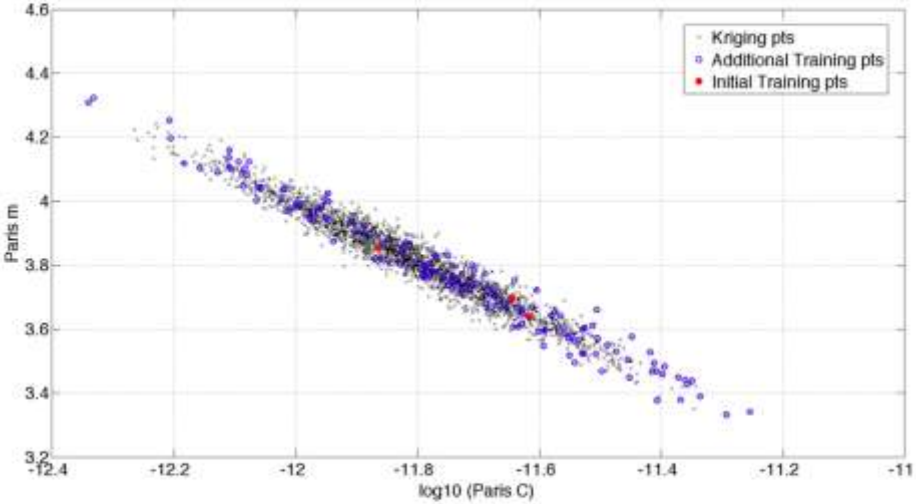


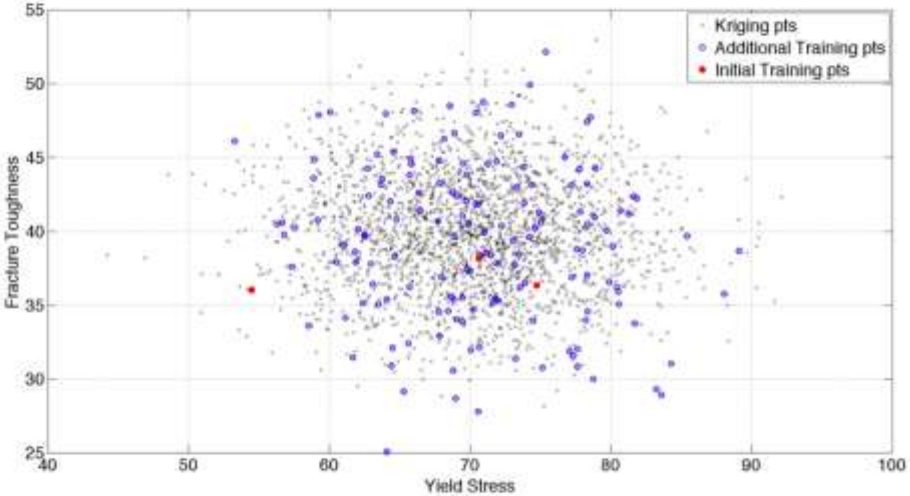


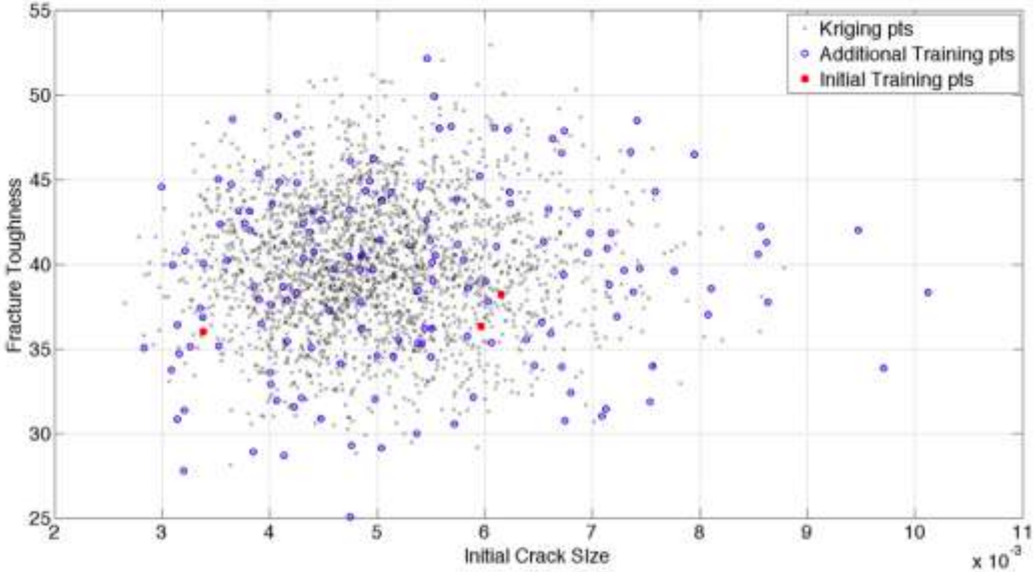
If an inspection occurs at time t, crack size Kriging surfaces are created at each inspection time

Adaptive Kriging

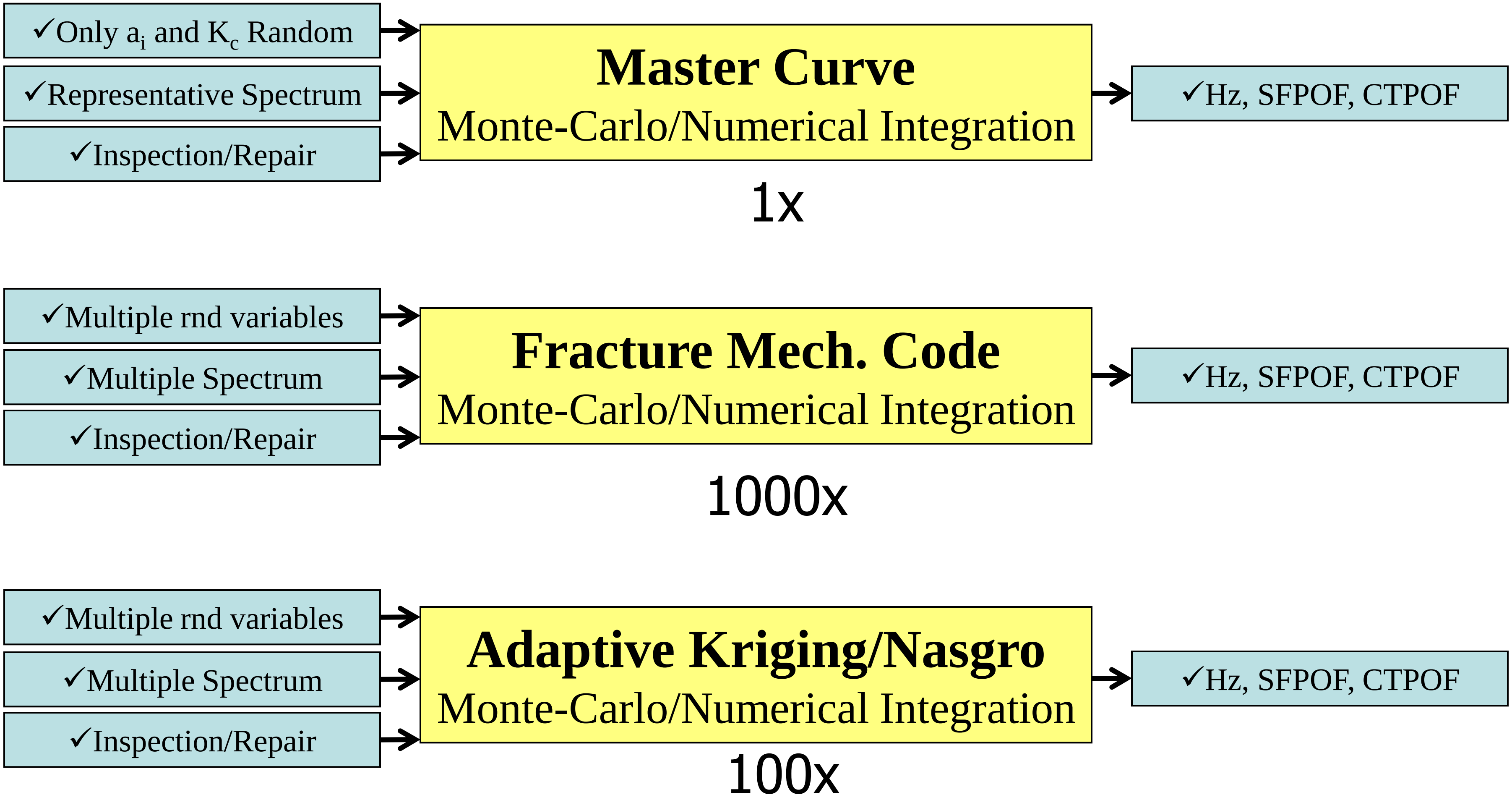
Multiple Random Variables







KrigingComparison



✓Only a_i and K_c Random

✓Representative Spectrum

✓Inspection/Repair

Master Curve

Monte-Carlo/Numerical Integration

✓Hz, SFPOF, CTPOF

1x

✓Multiple rnd variables

✓Multiple Spectrum

✓Inspection/Repair

Fracture Mech. Code

Monte-Carlo/Numerical Integration

✓Hz, SFPOF, CTPOF

1000x

✓Multiple rnd variables

✓Multiple Spectrum

✓Inspection/Repair

Adaptive Kriging/Nasgro

Monte-Carlo/Numerical Integration

✓Hz, SFPOF, CTPOF

100x

Example Problems

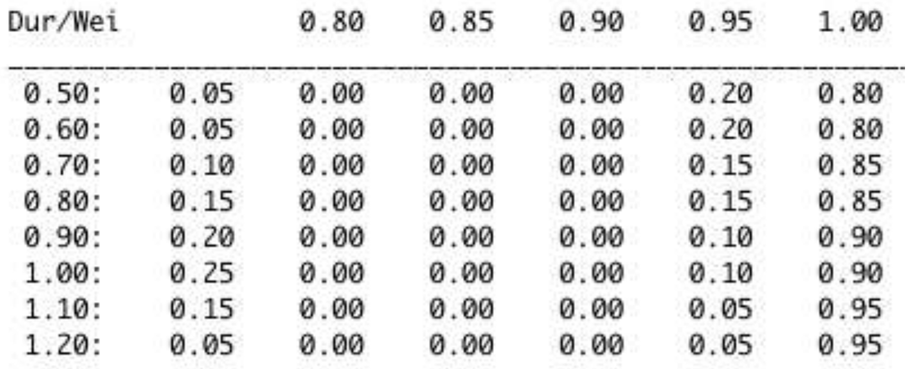
SMART$_{\text{DT}}$

SMall Aircraft Risk Technology Damage Tolerance Analysis

High Performance Aircraft no Inspection

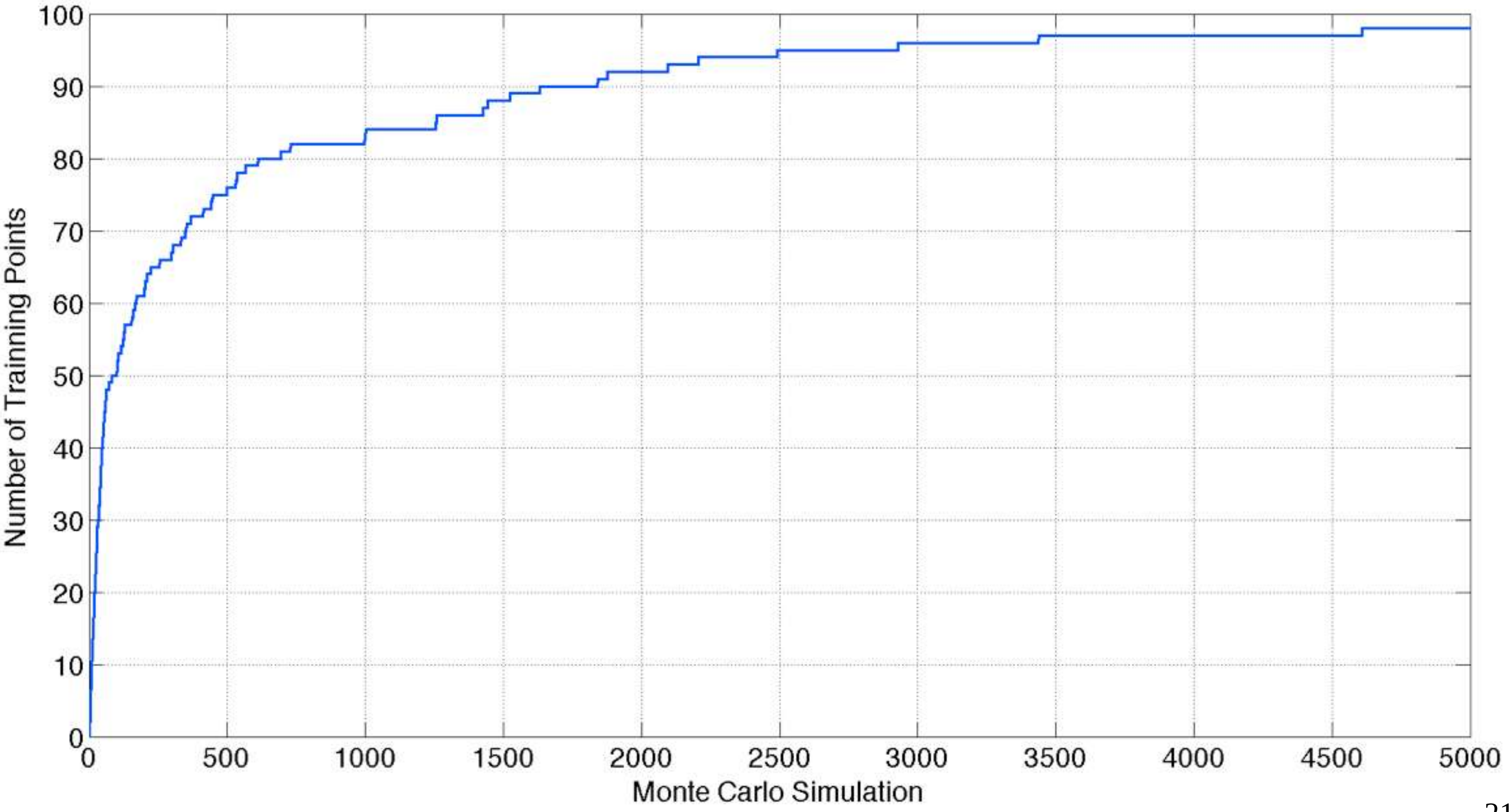
Quantity	Definition
Nasgro Crack Growth Model.	TC03 – Through crack in a hole
Geometric Variables	Width = 2.5 in. Thickness = 0.09 in. Hole Diameter = 0.10 in. Hole Offset = 0.5 in.
Fracture Toughness Distribution	Normal: Mean = 34.8ksi√in. Standard Deviation = 3.9 ksi√in.
Initial Crack Size Distribution	Lognormal Median = 0.00163 in. Mean = ln(median) = -6.420 Standard Deviation = 1.113
Extreme Value Distribution (Weibull)	Location = 5.0, Scale = 10.0, and Shape = 5.0
Material	Al-2024

High Performance Aircraft no Inspection

Variable	Value
Usage	Single Engine Unpressurized Basic Executive Usage
Design LLF Maneuver	3.8, -1.52
Design LLF Gust	3.155, -1.155
Ground Stress (psi)	-4,550
One-g stress (psi)	7,100
Flight Length and Velocity Matrix	
Flight Length and Weight Matrix	
Average Velocity (Vno/Vmo (Knots))	165

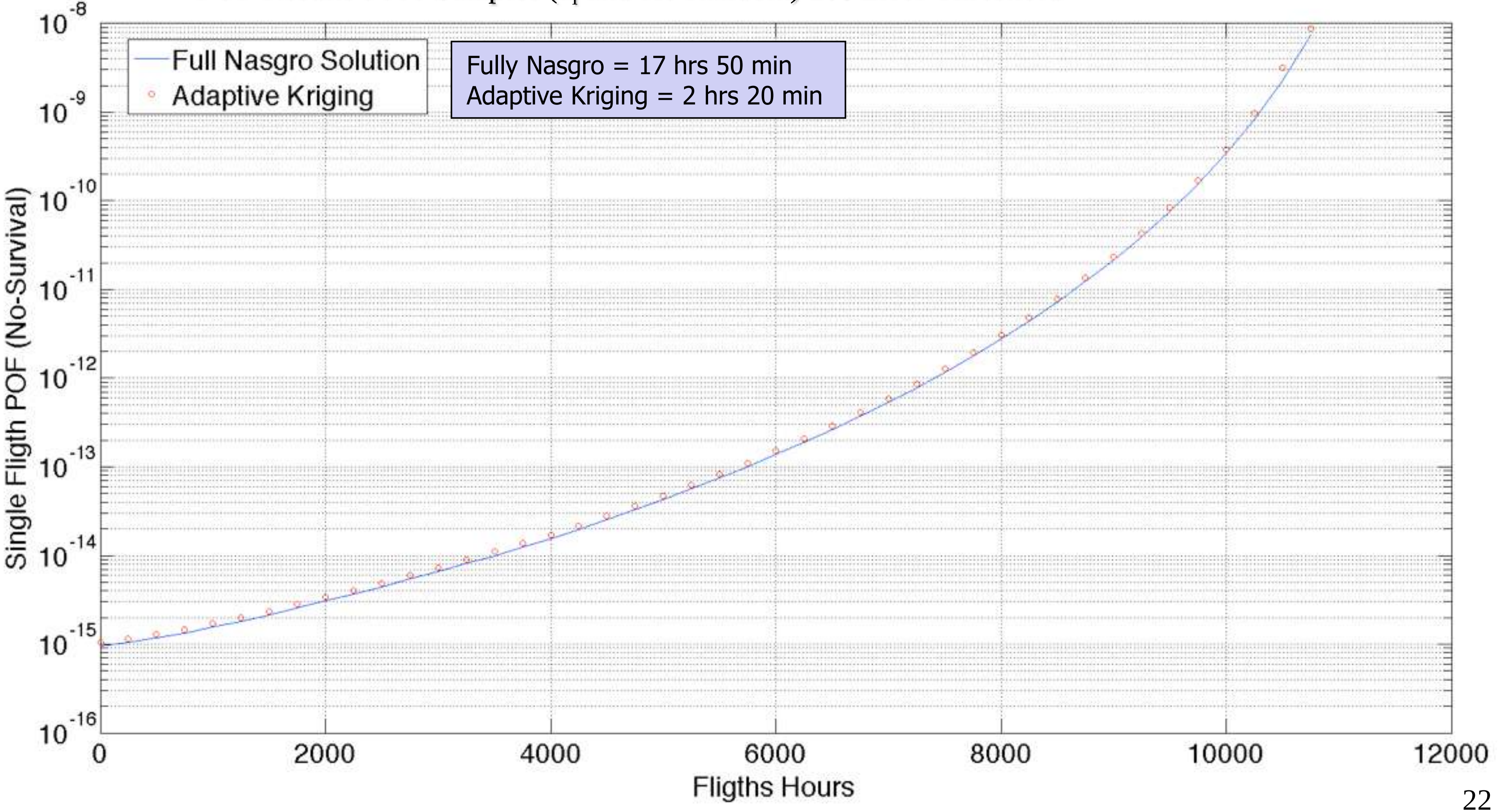
```
Dur/Wei              0.80    0.85    0.90    0.95    1.00
--------------------------------------------------------------
 0.50:       0.05    0.00    0.00    0.00    0.20    0.80
 0.60:       0.05    0.00    0.00    0.00    0.20    0.80
 0.70:       0.10    0.00    0.00    0.00    0.15    0.85
 0.80:       0.15    0.00    0.00    0.00    0.15    0.85
 0.90:       0.20    0.00    0.00    0.00    0.10    0.90
 1.00:       0.25    0.00    0.00    0.00    0.10    0.90
 1.10:       0.15    0.00    0.00    0.00    0.05    0.95
 1.20:       0.05    0.00    0.00    0.00    0.05    0.95
```

High Performance Aircraft no Inspection



High Performance Aircraft no Inspection

POF Results 5000 Samples (a_i and Kc Random) 2% Error Threshold



Fully Nasgro = 17 hrs 50 min
Adaptive Kriging = 2 hrs 20 min

Commuter Aircraft with Inspections

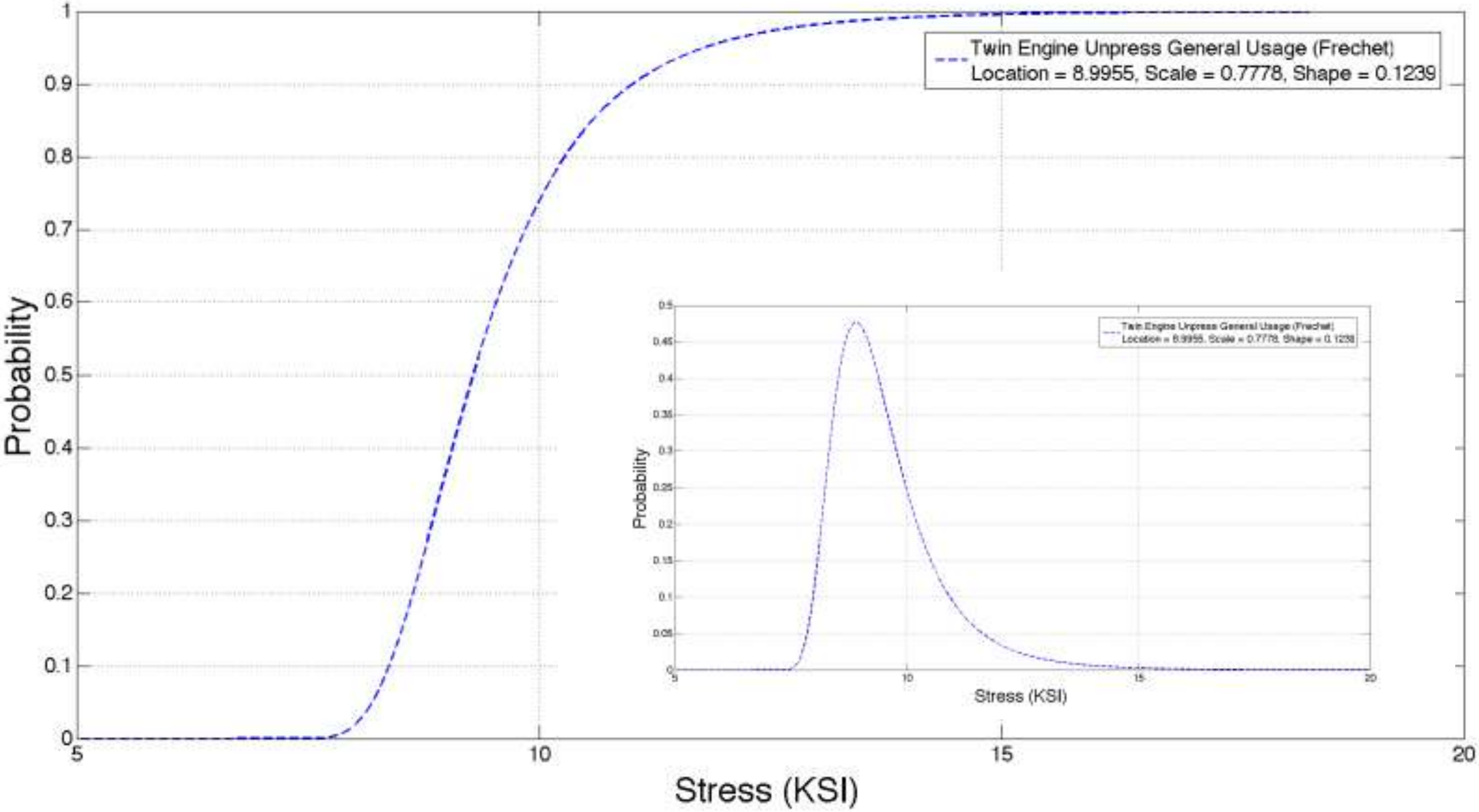
Quantity	Definition
Nasgro Crack Growth Model.	TC03 – Through crack in a hole
Geometric Variables	Width = 2.5 in. Thickness = 0.15 in. Hole Diameter = 0.10 in. Hole Offset = 0.5 in.
Fracture Toughness Distribution	Normal: Mean = 40.0 ksi√in. Standard Deviation = 4.0 ksi√in.
Initial Crack Size Distribution	Lognormal Median = 0.050 in. Mean = ln(median) = -2.995 Standard Deviation = 0.001
Material	Al-2024

Commuter Aircraft with Inspections

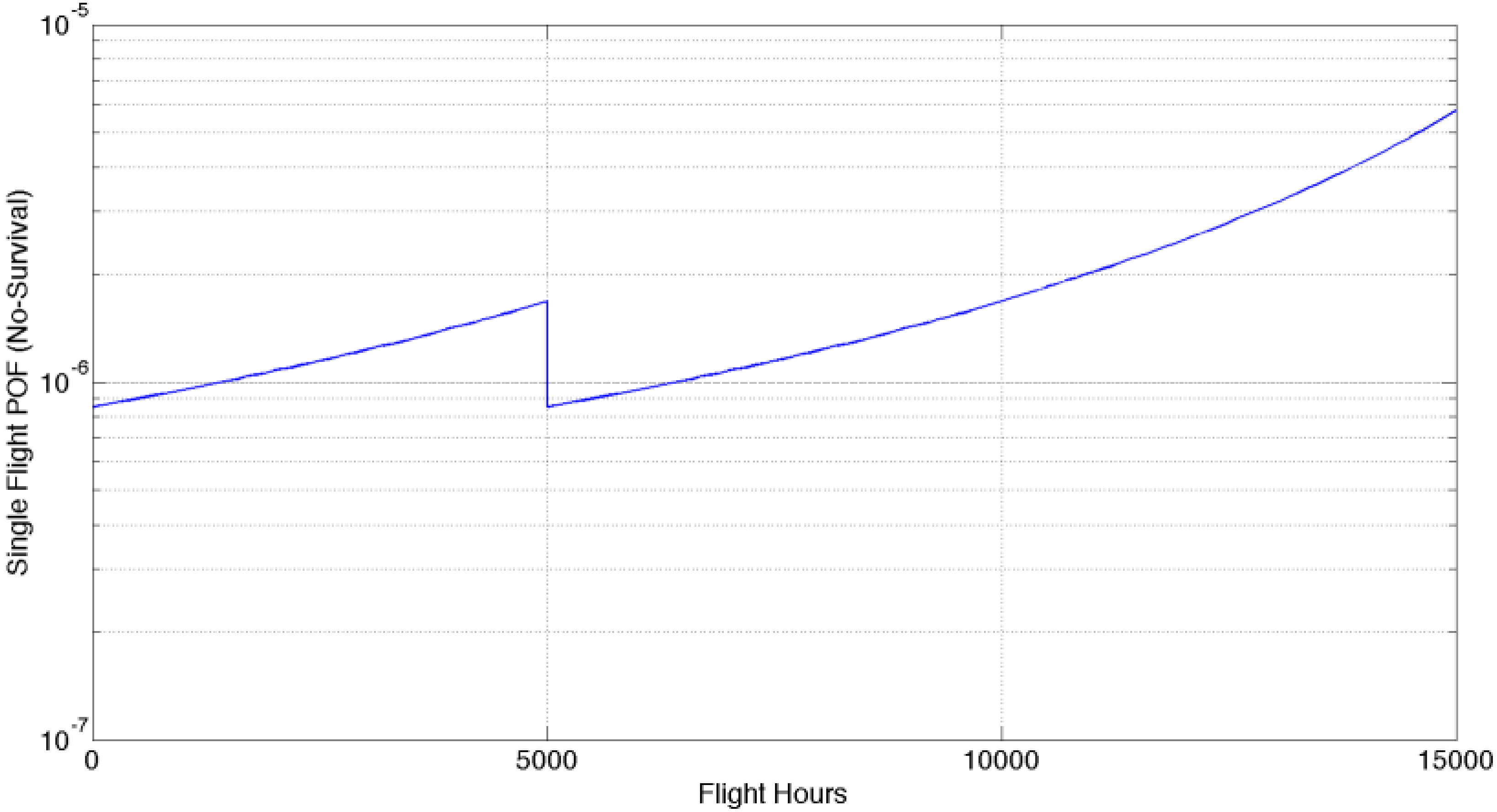
Variable	Value
Usage	Twin Engine Unpressurized Basic Executive Usage
Design LLF Maneuver	3.2, -1.5
Design LLF Gust	3.2, -1.2
Ground Stress (psi)	-4,000
One-g stress (psi)	5,100
Flight Length and Velocity Matrix	Deterministic (1 hr. Duration)
Flight Length and Weight Matrix	deterministic
Average Velocity (Vno/Vmo (Knots))	165

Quantity	Definition
Inspection Time	5,000
Probability of Inspection	1.0
Probability of Detection	Lognormal Median = 0.00390 in. Mean = ln(median) = -5.545 in. Standard Deviation = 1.113 in.
Repair Crack Size Distribution	Lognormal Median = 0.050 in. Mean = ln(median) = -2.995 Standard Deviation = 0.001

Commuter Aircraft with Inspections



Commuter Aircraft with Inspections



Conclusions & Discussion

- Probabilistic damage tolerance evaluation of General Aviation Aircraft is vital in order to provide important insight into the severity or criticality of a potential structural issue.
- Probabilistic damage tolerance evaluations are computational expensive, for that reason an adaptive Kriging metamodeling is used to improve the computational time.
- The methodology described in this presentation provides a tool to perform probabilistic damage tolerance evaluation for real aviation fleet applications.

Current Work

- Additional testing for different number of random variables.

Acknowledgements

- Probabilistic Damage Tolerance-Based Maintenance Planning for Small Airplanes, Sep. 2009-Aug. 2012, Federal Aviation Administration, Grant 09-G-016
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Questions?

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