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Probabilistic Damage Tolerance for Small Airplanes Using a Linear-Elastic Crack Growth Fracture Mechanics Surrogate Model

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Most general aviation (GA) aircraft are designed for safe-life based upon a crack initiation type failure mechanism, e.g., Miner's rule. However, newer GA aircraft have fatigue crack growth as a design option. In addition, it may be necessary to evaluate a field event such as a cracked structure to ascertain the remaining life. Therefore, a risk based probabilistic damage tolerance analysis (PDTA) program is needed in several aerospace situations. A comprehensive probabilistic damage tolerance method requires a combination of deterministic crack growth, inspection methods, probabilistic methods, and random variable modeling to provide a single probability-of-failure, cumulative probability-of-failure, and hazard rate with and without inspection. In this work, a general methodology to conduct probabilistic crack growth based damage tolerance methodology for small airplanes will be developed and incorporated in a computer software. Random variables can be included in the model using Monte Carlo Sampling (MCS) and efficient numerical integration algorithms. Probabilistic damage tolerance analysis involves mathematically complex models and computational expensive simulations, which makes these analyses very inefficient. In this work the computational weight will be reduced using an error based adaptive surrogate model; the surrogate model will include the most influential random variables. The surrogate model will be used as a temporary substitution for the original crack growth model. An example problem will be presented to demonstrate the methodology.

I. Introduction

In many applications, damage tolerance analysis is made using a deterministic approach, and general aviation is not the exception. However, due to the number of uncertainties presented in this area and the critical condition of some airplanes, a probabilistic approach is needed. [1,2]. Probabilistic damage tolerance method requires a combination of deterministic crack growth, inspection methods, probabilistic methods, and random variable modeling to provide a single probability-of-failure, cumulative probability-of-failure, and hazard rate.

Many military aircraft fleets (US DoD, UK MoD, and Canadian Forces) have adopted a risk management program/tool to ensure aircraft safety and airworthiness. Now more non-military agencies are adopting these practices to guarantee aircraft safety and maintain airworthiness.

A damage tolerance analysis (DTA) contains a number of elements such as: expected usage, structural material properties, crack growth and fracture material properties, crack size and aspect ratio, component specific usage-to-stress models, geometric factors, stress intensity

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factor calculations, and others. These elements have associated with them inherent uncertainty (as in material properties) or statistical uncertainty (as in loads, initial crack sizes, and geometry). Also, the assumptions contained in the analysis methods may result in modeling errors, i.e., the use of simplified models to represent complex behavior.

The objective of this research is to develop a comprehensive probabilistic methodology such that FAA engineers can conduct a risk assessment of a GA structural issue in support of policy decisions. The underlying structural degradation mechanism will be crack growth as governed by linear elastic fracture mechanics (LEFM).

A comprehensive probabilistic damage tolerance method requires a combination of an efficient deterministic crack growth, probabilistic methods, random variable modeling, and inspection and repair methods to provide a cumulative probability-of-failure and single probability-of-failure with and without inspection. The end result will be a software program that can be used by FAA engineers to evaluate a structural configuration subject to crack growth and assess the effects of inspection and repair.

Probabilistic damage tolerance analysis consists of running complex crack growth models to compute crack sizes and residual strength as a function of time. Despite continual advances in computational efficiency in terms of power and speed, running these types of complex computer codes remains non-trivial. Single evaluations of a crack growth model can take from few seconds to a few minutes, but when several evaluations are needed to compute the Single Flight Probability of Failure (SFPOF) several hours might be needed. To help to improve the computational time, metamodeling techniques are used. Metamodeling is a collection of statistical and mathematical techniques that provide cheap evaluations of complex and computationally expensive simulation codes. Metamodeling produces approximate responses, called metamodels, of an unknown function describing a particular behavior affected by known independent variables.

II. Methodology

The methodology in this work has four main ingredients: aircraft load generation, extreme value maximum load per flight distribution (EVD) generation, surrogate fracture mechanics crack growth, and the probabilistic methods to compute the probability of failure at any time in the aircraft life. Figure 1 shows schematically the PDTA process.

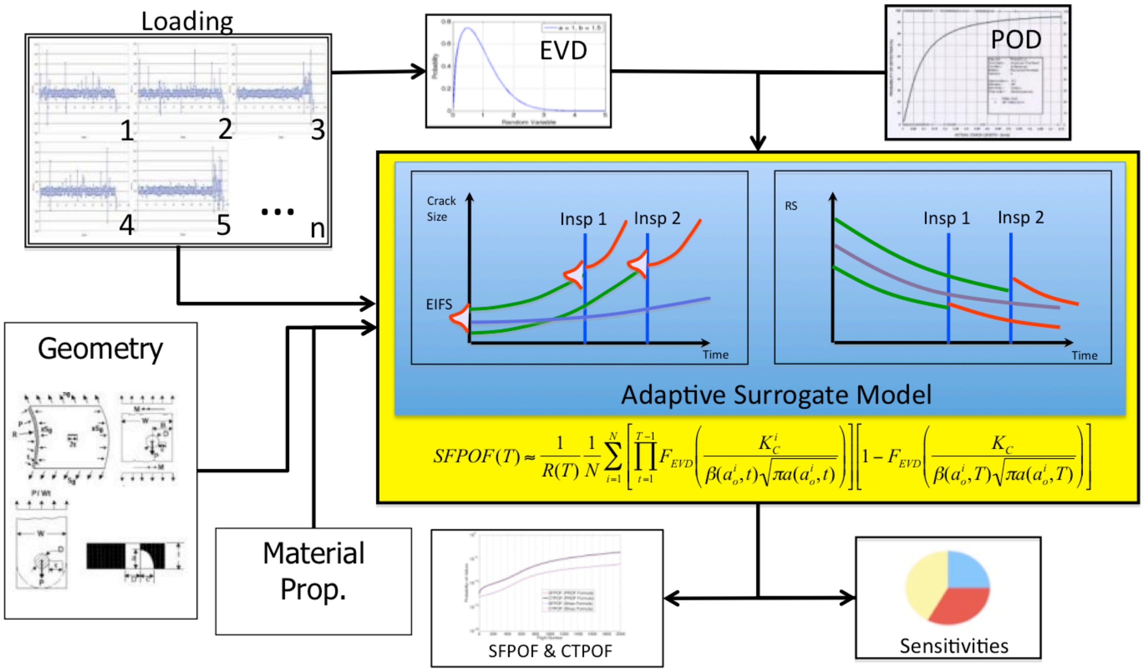

Figure 1. Schematic for Probabilistic Damage Tolerance Analysis

A. Load generation

The computer code generates a realistic load spectrum accounting for five different flight regimes: Maneuver, Gust, Taxi, Landing and Rebound, and Ground-Air-Ground (GAG)). Incomplete cycles at any current flight are saved and added in future flights. The stresses per flight and the flights are randomized.

The input parameters used to generate a load spectrum are given in Table 1 and the steps to generate the spectrum are as follows:

1. Provide input parameters (a summary of the input parameters is presented in Table 1).
2. Generate random realizations of the parameters: maneuver and gust exceedance curves, flight-length and aircraft velocity as per flight length-velocity and maximum aircraft velocity, and one-g-stress as per flight length-weight and maximum one-g-stress.
3. Calculate the number of occurrences for each of the flight stages using the methodology in references [1], [2], and [3].
4. After each of the stresses and occurrences are calculated for the current flight, incomplete cycles from previous flights are added to the current flight stresses. Then, the complete stresses are extracted and the incomplete stresses from the current flight are saved for the next flight.
5. Randomize the load pairs within a flight generated in the previous steps.
6. Save the maximum load per flight to later estimate the extreme value distribution.
7. Repeat steps 1 through 6 for the given number of flights.

8. After all the flights have been generated, randomize the flights such that there is an equal probability of the high severity loads appearing at any flight during the crack growth analysis.

Table 1. Spectrum Variable Classification

Variable	**Description**
Number of Flights	Number of flight to be generated in the flight spectrums
Exceedance Curves	Usage exceedance curves
Maneuver Load Limit Factor	Maximum load limit factors for maneuver load
Gust Load Limit Factor	Maximum load limit factors for gust load
Maximum Ground Stress	Airplane ground stress in psi
Maximum One g Stress	One g stress of an airplane in psi.
Maximum A/C Velocity	Average Speed During Flight, VNO (Maximum aircraft safe cruise speed) or VMO (Maximum operating limit speed). In nautical miles.
Flight Length-Velocity Matrix	Probabilistic flight length and airspeed data
Flight Length-Weight Matrix	Probabilistic flight length and weight data

Figure 2 shows schematically the process to generate a flight spectrum. Figure 3 shows a spectrum example including all the flight stages for a single flight.

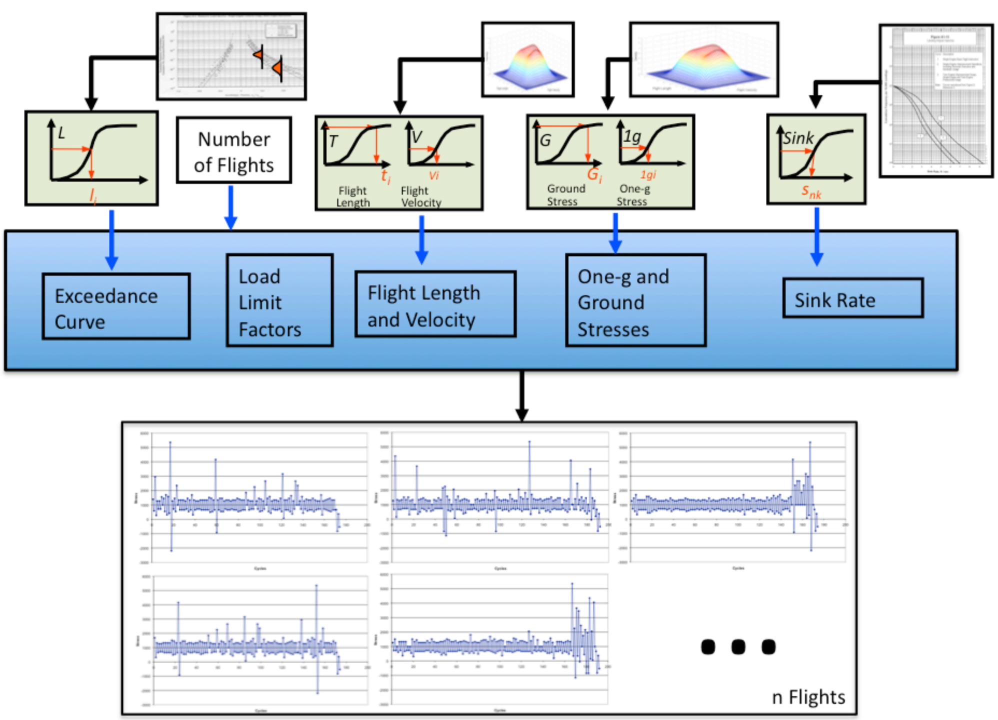

Figure 2. Schematic for the Spectrum Generation

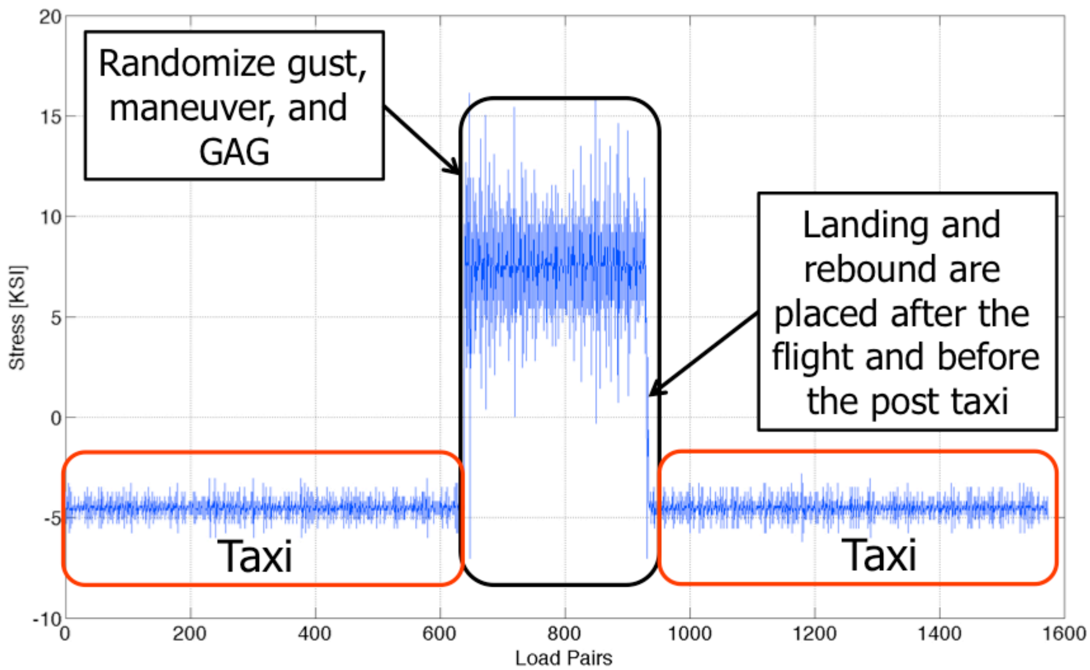

Figure 3. Spectrum Example

B. Extreme Value Maximum Load per Flight Distribution (EVD) Generation

An extreme value distribution (EVD) of the maximum load per flight of a load spectrum is critical for a probabilistic damage tolerance analysis of a general aviation aircraft. The EVD parameters are important because the structural integrity of the aircraft depends upon the maximum load seen by the structure during a specified number of flights.

In probabilistic damage tolerance analysis, the EVD must be generated from the same loading used for crack growth analysis. In this program, the maximum load per flight is extracted in sets of a fixed number of flights, and the software continues generating sets of flights until the parameters converge as shown schematically in Figure 4.

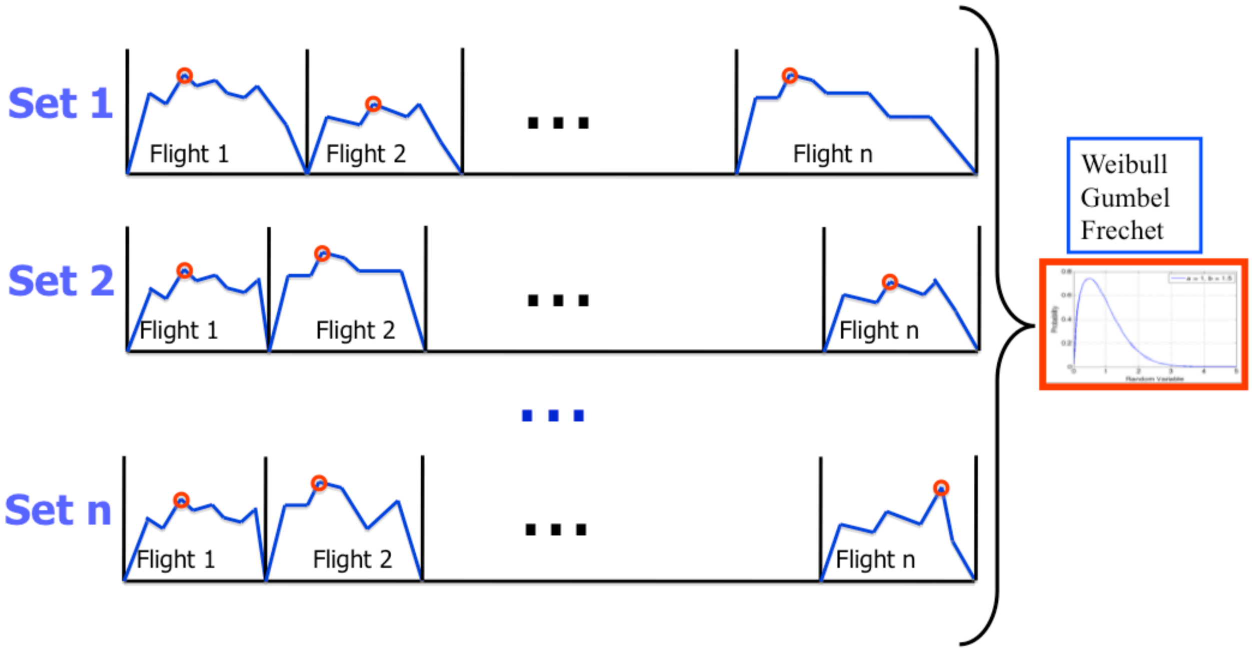

Figure 4. Extreme Value Distribution Generation Schematic

Using these sets of maximum load per flight the EVD can be calculated using the generalized extreme value theory. The generalized extreme value theory can be explained as follows: Suppose X1, X2, ...,Xp is a sequence of independent random variables having a common distribution function F(x). If Mp represents the maximum of the process over n observations, then as per extreme value theory, the distribution of Mp can be derived exactly for all the values of p [0]:

$$\begin{aligned} P\{M_n \le z\} &= P\{X_1 \le z, X_2 \le z, \ldots, X_n \le z\} \\ &= P\{X_1 \le z\} \times P\{X_2 \le z\} \times \ldots \times P\{X_n \le z\} \\ &= F(z)^n \end{aligned} \tag{1}$$

Therefore, if the probability density function (PDF) or the distribution function of a random variable is given, then an EVD of the variable over p samples can be estimated using Eq. 1. This may not be immediately helpful in practice because the PDF of aircraft loading is not available in a closed-form equation. However this principle provides the exact solution for a standard distribution such as uniform, normal, or Weibull distribution. When the PDF of the parent distribution is not available and the above approach cannot be used, the following approach can be employed.

From the extreme value theory, it is known that the asymptotic form of extreme value of maximum data as p → ∞ can take one of three forms: Gumbel, Frechet, Weibull (Types I, II, and III). The three possible models for the maximum can be encapsulated in the generalized extreme value model as [4]:

$$F(x;\mu,\sigma,\xi) = P = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \quad (2)$$

The distribution in Eq. 2 $F(Q_x;\mu,\sigma,\xi)$ is known as the generalized extreme value distribution. Here μ, σ, and ξ indicate the location, scale, and shape parameters of the generalized extreme value distribution, respectively. The value of the shape parameter determines the type of the distribution. The extreme value distribution converges to Weibull, Gumbel, or Frechet if the shape parameter (ξ) value is less than zero, equal to zero, or greater than zero, respectively.

The inverse of the generalized extreme value distribution, also known as the quantile function, for $P \in (0,1)$ is:

$$F^{-1}(P;\mu,\sigma,\xi) = x = \mu - \left(\frac{\sigma}{\xi}\right) + \left(\frac{\sigma}{\xi}\right) \cdot \left[\ln(P)\right]^{-\xi} \quad (3)$$

For a given value of x and its probability, the inverse function is an equation with three unknowns: location, scale, and shape. For three equations, it is possible to solve for the three unknowns. Sorting all of the maximum-load-per-flight elements will produce an empirical CDF. The position of a given x value within the sort is its probability. For example, the median value in the sorted array has a probability of 0.50. Thus it is possible to choose three such values, and solve for the parameters of the EVD. For example, the code can choose the values associated with p={0.50, 0.25, 0.125). The PDTA code chooses at least seven distinct sets of three values, solves the three equations for μ,σ,ξ, and averages the results to obtain good estimates of μ,σ,ξ.

This is called the Method of Quantiles. The average of the seven distinct points is the starting point for a minimization method, the Nelder-Mead algorithm, that finds a set of parameters that minimizes the total absolute error while leaning toward a conservative probability-of-survival. The EVD fits using this method tend to have correlations above 99.5%.

After this process the three EVD parameters are stored for use in the probability of failure calculations.

Figure 5 shows probability density functions for different general aviation aircraft usages.

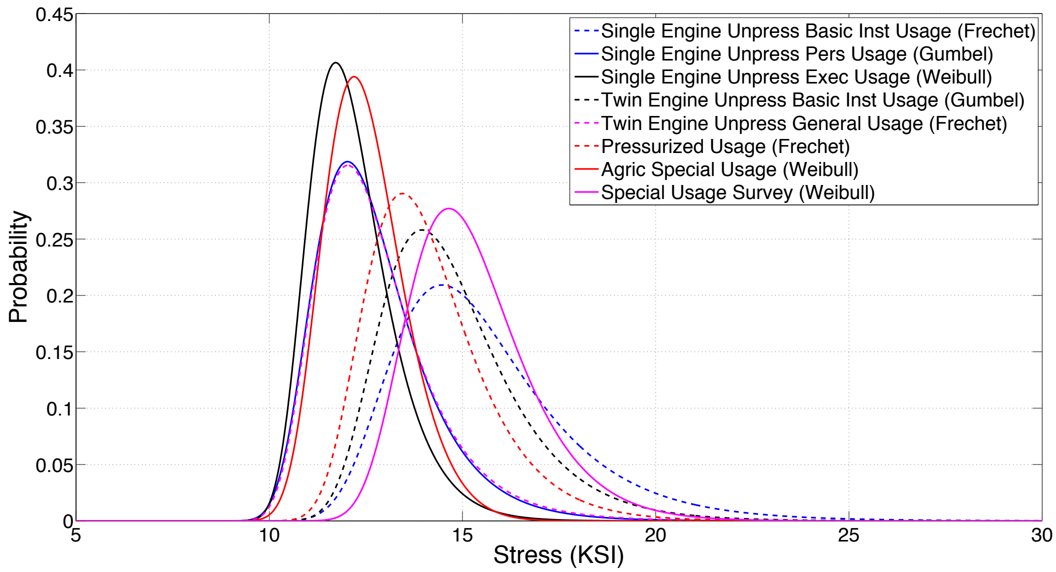

Figure 5. EVD PDF Distributions

C. Surrogate Fracture Mechanics Crack Growth

The mathematical statement of the problem is to determine the crack growth as a function of cycles by integrating the equations for the crack growth rate. That is, solving a set of coupled first order differential equations

$$\begin{aligned}\frac{\partial a}{\partial N} &= f(\Delta K, a, c)\\ \frac{\partial c}{\partial N} &= f(\Delta K, a, c)\end{aligned} \tag{4}$$

where f represents a crack growth law, e.g. Paris, Walker, or Nasgro. The crack shape is represented as an ellipse (or partial ellipse) and a and c represent major and minor axes. (Depending on the crack shape, either "a" or "c" may be the major axis). The stress intensity factor is decomposed as $K = \beta(a)\sigma\sqrt{\pi a}$, where β denotes the geometry correction factor.

The challenge in solving Eq. (4) is to integrate the differential equations quickly and accurately. Robustness is important as the crack geometry may transition during crack growth, e.g., surface-to-corner-to-through crack. Efficiency is essential since the equations must be solved many times during a probabilistic analysis. The NASGRO software [5], developed by NASA and Southwest Research Institute (SwRI), in combination with an error based surrogate model (Kriging metamodel) will be used in this computer code as the crack growth engine.

The Kriging metamodel is of the form (See reference [6]):

$$y(x) \approx f(x) + Z(x) \tag{5}$$

where $y(x)$ is the unknown function of interest, $f(x)$ is a known (usually polynomial) function of $\boldsymbol{x}$, and $Z(x)$ is the realization of a stochastic process with mean zero, variance σ^2, and non-zero covariance. The $f(x)$ term in Eqn. (6) is similar to a polynomial model and provides a "global" model of the design space.

While $f(x)$ "globally" approximates the design space, $Z(x)$ creates "localized" deviations so that the Kriging model interpolates the n_s sampled data points.

The covariance matrix of $Z(x)$ is given by Eqn. (7).

$$Cov\left[Z(x^i), Z(x^j)\right] = \sigma^2 \mathbf{R}(x^i, x^j) \tag{6}$$

In Eqn. (7), $\mathbf{R}$ is the correlation matrix, and $R(x^i, x^j)$ is the correlation function between any two of the n_s sampled data points x^i and x^j. $\mathbf{R}$ is a $(n_s \text{ x } n_s)$ symmetric matrix with ones along the diagonal. The correlation function $R(x^i, x^j)$ is specified by the user; references [6], [7], and [8] discuss several correlation functions which may be used.

Predicted estimates, $\hat{y}(x)$, of the response $y(x)$ at untried values of x are given by:

$$\hat{y} = \hat{\beta} + \mathbf{r}^T(x)\mathbf{R}^{-1}\left(\mathbf{y} - \mathbf{f}\hat{\beta}\right) \tag{7}$$

where $\mathbf{y}$ is the column vector of length n_s which contains the sample values of the response, and $\mathbf{f}$ is a column vector of length n_s which is filled with ones when $f(x)$ is taken as a constant. In Eqn. (8), $\mathbf{r}^T(x)$ is the correlation vector of length n_s between an untried $\boldsymbol{x}$ and the sampled data points $\left\{x^1, \ldots, x^{n_s}\right\}$:

$$\mathbf{r}^T(x) = \left[R(x, x^1), R(x, x^2), \ldots, R(x, x^{n_s})\right]^T \tag{8}$$

In Eqn. (8), $\hat{\beta}$ is estimated using Eqn. (10)

$$\hat{\beta} = \left(\mathbf{f}^T\mathbf{R}^{-1}\mathbf{f}\right)^{-1}\mathbf{f}^T\mathbf{R}^{-1}\mathbf{y} \tag{9}$$

The estimate of the variance, σ^2, between the underlying global model $\hat{\beta}$ and $\mathbf{y}$, is estimated as:

$$\widehat{\sigma}^2 = \frac{\left(\mathbf{y} - \mathbf{f}\widehat{\beta}\right)^T R^{-1}\left(\mathbf{y} - \mathbf{f}\widehat{\beta}\right)}{n_s} \tag{10}$$

Figure 6 shows schematically the surrogate model process and it is explained briefly as follows.

1. Initial realizations of the random variables are produced and the initial training points are generated using the NASGRO software.

2. The response surfaces for residual strength and initial crack size are constructed based on the initial training points.

3. A new realization of the random variables is generated.

4. The surrogate model is evaluated for the random realization generated in step 3.

5. The error giving by the Kriging surrogate model is compared against an user defined threshold error.

    a. If the error is not acceptable, the random realization generated in step 3 is evaluated using NASGRO and the response surface is updated with the new training point.

    b. If the error is acceptable, the individual SFPOF is computed and the inspection is applied to the sample.

6. Repeat steps 3 through 5 for the given number of samples.

7. Compute the SFPOF, Hz, CTPOF for the total number of samples.

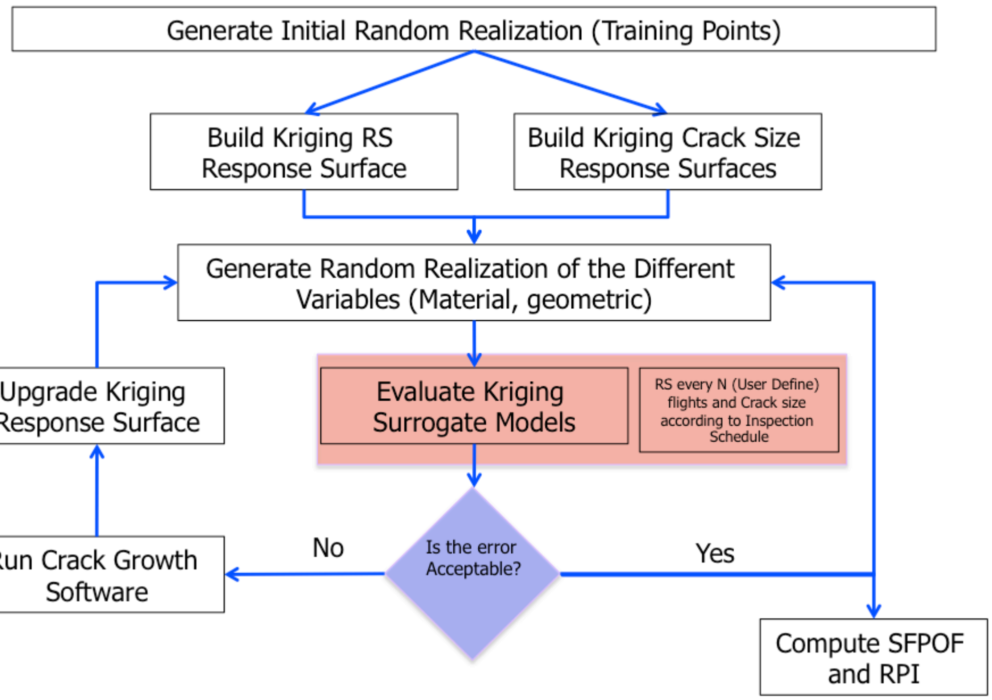

Figure 6. Surrogate Model Flowchart

D. Probabilistic Methods

The probability-of-failure during a single flight assuming no failures before that flight can be determined as the probability that the maximum load experienced during the flight will exceed the residual strength of the structure. Considering only three random variables for simplicity, this is written mathematically as

$$SFPOF(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a(t))g(K_C)h(\sigma)dadK_C d\sigma \tag{11}$$

where a denotes crack size, t–flight hours, K_C–fracture toughness, σ–maximum load experienced per flight, and *f, g, h* are the corresponding probability density functions. Additional random variables can be added in a similar fashion.

Eq. 11 can be further simplified by analytically integrating $h(\sigma)$ in terms of the other random variables, that is, determine the cumulative distribution function $H(\sigma)$ that defines the probability of the maximum load being less than the residual strength, $\sigma \geq K_C / \beta(a)\sqrt{\pi a}$. The probability of the maximum load exceeding the residual strength is equal to 1 minus $H(\sigma)$ at $K_C / \beta(a)\sqrt{\pi a}$. Eq. 11 becomes

$$SFPOF(t) = \int_{-\infty}^{\infty}\int_{0}^{\infty} f(a,t)g(K_C)(1 - H(K_C / \beta(a)\sqrt{\pi a}))dadK_C \tag{12}$$

where H is the CDF of the maximum load (F_{EVD}). Eq. 12 is now a two dimensional integral. Rewriting in terms of the initial crack size yields

$$SFPOF(t) = \int_0^{\infty}\int_{-\infty}^{\infty}\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right] f_{a_0}(a_0) f_{K_c}(K_c)\, da_0 dK_c \tag{13}$$

Eq. 13 is an example of conditional expectation. In this equation, the function $F_{EVD}(K_C / \beta(a(a_o,t))\sqrt{\pi a(a_0,t)})$ is the probability of the maximum load exceeding the residual strength. Thus, $SFPOF(t)$ is the expected value of 1-F_{EVD}. Significant variables not considered in this approach are crack growth variability and geometric variations. Additional random variables can be added to Eq. 13.

The $SFPOF(t)$ using sampling and accounting for any number of random variables is presented in Eq. 14.

$$SFPOF(t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a_o^i,T)\sqrt{\pi a(a_o^i,T)}}\right)\right] \tag{14}$$

Finally, the hazard function can be expressed as:

$$Hz(T) \approx \frac{1}{R(T)}\frac{1}{N}\sum_{i=1}^{N}\left[\prod_{t=1}^{T-1} F_{EVD}\left(\frac{K_C^i}{\beta(a_o^i,t)\sqrt{\pi a(a_o^i,t)}}\right)\right]\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a_o^i,T)\sqrt{\pi a(a_o^i,T)}}\right)\right] \tag{15}$$

where $\left[1 - F_{EVD}\left(K_C^i / \beta(a_o,T)\sqrt{\pi a(a_o,T)}\right)\right]$ represents the failure during the flight (T), $\left[\prod_{t=1}^{T-1} F_{EVD}\left(K_C^i / \beta(a_o^i,t)\sqrt{\pi a(a_o^i,t)}\right)\right]$ represents the probability of survival during the previous flights (T-1), and $R(T)$ is the reliability considering all random variables.

SFPOF has been also formulated in references [9] through [12].

The cumulative probability-of-failure calculated by sampling can be expressed as:

$$CTPOF(T) \approx \frac{1}{N}\sum_{i=1}^{N}\left[1 - \prod_{t=1}^{T}(1 - SFPOF(t))\right] \tag{16}$$

and the reliability term from Eq. 15 can be computed from Eq. 16.

$$R(T) = 1 - CTPOF(T) \tag{17}$$

III. Example Problem

Table 2 presents the crack growth parameters for a through crack in a hole. The random variables included initial crack size (Lognormal Distribution), fracture toughness (Normal Distribution), and loading (Gumbel Distribution). Table 3 presents the loading variables.

Table 2. Example Problem Definition

Quantity	**Definition**
Nasgro Crack Growth Model.	TC03 – Thought crack in a hole
Geometric Variables	Width = 2.5 in. Thickness = 0.09 in. Hole Diameter = 0.10 in. Hole Offset = 0.5 in.
Fracture Toughness Distribution	Normal: Mean = 34.8ksi√in. Standard Deviation = 3.9 ksi√in.
Initial Crack Size Distribution	Lognormal Median = 0.00163 in. Mean = ln(median) = -6.420 Standard Deviation = 1.113 ksi√in.
Extreme Value Distribution (Weibull)	Location = 5.0, Scale = 10.0, and Shape = 5.0
Material	Al-2024

Table 3. Example Problem Loading Variables

Variable	**Value**
Usage	Single Engine Unpressurized Basic Instructional Usage
Design LLF Maneuver	3.8, -1.52
Design LLF Gust	3.155, -1.155
Ground Stress (psi)	-4,550
One-g stress (psi)	7,100
Flight Length and Velocity Matrix	Dur/Vel 0.80 0.85 0.90 0.95 1.00 0.50: 0.05 0.05 0.10 0.10 0.10 0.65 0.60: 0.05 0.05 0.05 0.05 0.15 0.70 0.70: 0.10 0.00 0.05 0.05 0.15 0.75 0.80: 0.15 0.00 0.05 0.05 0.10 0.80 0.90: 0.20 0.00 0.00 0.00 0.10 0.90 1.00: 0.25 0.00 0.00 0.05 0.05 0.90 1.10: 0.15 0.00 0.00 0.00 0.05 0.95 1.20: 0.05 0.00 0.00 0.00 0.05 0.95
Flight Length and Weight Matrix	Dur/Wei 0.80 0.85 0.90 0.95 1.00 0.50: 0.05 0.00 0.00 0.00 0.20 0.80 0.60: 0.05 0.00 0.00 0.00 0.20 0.80 0.70: 0.10 0.00 0.00 0.00 0.15 0.85 0.80: 0.15 0.00 0.00 0.00 0.15 0.85 0.90: 0.20 0.00 0.00 0.00 0.10 0.90 1.00: 0.25 0.00 0.00 0.00 0.10 0.90 1.10: 0.15 0.00 0.00 0.00 0.05 0.95 1.20: 0.05 0.00 0.00 0.00 0.05 0.95
Average Velocity (Vno/Vmo (Knots))	165

To calculate the Single Flight Probability of Failure (SFPOF) as presented in Eq. 14, the code was run using 5,000 Monte Carlo Samples, a surrogate model with ten initial training points, and an user defined error threshold equal to five percent.

Figure 7 shows the total number of training points (NASGRO evaluations) used as a function of the total number of Monte Carlo Samples. From Figure 7 it can be observed that for 5,000 samples and an error threshold equal to five percent, only ninety-eight NASGRO evaluations were needed which truly reduces the computational weight.

Figure 8 shows a comparison between the SFPOF if NASGRO is evaluated 5,000 times (Exact- Black line – Running all the 5,000 evaluations using Nasgro) and if the surrogate model is used (Kriging-Red Dots). The total time for the fully NASGRO run was 18 hours. The total time for the Kriging model was 2.3 hours.

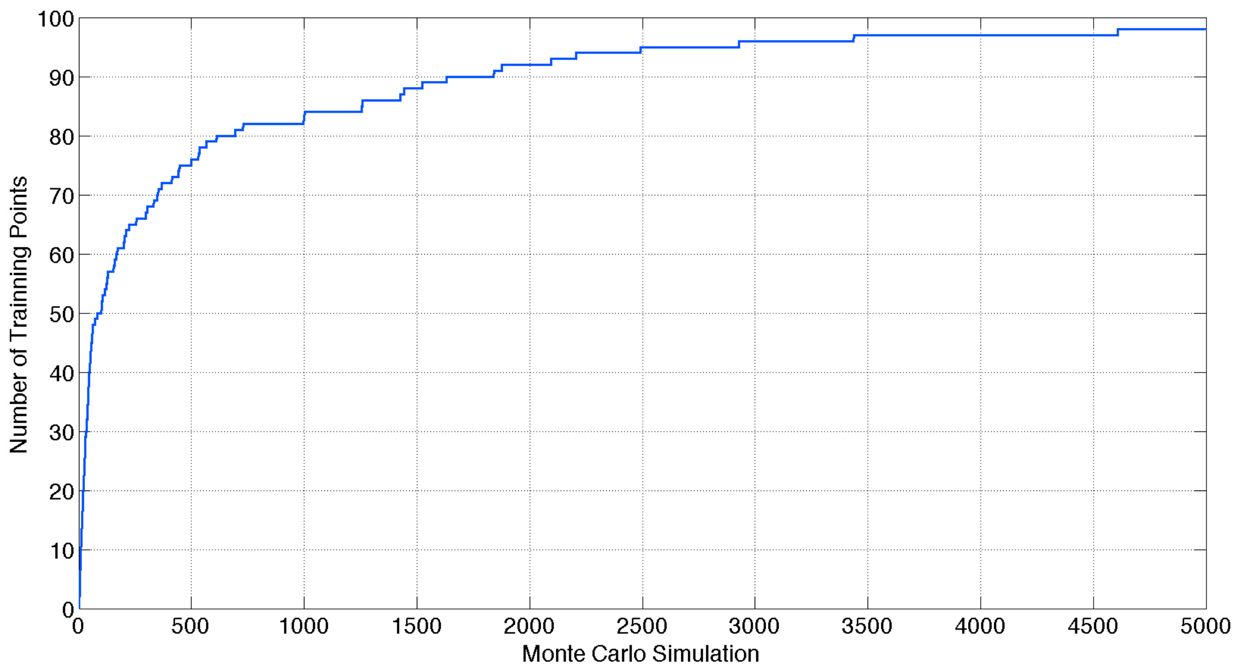

Figure 7. Surrogate Model Number of Training Points

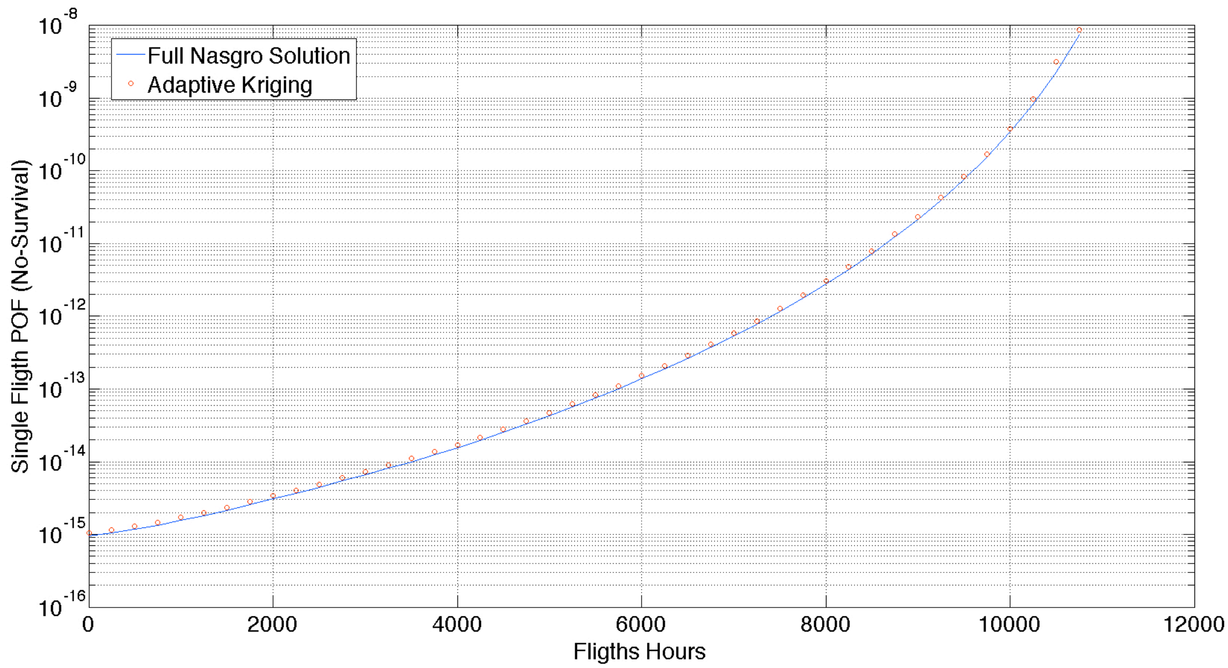

Figure 8. SFPOF Calculation Results

IV. Conclusions

Probabilistic damage tolerance evaluation of General Aviation Aircraft is vital in order to provide important insight into the severity or criticality of a potential structural issue. The methodology described above provides a tool to perform probabilistic damage tolerance evaluation for real general aviation applications. The methodology includes loading generation, Extreme Value Distribution generation from the same loading using to perform the crack growth analysis, crack growth analysis incorporated with a adaptive Kriging metamodeling subroutine, and probabilistic methods to compute probabilities of failure and hazard rate.

Probabilistic damage tolerance evaluations are computational expensive, for that reason an adaptive Kriging metamodeling is helping to improve the computational time by almost 16 hours (8X times faster) for the example problem presented.

Future work includes the addition of more random variables to the Kriging subroutine and implement inspections and repair to the Kriging model.

V. Acknowledgements

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