Multiple Repair Scenarios in Aircraft Fleets Using the Weighted Branch Integration (WBI) Method

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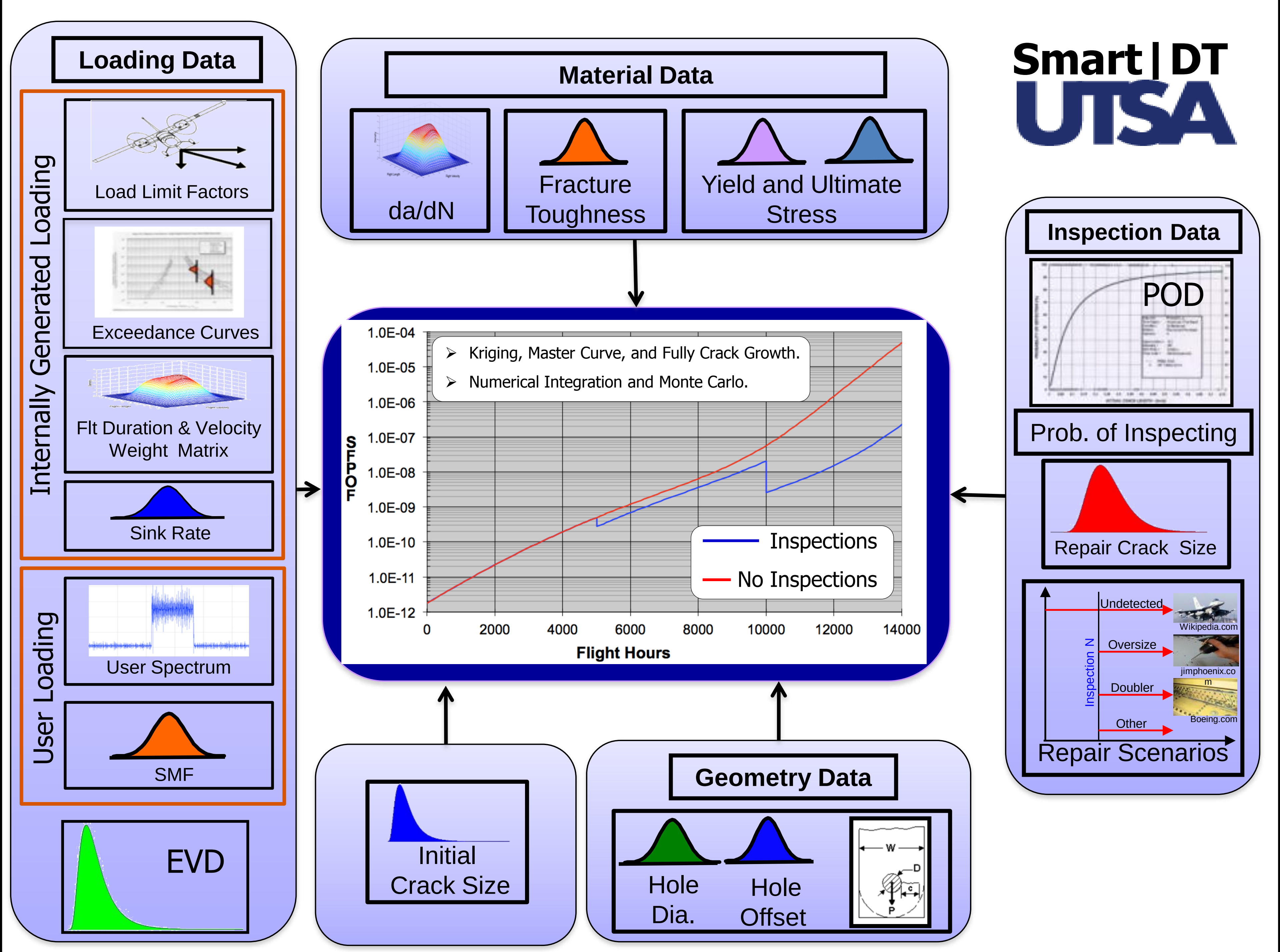
Outline

- ✓ SMART|DT Overview
- ✓ Methodology Flowchart
- ✓ Single Flight Probability of Failure (SFPOF)
- ✓ Weighted Branch Integration (WBI) Method
  - ✓ Inspection and Repair
  - ✓ Multiple Repair Scenarios within/between inspections
- ✓ Example Problem
  - ✓ Example – Multiple Repair Scenarios



Smart | DT
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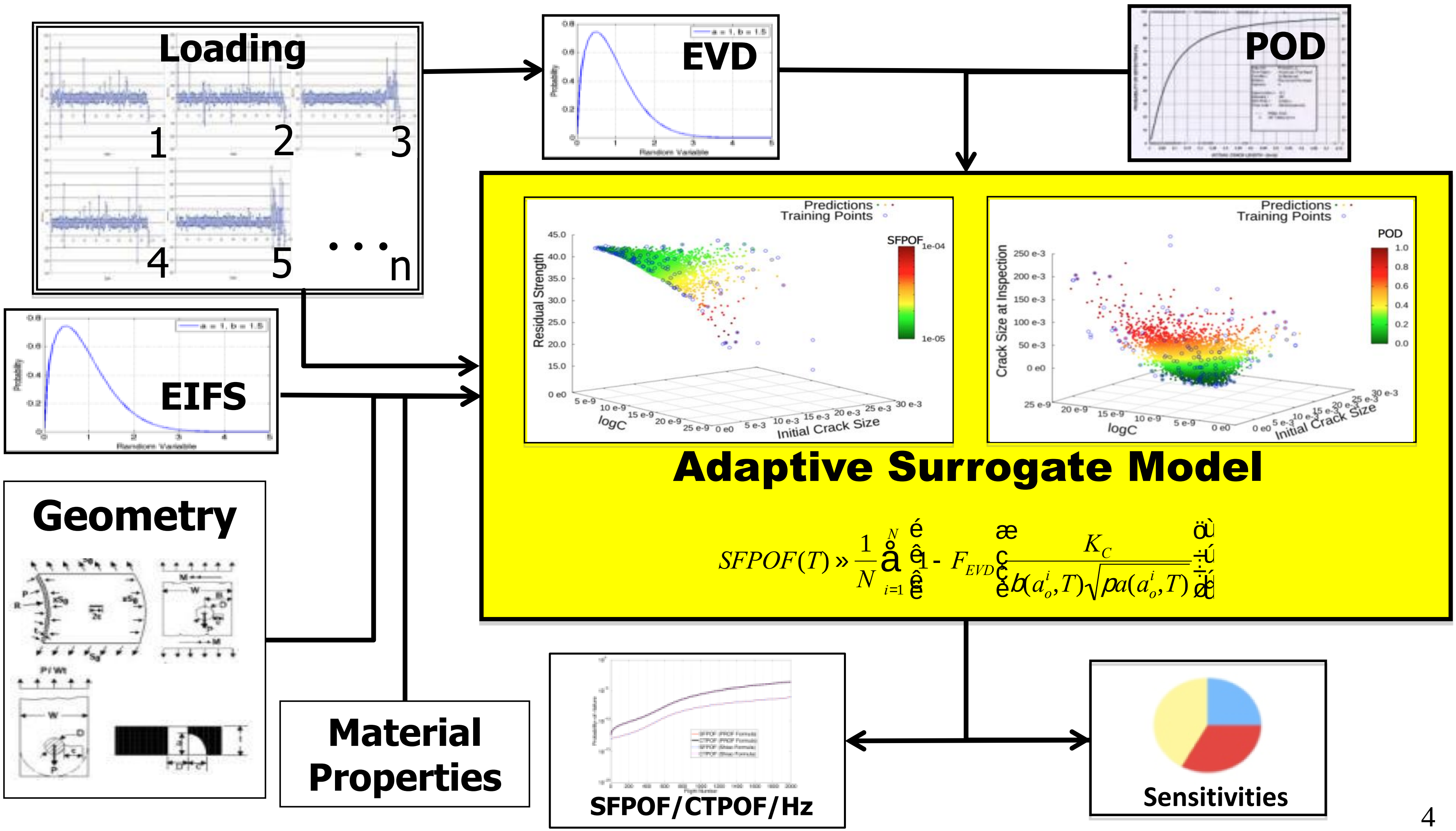
Loading Data
Internally Generated Loading
Load Limit Factors
Exceedance Curves
Flt Duration & Velocity Weight Matrix
Sink Rate
User Loading
User Spectrum
SMF
EVD
Material Data
da/dN
Fracture Toughness
Yield and Ultimate Stress
Kriging, Master Curve, and Fully Crack Growth.
Numerical Integration and Monte Carlo.
SFPOF
Flight Hours
Inspections
No Inspections
Inspection Data
POD
Prob. of Inspecting
Repair Crack Size
Undetected
Oversize
Doubler
Other
Inspection N
Wikipedia.com
jimphoenix.com
Boeing.com
Repair Scenarios
Initial Crack Size
Geometry Data
Hole Dia.
Hole Offset

Damage Tolerance Methodology



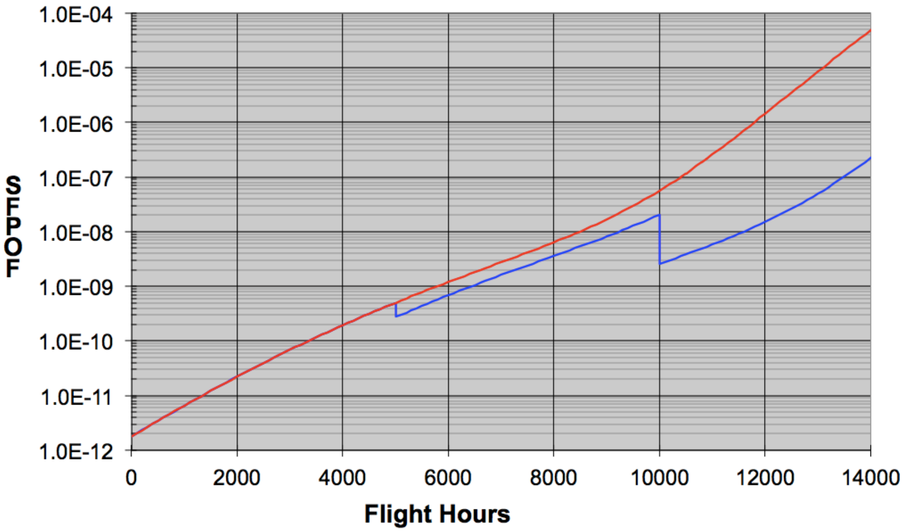
Probability of Failure

SMART$_{DT}$

SMall Aircraft Risk Technology - Damage Tolerance Analysis

Why SFPOF is Important

- It is used as a risk metric.
- Ensures structural integrity.
- It is used to schedule inspections and evaluate repairs.
- It is used for fleet management



Probability of Failure

The probability-of-failure is the probability that maximum value of the applied stress (during the next flight) will exceed the residual strength σ_{RS} of the aircraft component

$$P_f = P\left[\sigma_{Max} > \left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right] = P\left[\sigma_{Max} > \sigma_{RS}\right]$$

The CDF of the maximum stress in a flight (F_{EVD}) can be determined using extreme value theory

$$P_f(t | a_i K_C) = 1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)$$

Lincoln formulation – “conditional” Single Flight POF

$$SFPOF(t) = \int_0^{\infty}\int_{-\infty}^{\infty}\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right] f_{a_0}(a_0) f_{K_c}(K_c)\ \mathrm{d}a_0\,\mathrm{d}K_c$$

Single Flight Probability of Failure (SFPOF)

Hazard Fn.

Single flight probability of failure formula can be considered as a hazard function

$$Hz(T) = \frac{1}{R(T)} \int_{-\infty}^{\infty}\int_{0}^{\infty} \left[\prod_{t=1}^{T-1} F_{EVD}\left(\frac{K_C}{b(a_o,t)\sqrt{\pi a(a_o,t)}}\right)\right]\left[1 - F_{EVD}\left(\frac{K_C}{b(a_o,T)\sqrt{\pi a(a_o,T)}}\right)\right] f_{a_0}(a_0)\, f_{K_c}(K_c)\; \mathrm{d}a_0\, \mathrm{d}K_c$$

Probability of survival until flight *T-1*

Probability of failure during the flight *T*

R(T) is the reliability considering all random variables

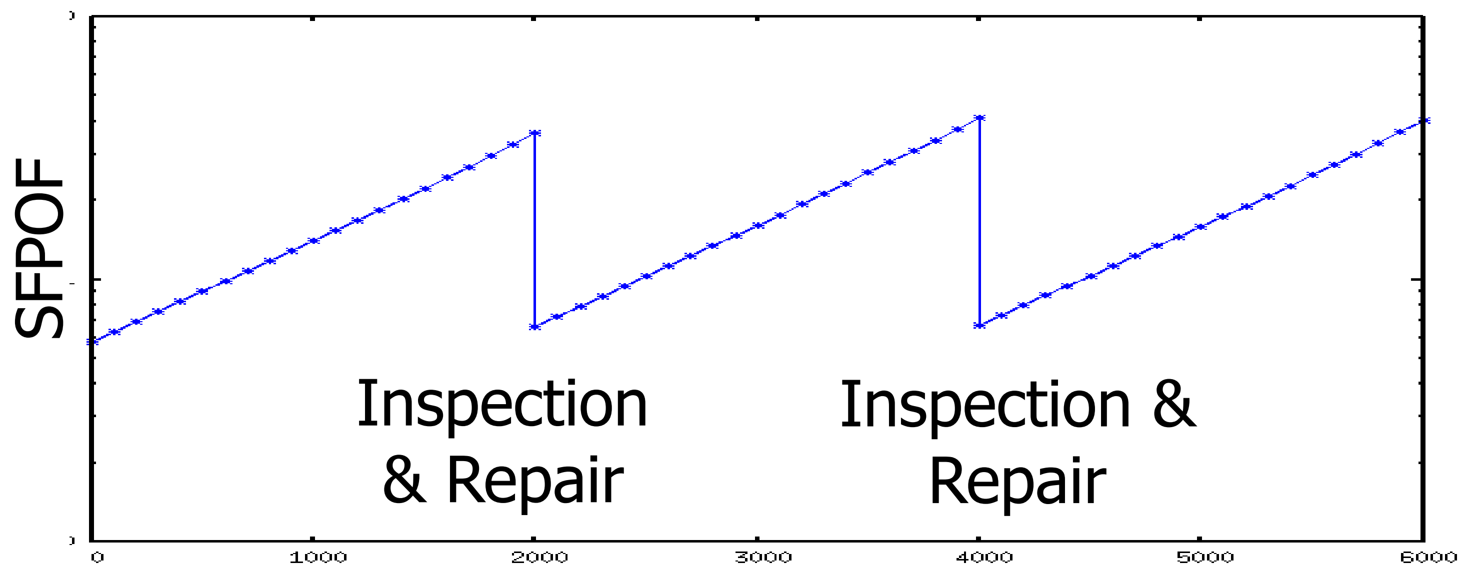
For any number of random variables

$$Hz(T) = \frac{1}{R(T)} \int_{-\infty}^{\infty} \left[\prod_{t=1}^{T-1} F_{EVD}\left(\frac{K_C}{b(a_o,t)\sqrt{\pi a(a_o,t)}}\right)\right]\left[1 - F_{EVD}\left(\frac{K_C}{b(a_o,T)\sqrt{\pi a(a_o,T)}}\right)\right] f_{\mathbf{X}}(\mathbf{x})\; \mathrm{d}\mathbf{x}$$

POF Difficult Integral?

$$POF(t) = \int_{-\infty}^{\infty} \left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}} \right) \right] f_{\mathbf{x}}(\mathbf{x})\ d\mathbf{x}$$

- Small probabilities: (1E-14 - 1E-5)
- Time dependent: multiple integrals
- Inspections and Multiple repairs leads to Multimodal pdf’s
- Multiple Random Variables



Weighted Branch Integration (WBI)

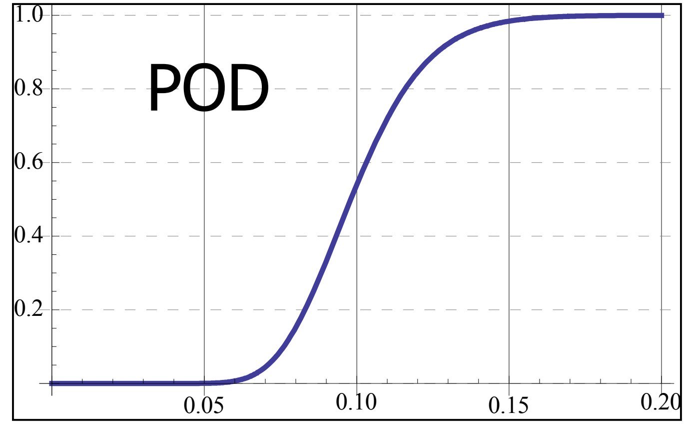
SMART$_{DT}$

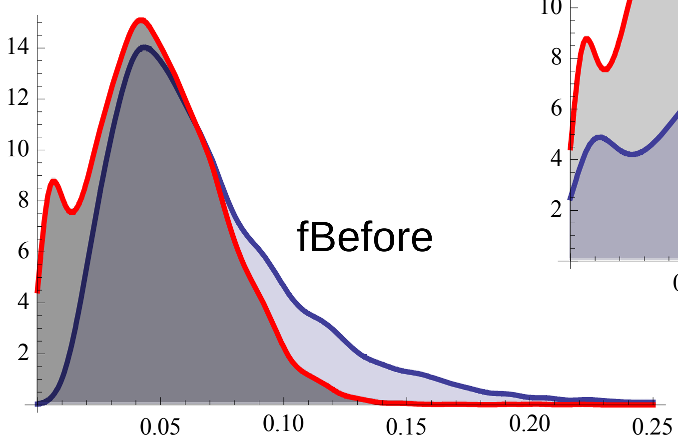
SMall Aircraft Risk Technology - Damage Tolerance Analysis

Inspections

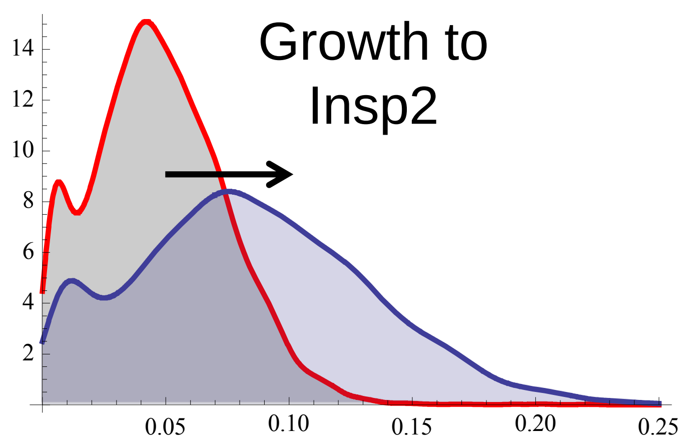
$$f_{After}(a) = p_{\infty} f_R(a) + (1 - POD(a)) f_{Before}(a)$$

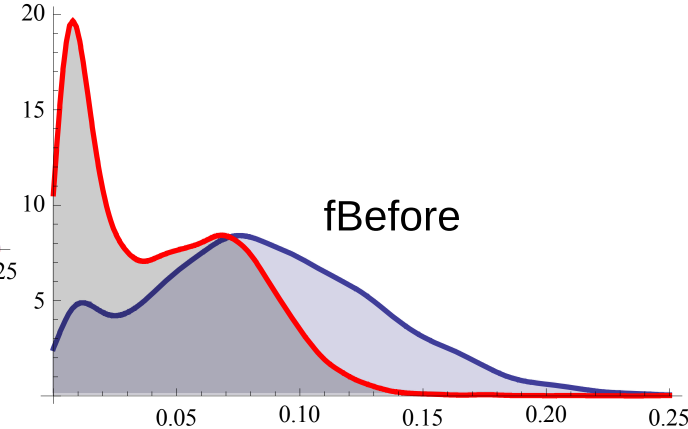
A percentage of cracks are detected and repaired. This leads to multi-modal PDFs.





Insp 1





Insp 2

Methodology

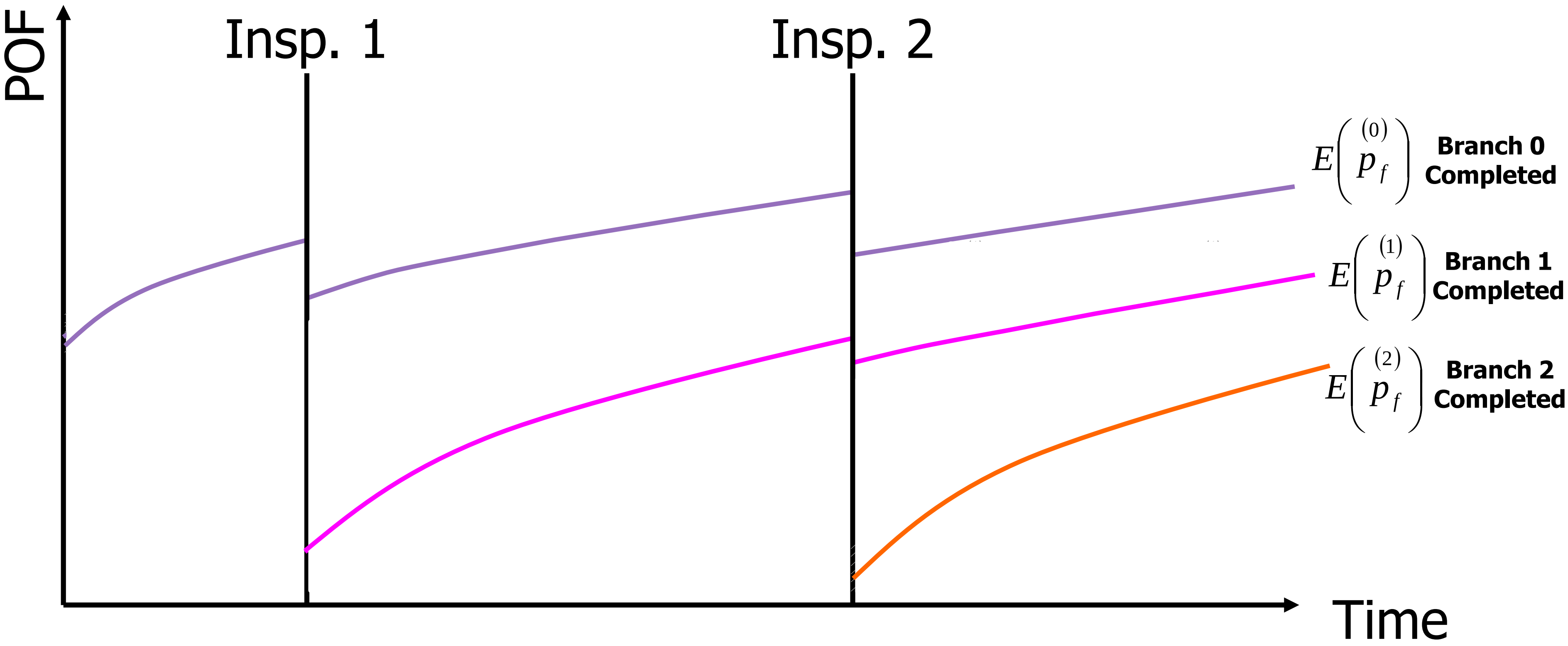
WBI Monte Carlo/NI

- Decomposes the probability integral into a series of integrals with unimodal crack size PDFs.
- Each integral is represented as a "branch" of the analysis where a branch represents a repair scenario.
- Each branch computed independently.
- Overall POF determined as a sum from all branches.

Material, geometry, and crack growth properties can be changed for each branch (different repair scenarios can be analyzed).

Implementation

Monte Carlo – Simple Repair



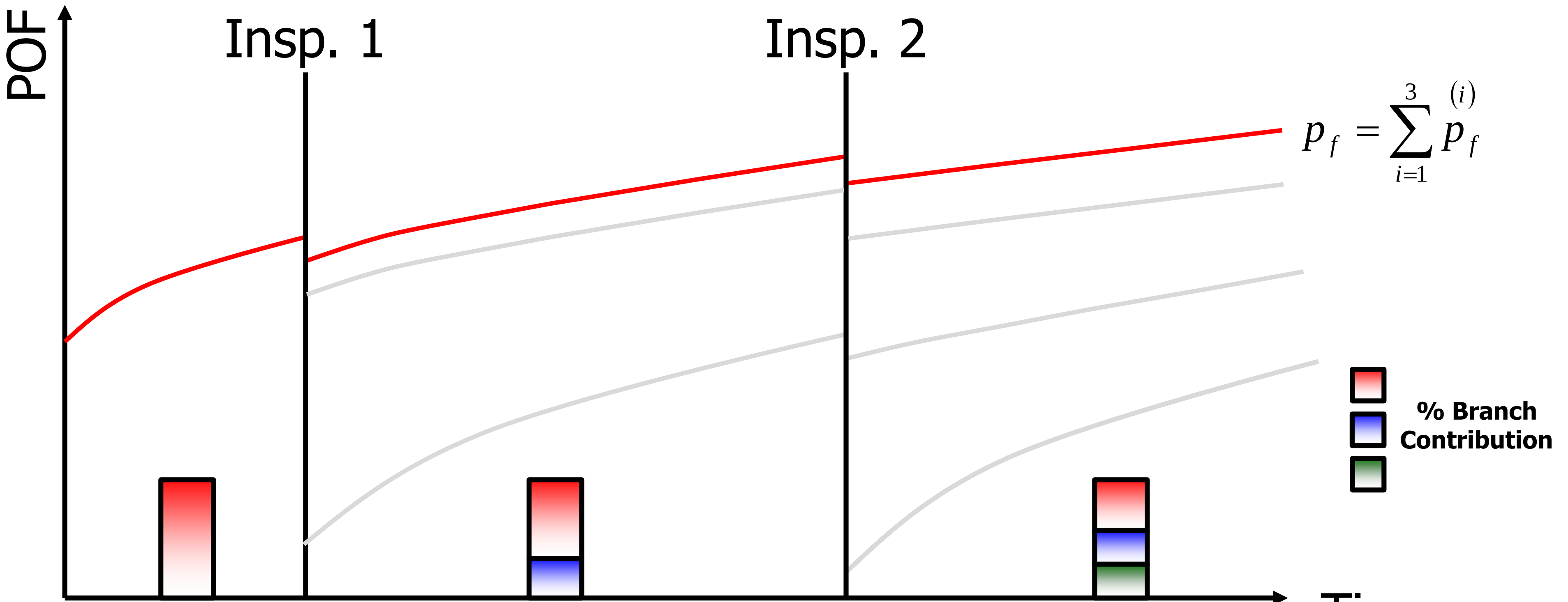
$N = \text{User Samples}$

$$p_{\det}^{(1)} = \frac{1}{N}\sum POD_1(a_j^0)$$

$$p_{\det}^{(2)} = \frac{1}{N}\sum POD_2(a_j^0) + \frac{1}{N}\sum POD_1(a_j^1)$$

Implementation

Monte Carlo – Simple Repair

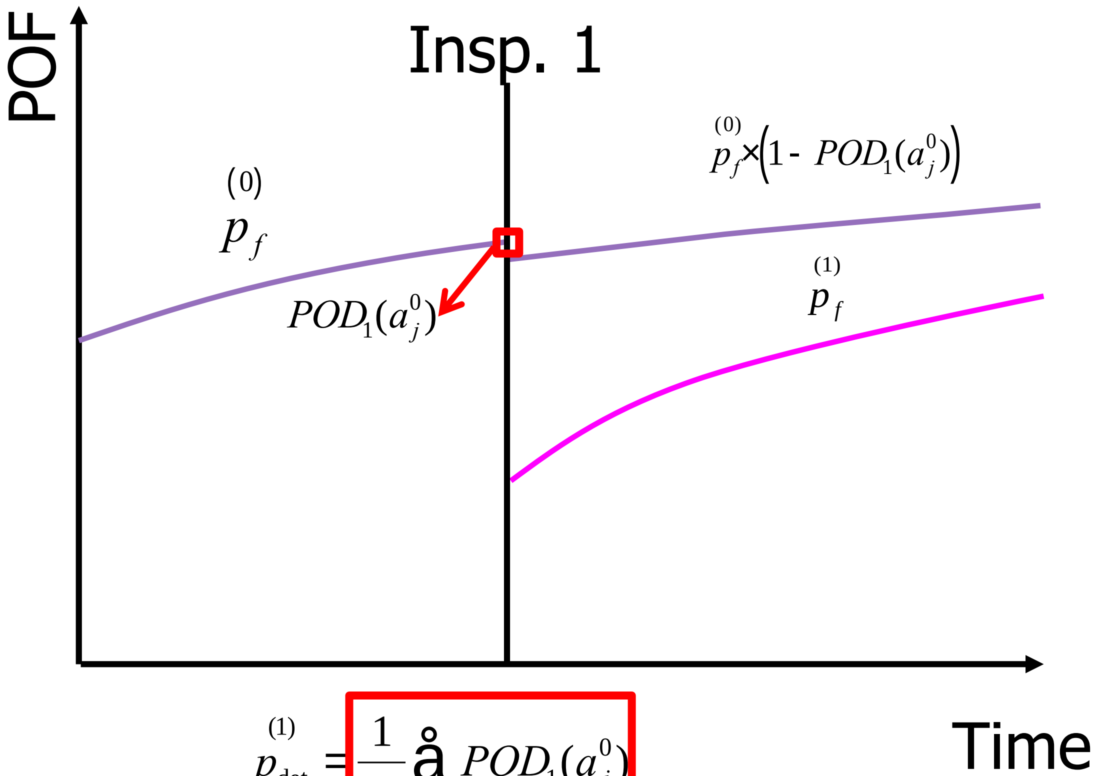


$$p_{\det}^{(1)} = \frac{1}{N_1} \sum POD_1(a_j^0)$$

$$p_{\det}^{(2)} = \frac{1}{N_1} \sum POD_2(a_j^0) + \frac{1}{N_2} \sum POD_1(a_j^1)$$

Add the contribution from each branch into the final POF

WBI
Post Processing



$$p_{\det}^{(1)} = \frac{1}{N_1} \sum POD_1(a_j^0)$$

1. Compute WBI only for the baseline including the inspection points
2. Record *pdet* (smart output)
3. Compute WBI for the repair branches
4. Compute final *pof* a weighted sum of the results from all branches for each time

WBI

Post Processing Example I

FLT NUM	POF (BL)	POF (Rep.)	POF (Rep.)*pdet	POF
0	1.53E-14	0.0	0.0	1.53E-14
1000	2.79E-13	0.0	0.0	2.79E-13
2000	2.98E-12	0.0	0.0	2.98E-12
3000	1.98E-11	0.0	0.0	1.98E-11
4000	8.89E-11	0.0	0.0	8.89E-11
5000	3.02E-10	0.0	0.0	3.02E-10
6000	8.67E-10	0.0	0.0	8.67E-10
7000	2.31E-09	0.0	0.0	2.31E-09
7500	3.76E-09	0.0	0.0	3.76E-09
7500	5.65E-10	1.53E-14	6.49E-15	5.65E-10
8000	9.35E-10	7.02E-14	2.97E-14	9.36E-10
9000	2.53E-09	9.73E-13	4.11E-13	2.53E-09
10000	7.27E-09	8.11E-12	3.43E-12	7.27E-09
11000	2.41E-08	4.37E-11	1.85E-11	2.41E-08
12000	9.32E-08	1.68E-10	7.12E-11	9.33E-08

```
*********************************************
********** PDTA Code Progress   **************
*********************************************

  Branch Number =    1 (Samples =  1000000)
  Sample no.    100000     10 % complete.
  Sample no.    200000     20 % complete.
  Sample no.    300000     30 % complete.
  Sample no.    400000     40 % complete.
  Sample no.    500000     50 % complete.
  Sample no.    600000     60 % complete.
  Sample no.    700000     70 % complete.
  Sample no.    800000     80 % complete.
  Sample no.    900000     90 % complete.
  Sample no.   1000000    100 % complete.

  Inspection:   1 Cracks Detected:   42.27%.

*********************************************
********* PDTA analysis complete ************
*********************************************


Warning statements were generated.
Please see hnd_insp.wrn for more information.
Total CPU  time =      10.327 secs
Total wall time =      10.000 secs
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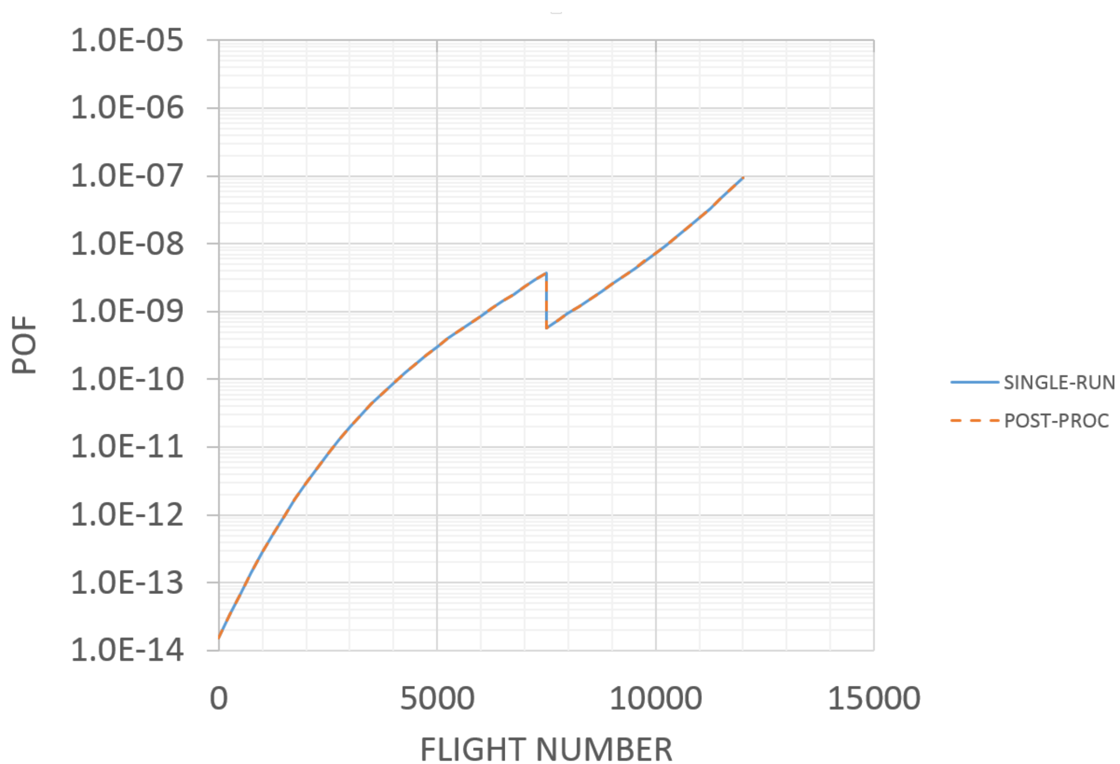
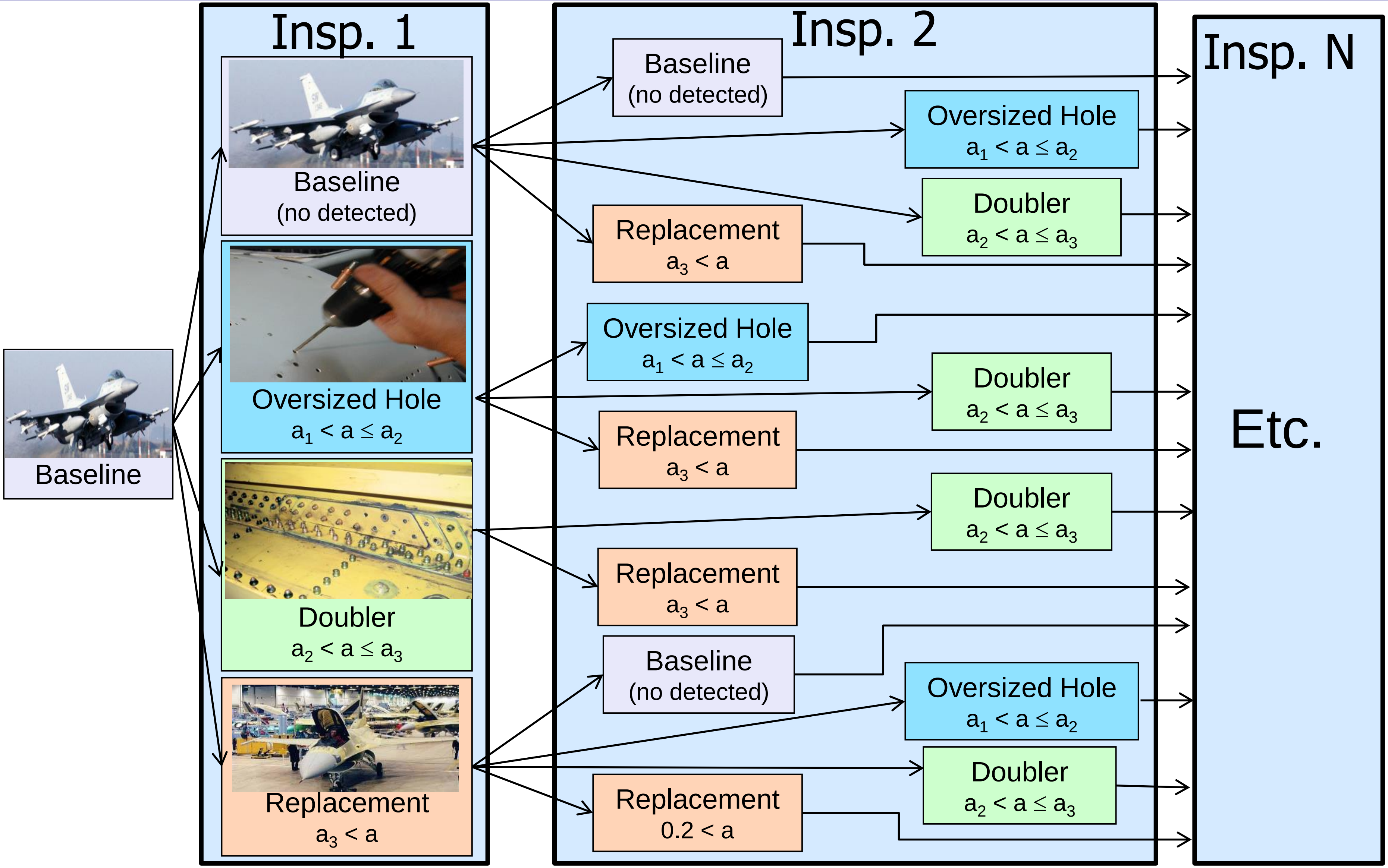


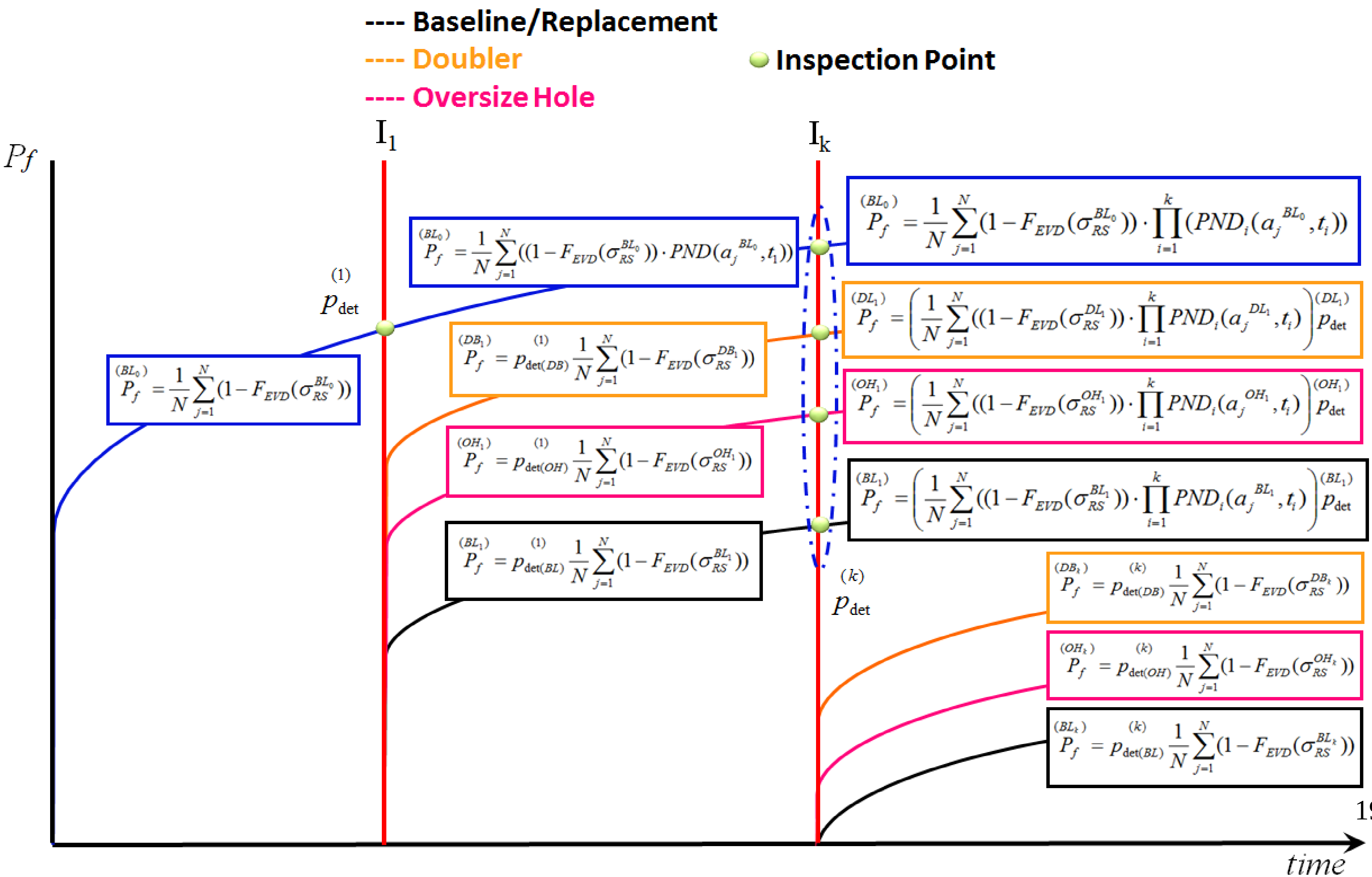
Multiple Repair scenarios

Multiple Repair Example



WBI Implementation

Monte Carlo – Multiple Repair



External Crack Growth (ECG) Software Link

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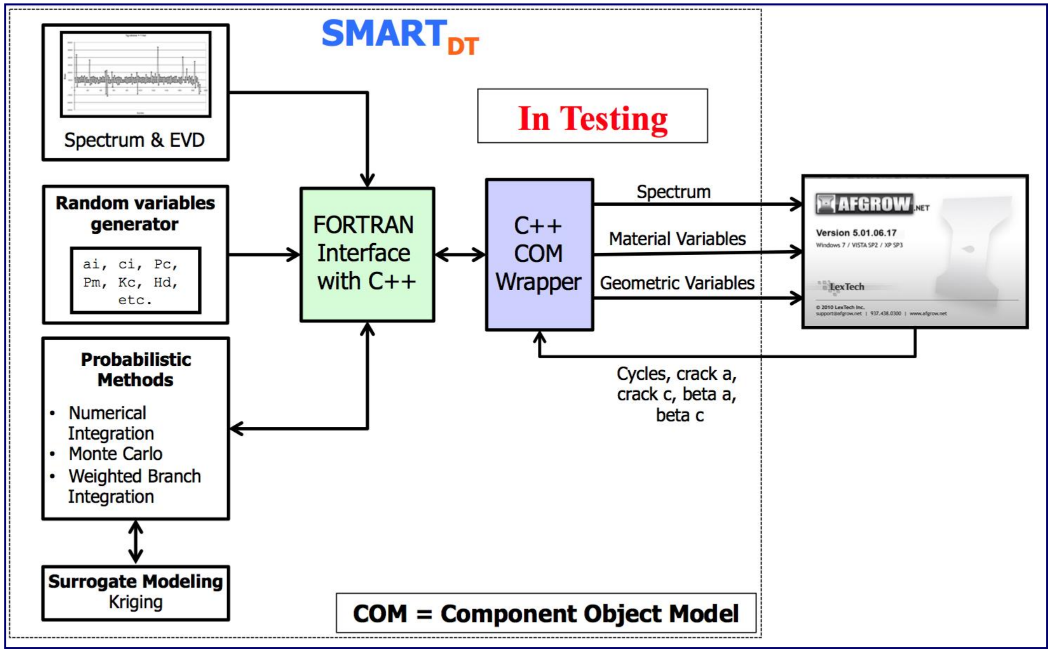
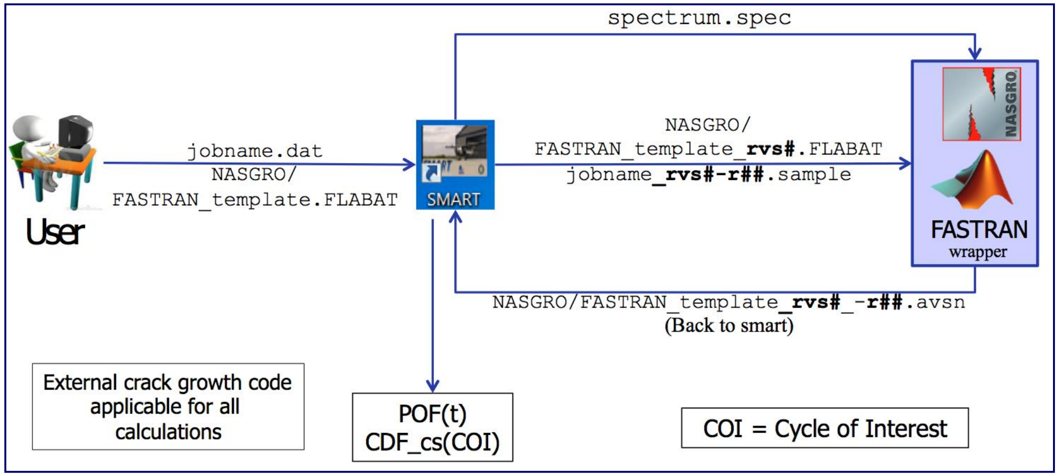
ECG Software Link for Single and Multiple Repairs

AFGROW Interface: COM driven



NASGRO/FASTRAN Interface: File based I/O



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Example Problem

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SMall Aircraft Risk Technology Damage Tolerance Analysis

Multiple Repair Scenarios

	Baseline/ Replacement	Hole Oversize	Doubler
Initial Crack Size	W~(3.5,0.01)	W~(3.5,0.01)	No-Repaired
Crack Growth Curve	Baseline CG curve	Hole Oversize CG curve	Doubler CG curve
EVD	G~(10.5,0.8)	G~(10.45,0.89)	G~(10.45,0.89)
POD	Baseline curve	Baseline curve	Doubler curve
POI	1.0	1.0	1.0
Repair Range	> 0.2	0.0-0.003	0.003-0.2

Service Life = 10,000 hours
Inspections every = 1,000 flight hours

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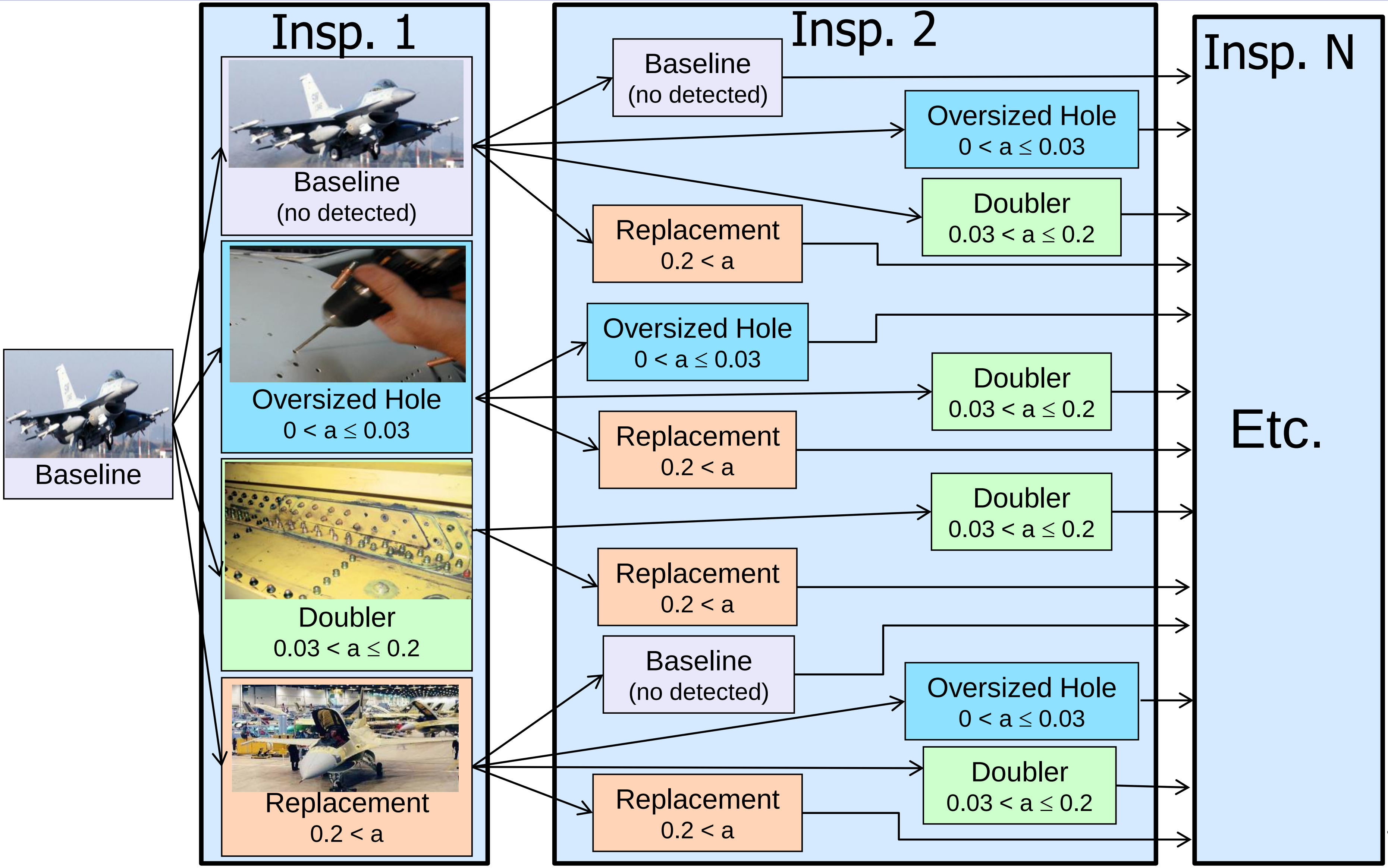
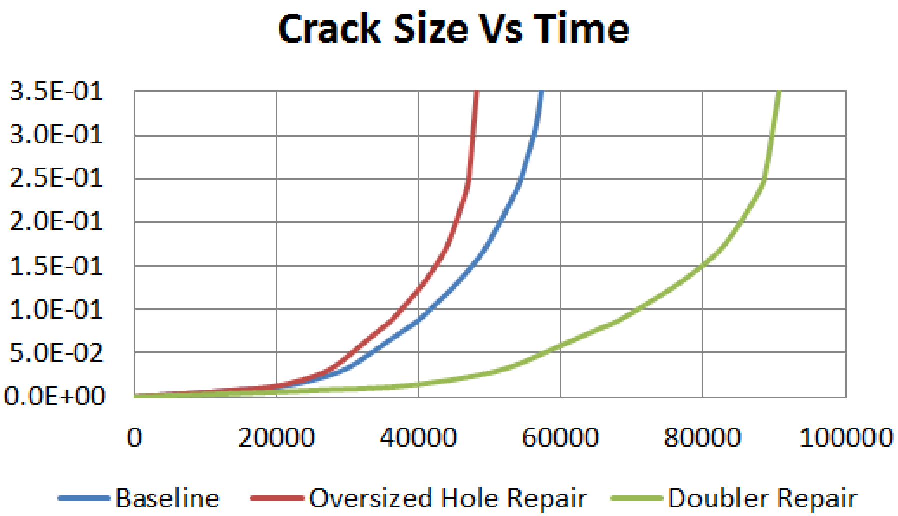
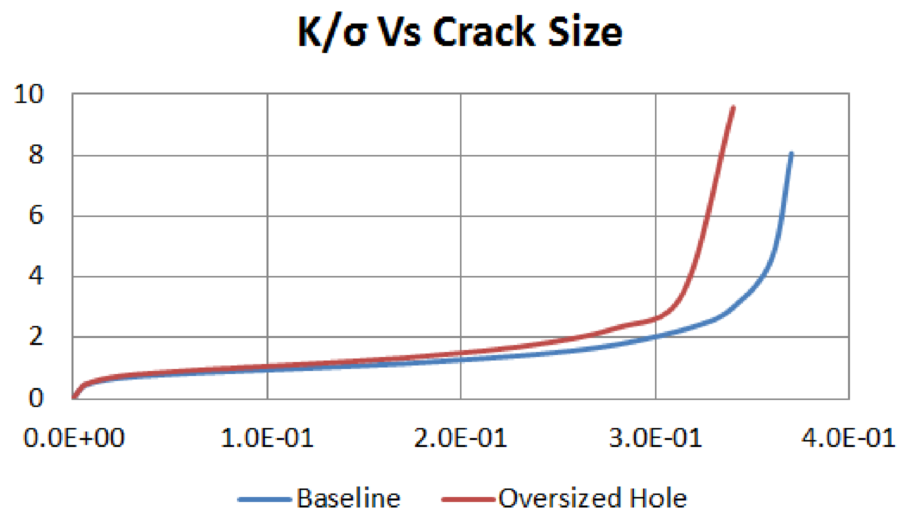
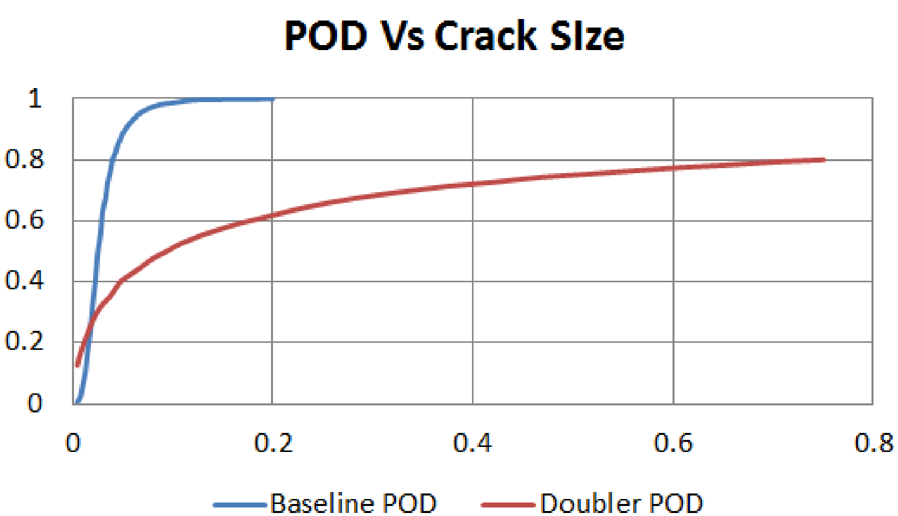
Example

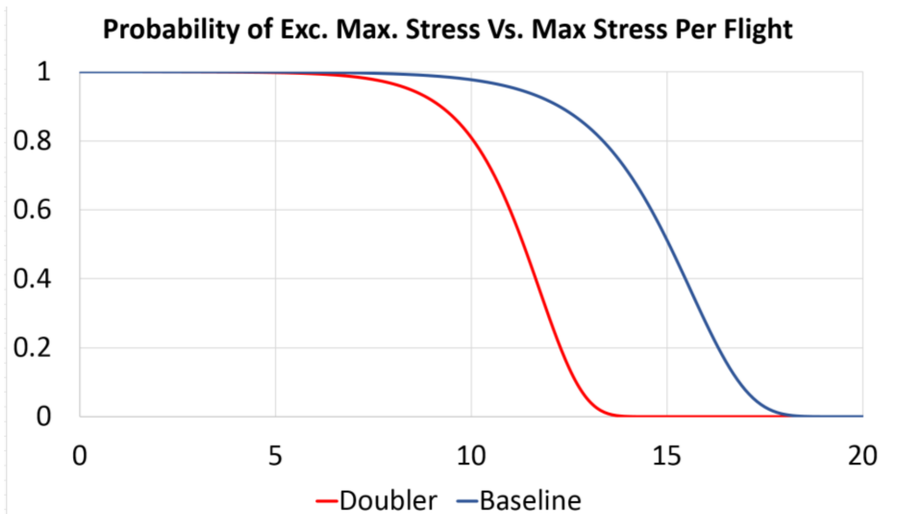


Multiple Repair Scenarios

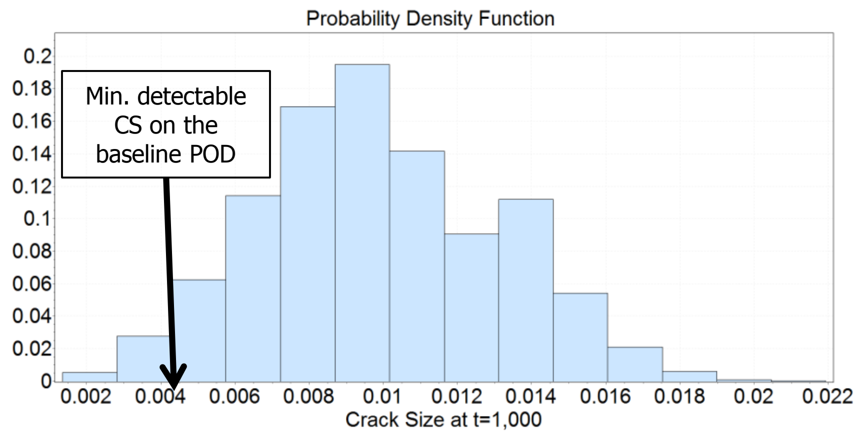


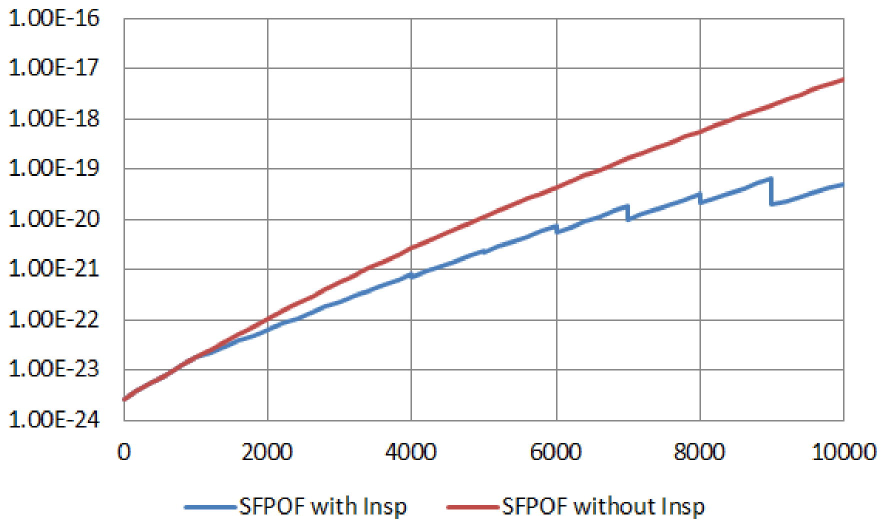






Multiple Repair Scenarios (Results)






```
Inspection:  1. Cracks Detected:   6.48%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:   6.47% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   0.01% - ( 0.020 < a < 1000.000 )
Inspection:  2. Cracks Detected:   8.84%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:   8.73% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   0.11% - ( 0.020 < a < 1000.000 )
Inspection:  3. Cracks Detected:  11.23%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  10.75% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   0.48% - ( 0.020 < a < 1000.000 )
Inspection:  4. Cracks Detected:  13.61%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  12.09% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   1.52% - ( 0.020 < a < 1000.000 )
Inspection:  5. Cracks Detected:  15.76%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  12.19% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   3.57% - ( 0.020 < a < 1000.000 )
Inspection:  6. Cracks Detected:  17.37%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  12.05% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   5.32% - ( 0.020 < a < 1000.000 )
Inspection:  7. Cracks Detected:  18.11%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  11.97% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   6.15% - ( 0.020 < a < 1000.000 )
Inspection:  8. Cracks Detected:  18.68%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  12.14% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   6.54% - ( 0.020 < a < 1000.000 )
Inspection:  9. Cracks Detected:  18.96%
  Repair ID:  1. Cracks Detected:   0.00% - ( 0.000 < a <    0.003 )
  Repair ID:  2. Cracks Detected:  12.29% - ( 0.003 < a <    0.020 )
  Repair ID:  3. Cracks Detected:   6.67% - ( 0.020 < a < 1000.000 )
```

Thank you!!

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