PROBABILISTIC DAMAGE TOLERANCE ANALYSIS FOR SMALL AIRPLANES

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ABSTRACT

Most general aviation (GA) aircraft are designed for safe-life based upon a crack initiation type failure mechanism, e.g., Miner's rule. However, newer GA aircraft have fatigue crack growth as a design option. In addition, it may be necessary to evaluate a field event such as a cracked structure to ascertain the remaining life. Therefore, a probabilistic damage tolerance analysis (PDTA) program has been developed. A PDTA approach also provides a mechanism whereby inspection and maintenance operations can be included into the simulation, thus providing engineers the opportunity to assess the benefits of maintenance actions.

A comprehensive probabilistic damage tolerance method requires a combination of deterministic crack growth, probabilistic methods, random variable modeling, and inspection and repair methods to provide a cumulative probability-of-failure and single flight probability-of-failure with and without inspection.

This paper describes the probabilistic methodology to be utilized in a computer software program that performs risk assessment of small airplanes employing NASGRO® or a user-selected code as the crack growth engine. The methodology can assess a range of random variables and calculate the extreme value distribution (EVD) of maximum load per flight from a general aviation (GA) spectrum. The main objective is to develop a comprehensive probabilistic methodology such that Federal Aviation Administration (FAA) engineers can conduct a risk assessment of GA structural issues in support of policy decisions.

This work presents a case study of a high performance single-engine airplane with 4,000 pounds of maximum take off weight to demonstrate the current methodology. The example uses NASGRO® as the crack growth engine and calculates cumulative probability-of-failure and single flight probability-of-failure without inspection.

METHODOLOGY OVERVIEW

The methodology in this work has four main ingredients: aircraft load generation, extreme value maximum load per flight distribution (EVD) generation, fracture mechanics crack growth, and the probabilistic methods (Monte Carlo sampling and numerical integration) to compute the probability of failure at any time in the aircraft life. Figure 1 shows schematically the PDTA process and it is explained briefly as follows.

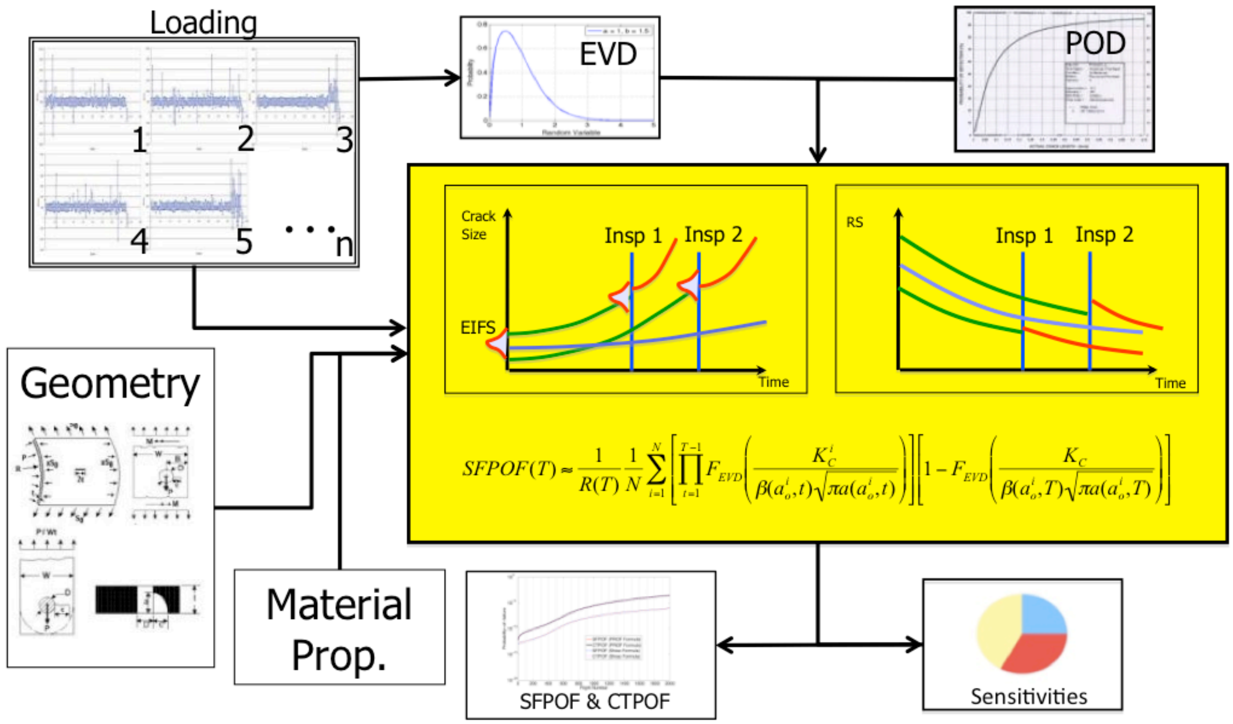


Figure 1. Schematic for Probabilistic Damage Tolerance Analysis

LOAD GENERATION

The PDTA code generates a realistic load spectrum accounting for five different flight regimes: Maneuver, Gust, Taxi, Landing and Rebound, and Ground-Air-Ground (GAG)). Incomplete cycles at any current flight are saved and added in future flights. The stresses per flight and the flights are randomized.

The input parameters used to generate a load spectrum are given in Table 1 and the steps to generate the spectrum are as follows:

1. Provide input parameters (a summary of the input parameters is presented in Table 1).
2. Generate random realizations of the parameters: maneuver and gust exceedance curves, flight-length and aircraft velocity as per flight length-velocity and maximum aircraft velocity, and one-g-stress as per flight length-weight and maximum one-g-stress.

3. Calculate the number of occurrences for each of the flight stages using the methodology in references [1], [2], and [3].
4. After each of the stresses and occurrences are calculated for the current flight, incomplete cycles from previous flights are added to the current flight stresses. Then, the complete stresses are extracted and the incomplete stresses from the current flight are saved for the next flight.
5. Randomize the load pairs within a flight generated in the previous steps.
6. Save the maximum load per flight to later estimate the extreme value distribution.
7. Repeat steps 1 through 6 for the given number of flights.
8. After all the flights have been generated, randomize the flights such that there is an equal probability of the high severity loads appearing at any flight during the crack growth analysis.

Table 1. Spectrum Variable Classification

Variable	**Description**
Number of Flights	Number of flight to be generated in the flight spectrums
Taxi Stage	Include or exclude taxi stage in the flight spectrum
Exceedance Curves	Usage exceedance curves
Maneuver Load Limit Factor	Maximum load limit factors for maneuver load
Gust Load Limit Factor	Maximum load limit factors for gust load
Maximum Ground Stress	Airplane ground stress in psi
Maximum One g Stress	One g stress of an airplane in psi.
Maximum A/C Velocity	Average Speed During Flight, **V**NO (Maximum aircraft safe cruise speed) or **V**MO (Maximum operating limit speed). In nautical miles.
Flight Length-Velocity Matrix	Probabilistic flight length and airspeed data
Flight Length-Weight Matrix	Probabilistic flight length and weight data

Figure 2 shows schematically the process to generate a flight spectrum. Figure 3 shows a spectrum example including all the flight stages for a single flight.

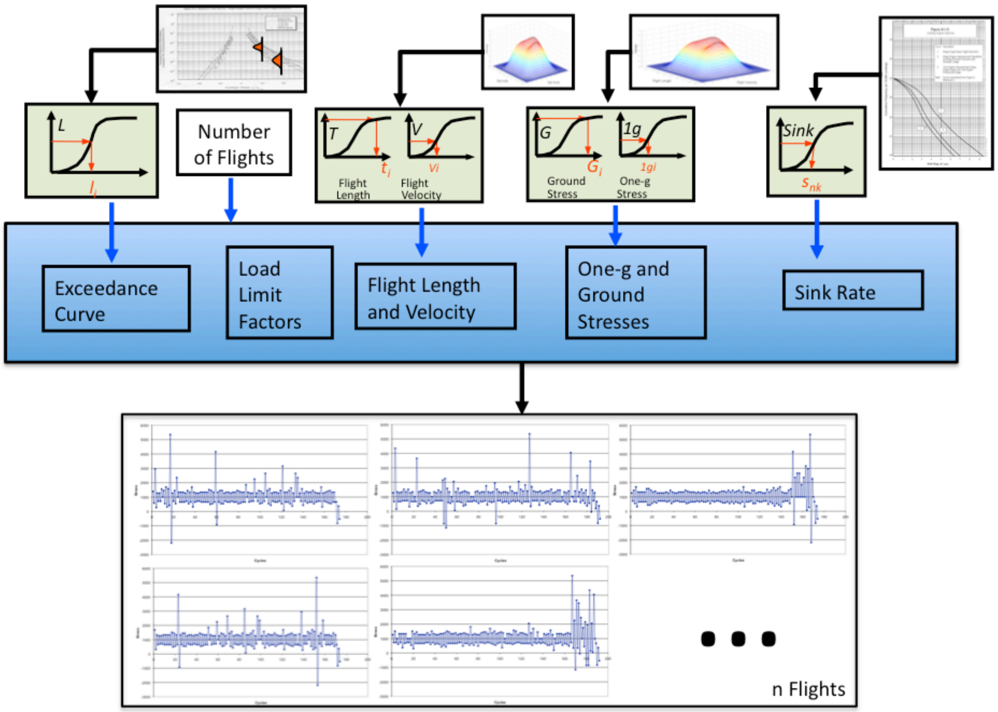


Figure 2. Schematic for the Spectrum Generation

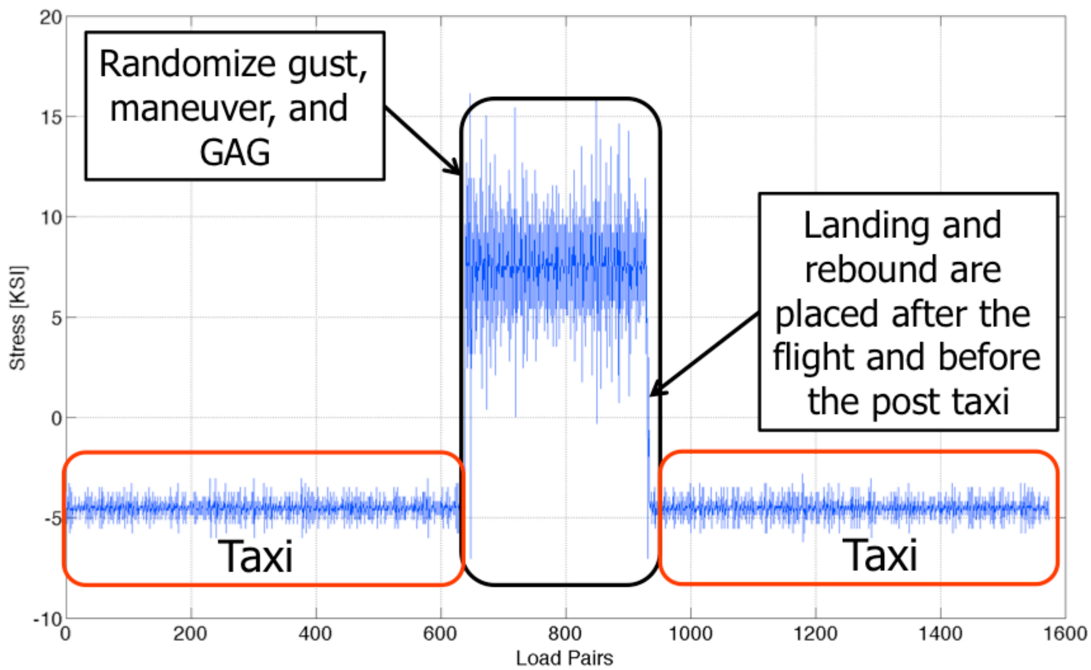


Figure 3. Spectrum Example

EXTREME VALUE DISTRIBUTION (EVD) GENERATION

An extreme value distribution (EVD) of the maximum load per flight of a load spectrum is critical for a probabilistic damage tolerance analysis of a general aviation aircraft. The EVD parameters are important because the structural integrity of the aircraft depends upon the maximum load seen by the structure during a specified number of flights.

In probabilistic damage tolerance analysis, the EVD must be generated from the same loading used for crack growth analysis. In this program, the maximum load per flight is extracted in sets of a fixed number of flights, and the software continues generating sets of flights until the parameters converge as shown schematically in Figure 4.

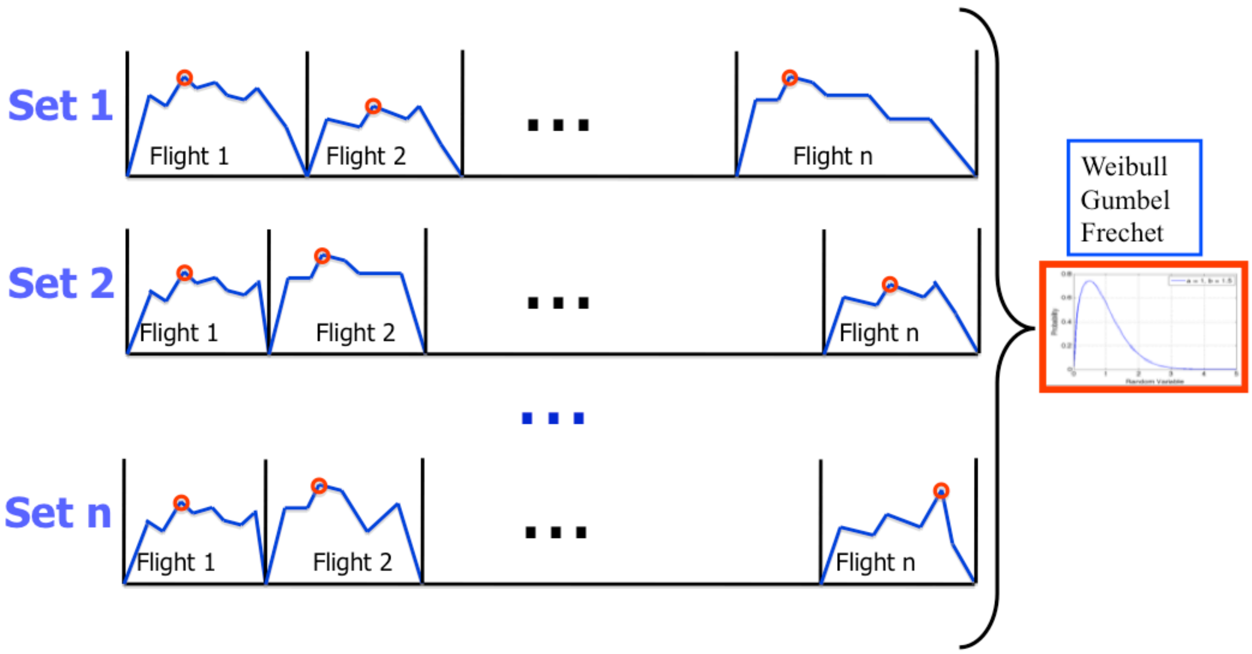


Figure 4. Extreme Value Distribution Generation Schematic

Using these sets of maximum load per flight the EVD can be calculated using the generalized extreme value theory. The generalized extreme value theory can be explained as follows: Suppose $X_1, X_2, \ldots, X_p$ is a sequence of independent random variables having a common distribution function $F(x)$. If M_p represents the maximum of the process over n observations, then as per extreme value theory, the distribution of M_p can be derived exactly for all the values of p [4]:

$$
\begin{aligned}
P\{M_n \le z\} &= P\{X_1 \le z, X_2 \le z, \ldots, X_n \le z\} \\
&= P\{X_1 \le z\} \times P\{X_2 \le z\} \times \ldots \times P\{X_n \le z\} \\
&= F(z)^n
\end{aligned} \tag{1}
$$

Therefore, if the probability density function (PDF) or the distribution function of a random variable is given, then an EVD of the variable over p samples can be estimated using Eq. 1. This may not be immediately helpful in practice because the PDF of aircraft loading is not available in a closed-form equation. However this principle provides the exact solution for a standard distribution such as uniform, normal, or Weibull distribution. When the PDF of the parent distribution is not available and the above approach cannot be used, the following approach can be employed.

From the extreme value theory, it is known that the asymptotic form of extreme value of maximum data as $p \rightarrow \infty$ can take one of three forms: Gumbel, Frechet, Weibull (Types I, II, and III). The three possible models for the maximum can be encapsulated in the generalized extreme value model as [5]:

$$
F(x;\mu,\sigma,\xi) = P = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\} \tag{2}
$$

The distribution in Eq. 2 $F(Q_x;\mu,\sigma,\xi)$ is known as the generalized extreme value distribution. Here μ, σ, and ξ indicate the location, scale, and shape parameters of the generalized extreme value distribution, respectively. The value of the shape parameter determines the type of the distribution. The extreme value distribution converges to Weibull, Gumbel, or Frechet if the shape parameter (ξ) value is less than zero, equal to zero, or greater than zero, respectively.

The inverse of the generalized extreme value distribution, also known as the quantile function, for $P \in (0,1)$ is:

$$
F^{-1}(P;\mu,\sigma,\xi) = x = \mu - \left(\frac{\sigma}{\xi}\right) + \left(\frac{\sigma}{\xi}\right)\cdot\left[\ln(P)\right]^{-\xi} \tag{3}
$$

For a given value of x and its probability, the inverse function is an equation with three unknowns: location, scale, and shape. For three equations, it is possible to solve for the three unknowns. Sorting all of the maximum-load-per-flight elements will produce an empirical CDF.

The position of a given x value within the sort is its probability. For example, the median value in the sorted array has a probability of 0.50. Thus it is possible to choose three such values, and solve for the parameters of the EVD. For example, the code can choose the values associated with p={0.50, 0.25, 0.125). The PDTA code chooses at least seven distinct sets of three values, solves the three equations for μ,σ,ξ, and averages the results to obtain good estimates of μ,σ,ξ.

This is called the Method of Quantiles. The average of the seven distinct points is the starting point for a minimization method, the Nelder-Mead algorithm, that finds a set of parameters that minimizes the total absolute error while leaning toward a conservative probability-of-survival. The EVD fits using this method tend to have correlations above 99.5%.

After this process the three EVD parameters are stored for use in the probability of failure calculations.

Figure 5 and Figure 6 show probability density functions and cumulative density function for differenten general aviation aircraf usages.

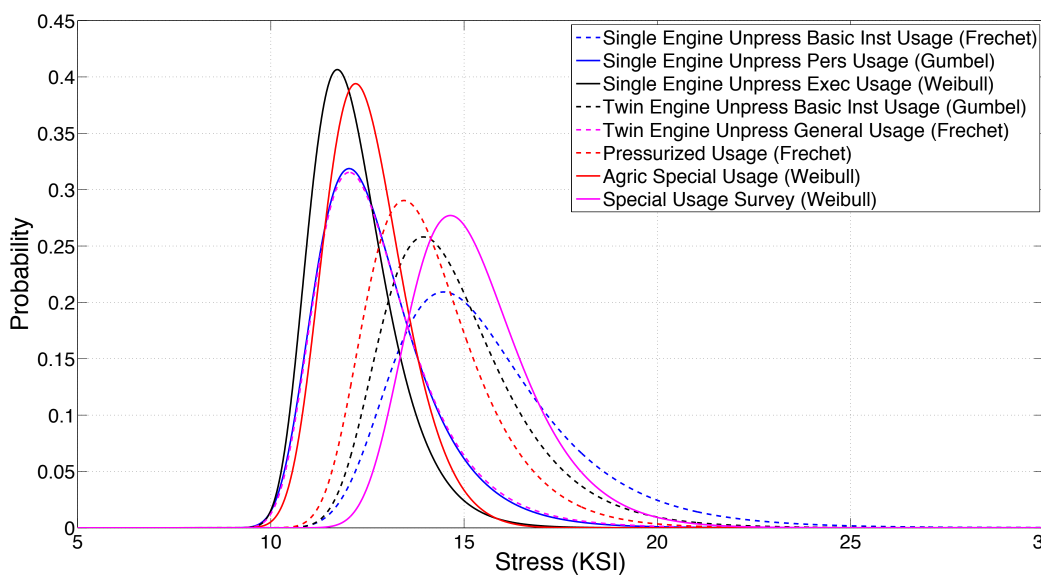


Figure 5. EVD PDF Distributions

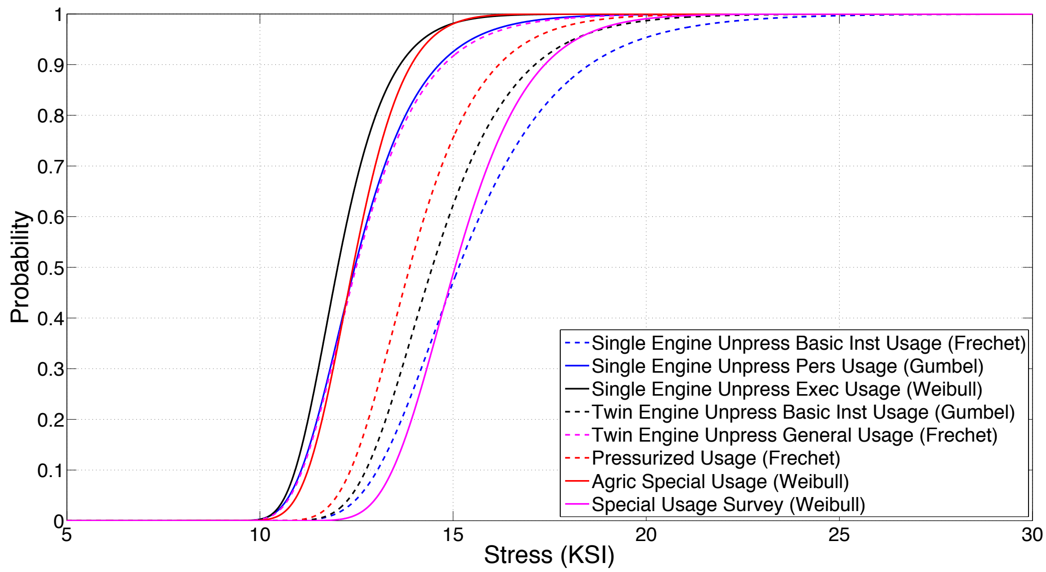


Figure 6. EVD CDF Distributions

FRACTURE MECHANICS

Crack growth analysis of structures requires accurate evaluation of stress intensity factors within realistic stress fields. Significant effort has been spent on characterizing stress intensity factors for various geometries and developing software that can integrate the differential equations accurately. The NASGRO software [4], developed by NASA and Southwest Research Institute (SwRI), is a world-class deterministic crack growth software program. NASGRO will be used in this program as the crack growth engine. SwRI has made some efficiency modifications to NASGRO in order for it to be used effectively on this research.

PROBABILISTIC METHODS

The probability-of-failure during a single flight assuming no failures before that flight can be determined as the probability that the maximum load experienced during the flight will exceed the residual strength of the structure. Considering only three random variables for simplicity, this is written mathematically as

$$SFPOF(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(a(t))g(K_C)h(\sigma)dadK_Cd\sigma \tag{4}$$

where a denotes crack size, t – flight hours, K_C – fracture toughness, σ – maximum load experienced per flight, and *f, g, h* are the corresponding probability density functions. Additional random variables can be added in a similar fashion.

Eq. 4 can be further simplified by analytically integrating $h(\sigma)$ in terms of the other random variables, that is, determine the cumulative distribution function $H(\sigma)$ that defines the probability of the maximum load being less than the residual strength, $\sigma \geq K_C / \beta(a)\sqrt{\pi a}$. The probability of the maximum load exceeding the residual strength is equal to 1 minus $H(\sigma)$ at $K_C / \beta(a)\sqrt{\pi a}$. Eq. 4 becomes

$$SFPOF(t) = \int_{-\infty}^{\infty}\int_{0}^{\infty} f(a,t)g(K_C)(1 - H(K_C / \beta(a)\sqrt{\pi a}))dadK_C \tag{5}$$

where H is the CDF of the maximum load (F_{EVD}). Eq. 5 is now a two dimensional integral. Rewriting in terms of the initial crack size yields

$$SFPOF(t) = \int_{0}^{\infty}\int_{-\infty}^{\infty} \left[1 - F_{EVD}\left(\frac{K_C}{\beta(a(a_o,t))\sqrt{\pi a(a_o,t)}}\right)\right] f_{a_0}(a_0) f_{K_c}(K_c) da_0 dK_c \tag{6}$$

Eq. 6 is an example of conditional expectation. In this equation, the function $F_{EVD}(K_C / \beta(a(a_o,t))\sqrt{\pi a(a_0,t)})$ is the probability of the maximum load exceeding the residual strength. Thus, $SFPOF(t)$ is the expected value of 1-F_{EVD}. Significant variables not considered in this approach are crack growth variability and geometric variations. Additional random variables can be added to Eq. 6.

The $SFPOF(t)$ using sampling and accounting for any number of random variables is presented in Eq. 7.

$$SFPOF(t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a_o^i,T)\sqrt{\pi a(a_o^i,T)}}\right)\right] \tag{7}$$

Finally, the hazard function can be expressed as:

$$SFPOF(T) \approx \frac{1}{R(T)}\frac{1}{N}\sum_{i=1}^{N}\left[\prod_{t=1}^{T-1} F_{EVD}\left(\frac{K_C^i}{\beta(a_o^i,t)\sqrt{\pi a(a_o^i,t)}}\right)\right]\left[1 - F_{EVD}\left(\frac{K_C}{\beta(a_o^i,T)\sqrt{\pi a(a_o^i,T)}}\right)\right] \tag{8}$$

where $\left[1 - F_{EVD}\left(K_C^i / \beta(a_o, T)\sqrt{\pi a(a_o, T)}\right)\right]$ represents the failure during the flight (T), $\left[\prod_{t=1}^{T-1} F_{EVD}\left(K_C^i / \beta(a_o^i, t)\sqrt{\pi a(a_o^i, t)}\right)\right]$ represents the probability of survival during the previous flights (T-1), and $R(T)$ is the reliability considering all random variables.

SFPOF has been also formulated in references [6] through [9].

<u>Cumulative probability of failure</u>

The cumulative probability-of-failure calculated by sampling can be expressed as:

$$CTPOF(T) \approx \frac{1}{N}\sum_{i=1}^{N}\left[1 - \prod_{t=1}^{T}(1 - SFPOF(t))\right] \tag{9}$$

And the reliability term from Eq. 8 can be computed from Eq. 9.

$$R(T) = 1 - CTPOF(T) \tag{10}$$

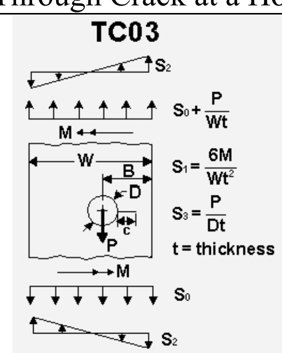
<u>CASE STUDY</u>

The case study presented in this paper considers a high performance single-engine airplane with 4,000 pounds of maximum take off weight analyzed using a single usage (single engine unpressurized executive usage). The airplane and flight characteristics used to generate the spectrum are presented in Table 2. The geometry considered for crack growth analysis is a through crack emanating from a hole; the geometric and material values for this case study are presented in Table 3. The random variables in this study are loading, initial crack size, fracture toughness, hole diameter, and Paris constants *C* and *m* and are presented in Table 3.

Table 2. Case Study Loading Variables

Variable	Value
Usage	Single Engine Unpressurized Executive Usage
Design LLF Maneuver	3.8, -1.6
Design LLF Gust	3.4, -1.4
Ground Stress (psi)	-1,950
One-g stress (psi)	6.200
Flight Length and Velocity Matrix	Dur/Vel 0.80 0.85 0.90 0.95 1.00 0.50: 0.05 0.05 0.10 0.10 0.10 0.65 0.60: 0.05 0.05 0.05 0.05 0.15 0.70 0.70: 0.10 0.00 0.05 0.05 0.15 0.75 0.80: 0.15 0.00 0.05 0.05 0.10 0.80 0.90: 0.20 0.00 0.00 0.00 0.10 0.90 1.00: 0.25 0.00 0.00 0.05 0.05 0.90 1.10: 0.15 0.00 0.00 0.00 0.05 0.95 1.20: 0.05 0.00 0.00 0.00 0.05 0.95
Flight Length and Weight Matrix	Dur/Wei 0.80 0.85 0.90 0.95 1.00 0.50: 0.05 0.00 0.00 0.00 0.20 0.80 0.60: 0.05 0.00 0.00 0.00 0.20 0.80 0.70: 0.10 0.00 0.00 0.00 0.15 0.85 0.80: 0.15 0.00 0.00 0.00 0.15 0.85 0.90: 0.20 0.00 0.00 0.00 0.10 0.90 1.00: 0.25 0.00 0.00 0.00 0.10 0.90 1.10: 0.15 0.00 0.00 0.00 0.05 0.95 1.20: 0.05 0.00 0.00 0.00 0.05 0.95
Average Velocity (Vno/Vmo (Knots))	153

Table 3. Case Study Geometric and Material Variables

Variable	Value
Geometry	Through Crack at a Hole
NASGRO Model (TC03)	
Width (W) [Inches]	1.5
Thickness (t) [Inches]	0.09
Hole Diameter (D) [Inches]	Mean = 0.156 Standard Deviation = 0.0156
Initial Crack Size (c) [Inches]	Mean = 0.05 Standard Deviation = 0.005
Material	Aluminum 2024
Fracture Toughness (K_C)	Mean = 34.4 Standard Deviation = 3.4
Paris Constants (C and m)	Mean log (C) =-8.09 Standard Deviation C= 0.142 Mean m =2.87 Standard Deviation C= 0.166 Correlation -0.99795

The EVD curve is a Frechet distribution according to the results in the optimization algorithm developed in this research and it is shown in Figure 7.

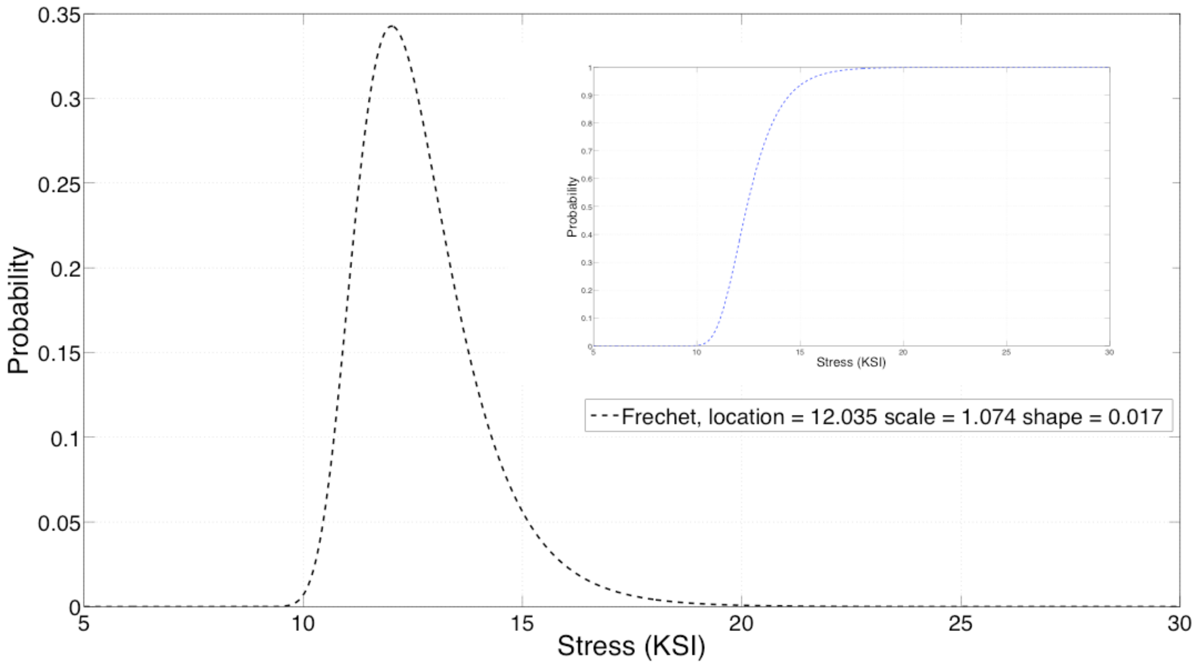


Figure 7. EVD Distribution

A total of 20,000 samples were run, . The crack growth curve for six samples chosen randomly from the total number of Monte Carlos Samples are shown in Figure 8.

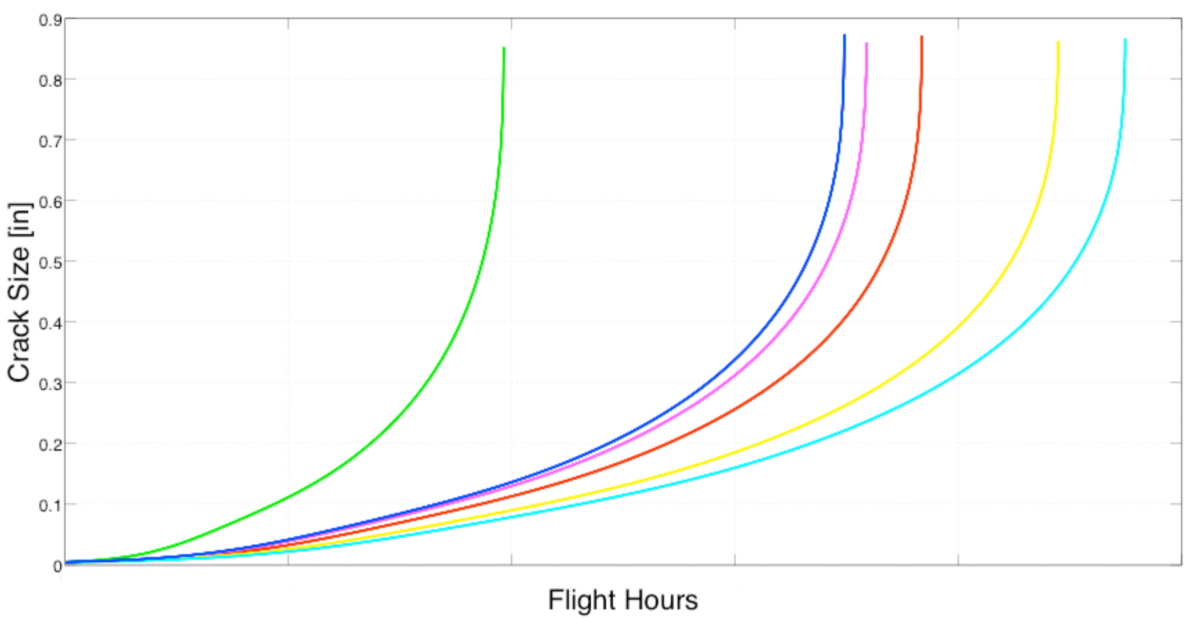


Figure 8. Crack Growth Curves

Finally, the single flight probability of failure, hazard rate, and cumulative probability of failure of the airplane were calculated are shown in Figure 9.

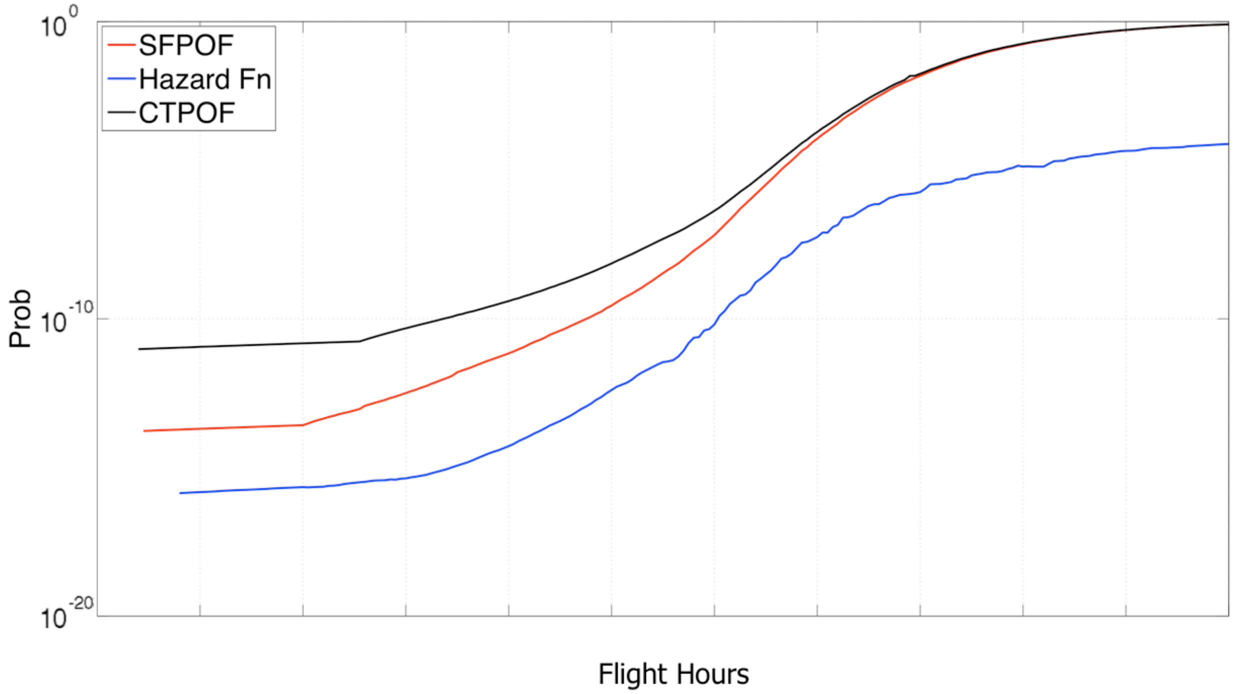


Figure 9. SFPOF and CTPOF

CONCLUSIONS:

Probabilistic damage tolerance evaluation of General Aviation aircraft is vital in order to provide insight into the severity or criticality of a potential structural issue. For this reason, probabilistic risk assessment methodology and computer software are being developed so that FAA engineers can perform a risk assessment of a structural issue.

A case study was run to demonstrate the methodology and its applicability to different scenarios represented by random variables. These were then used to calculate the single flight probability of failure, hazard rate, and cumulative probability of failure to assess the risk of an airplane based on the current condition and its predicted usage.

Future work in this software will be the addition of inspection capabilities and the inclusion of the numerical integration subroutines.

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